xthst: Testing slope homogeneity in Stata

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Motivation

- Different econometric methods are available if the parameter of interest (slope) is homogeneous or heterogeneous.
- Huge literature on homogeneous slopes. Examples: fixed effects, random effects, GMM, ...
- Methods for models with heterogeneous effects are available as well. Examples: SURE, mean group estimator, ...
- Incorrectly ignoring slope heterogeneity leads to biased results (Pesaran and Smith, 1995).
- Establishing slope homogeneity/heterogeneity key for model selection.
- This presentation: introducing the Delta test (Pesaran and Yamagata, 2008; Blomquist and Westerlund, 2013) for testing slope homogeneity in large panels using `xthst` (Bersvendsen and Ditzen (2020) and *forthcoming* in *The Stata Journal*).

Econometric Model

- Large panel data model with $N_g \to \infty$ cross-sectional units and $T \to \infty$ time periods.
- Slope coefficients can be heterogeneous:

$$y_{i,t} = \mu_i + \boldsymbol{\beta}'_{1i}\mathbf{x}_{1i,t} + \boldsymbol{\beta}'_{2i}\mathbf{x}_{2i,t} + \varepsilon_{i,t}, \tag{1}$$

- Effect of $\mathbf{x}_{1i,t}$ and $\mathbf{x}_{2i,t}$ on $y_{i,t}$ of main interest.
- We want to test if the effect of $\mathbf{x}_{2i,t}$ is the same across all cross-sectional units, namely if $\boldsymbol{\beta}'_{2i} = \boldsymbol{\beta}'_2 \; \forall i$.
- Assumption $\boldsymbol{\beta}_1$ heterogeneous and $\epsilon_{i,t}$ has heteroskedastic errors.

Testing slope homogeneity

Overview

- Hypothesis:

$$H_0 : \boldsymbol{\beta}_{2i} = \boldsymbol{\beta}_2 \text{ for all } i,$$

  against the alternative:

$$H_A : \boldsymbol{\beta}_{2i} \neq \boldsymbol{\beta}_2 \text{ for some } i.$$

- Tests available
  - F-Test requires homoskedasticity assumption, fixed N and requires $T > N$.
  - Hausman style tests valid only if $N > T$ and require strongly exogenous regressors (Pesaran et al., 1996; Pesaran and Yamagata, 2008).
  - Bootstrap approaches (Blomquist and Westerlund, 2016)
  - Delta Test (Pesaran and Yamagata, 2008) and HAC robust version (Blomquist and Westerlund, 2013).

Testing for slope homogeneity

Delta Test (Pesaran and Yamagata, 2008)

- Based on a standardised version of Swamy's test (Swamy, 1970).
- Compares the weighted difference between the cross-sectional unit specific estimate ($\beta_{2,i}$) and a weighted pooled estimate (β_{2WFE}):

$$\tilde{\Delta} = \frac{1}{\sqrt{N}} \left(\frac{\sum_{i=1}^{N} \tilde{d}_i - k_2}{\sqrt{2k_2}} \right) \tag{2}$$

with

$$\tilde{d}_i = (\hat{\boldsymbol{\beta}}_{2i} - \tilde{\boldsymbol{\beta}}_{2WFE})' \frac{\mathbf{X}_{2i}' \mathbf{M}_{1i} \mathbf{X}_{2i}}{\tilde{\sigma}_i^2} (\hat{\boldsymbol{\beta}}_{2i} - \tilde{\boldsymbol{\beta}}_{2WFE})$$

$$\mathbf{M}_{1i} = \mathbf{I}_{T_i} - \mathbf{Z}_{1i}(\mathbf{Z}_{1i}'\mathbf{Z}_{1i})^{-1}\mathbf{Z}_{1i}', \mathbf{Z}_{1i} = (\boldsymbol{\tau}_{T_i}, \mathbf{X}_{1i})$$

- β_{2WFE} is weighted by the cross-section unit specific variances.
- Under H_0, $\tilde{\Delta} \sim \mathcal{N}(0,1)$.

Testing for slope homogeneity

HAC Robust Delta Test (Blomquist and Westerlund, 2013)

- Standard delta test requires error not to be autocorrelated.
- Blomquist and Westerlund (2013) derive a HAC robust version.

$$\tilde{\Delta}_{HAC} = \sqrt{N}\left(\frac{N^{-1}S_{HAC} - k_2}{\sqrt{2k_2}}\right) \quad (3)$$

$$S_{HAC} = \sum_{i=1}^{N} T_i(\hat{\boldsymbol{\beta}}_{2i} - \hat{\boldsymbol{\beta}}_{2HAC})'(\hat{\mathbf{Q}}_{i,T_i}\hat{\mathbf{V}}_{i,T_i}^{-1}\hat{\mathbf{Q}}_{i,T_i})(\hat{\boldsymbol{\beta}}_{2i} - \hat{\boldsymbol{\beta}}_{2HAC})$$

- where
  - $\boldsymbol{\beta}_{2HAC}$ is a HAC robust estimator of the pooled coefficients $\boldsymbol{\beta}_2$
  - $\hat{\mathbf{Q}}_{i,T_i}$ is a projection matrix to partial the heteogenous variables out,
  - and $\hat{V}_{i,T}$ a robust variance estimator with kernel $\kappa()$ and bandwidth $B_{i,T}$.
- Under H_0, $\tilde{\Delta}_{HAC} \sim \mathcal{N}(0,1)$.

Testing for slope homogeneity

Cross-Sectional Dependence Robust version

- In large panels cross-sectional units likely to be correlated with each other, often modelled by common factor structure:

$$y_{i,t} = \mu_i + \beta'_{1i}\mathbf{x}_{1i,t} + \beta'_{2i}\mathbf{x}_{2i,t} + u_{i,t},$$
$$u_{i,t} = \gamma'_i \mathbf{f}_t + \varepsilon_{i,t},$$

- Following Pesaran (2006); Chudik and Pesaran (2015) the common factors $\mathbf{f}_t$ can be approximated by cross-sectional averages.
- We propose to defactor $\mathbf{y}_i$, $\mathbf{X}_{1i}$ and $\mathbf{X}_{2i}$ by using cross-sectional averages to remove strong cross-sectional dependence.
- Then use the defactored variables and construct the test statistic following (2) and (3).
- No formal derivation available so far, Monte Carlo results are encouraging.

xthst
Syntax

xthst *depvar indepvars* [*if*] [partial(*varlist_p*) noconstant crosssectional(*varlist_cr* [,cr_lags(*numlist*)]) ar hac bw(*integer*) whitening kernel(*kernel_options*) nooutput comparehac]

- *depvar* is the dependent variable of the model to be tested, *indepvars* the independent variables
- *varlist_p* are the variables to be partialled out ($\mathbf{X}_1$)
- *varlist_cr* are variables added as cross-sectional averages
- *hac* uses the HAC robust Delta test and *bw()* sets the bandwidth.
- *kernel_options* can be *qs*, *bartlett* or *truncated*.
- *ar* for pure autoregressive model.

Options Stored Values

xthst - HAC and kernel options

- xthst supports several kernel estimators for the variance/covariance estimator when using the HAC robust Delta test.

$$\hat{\mathbf{V}}_{i,T_i} = \hat{\mathbf{\Omega}}_i(0) + \sum_{j=1}^{T_i-1} \kappa(j/B_{i,T_i})[\hat{\mathbf{\Omega}}_i(j) + \hat{\mathbf{\Omega}}_i(j)'], \qquad (4)$$

- Possible kernel estimator for $\kappa()$ are: *Bartlett* (default), *Quadratic spectral* (QS) and the *Truncated*.
- If bandwidth is not manually chosen, xthst opts for a data-dependent selection based on the chosen kernel:

$$B_{i,T_i} = [c(\alpha_i(q)^2 T_i)^{1/(2q+1)}], \qquad (5)$$

where scalars c and q depend on the type of kernel (Andrews and Monahan, 1992; Newey and West, 1994; Bersvendsen and Ditzen, 2020).
- To reduce small sample bias, residuals for the variance estimator can be pre-whitened (Blomquist and Westerlund, 2013).

Monte Carlo Results

Overview

- Following Pesaran and Yamagata (2008) and Blomquist and Westerlund (2013):

$$y_{i,t} = \mu_i + \sum_{l=1}^{k} \beta_{l,i} x_{i,l,t} + u_{i,t}$$

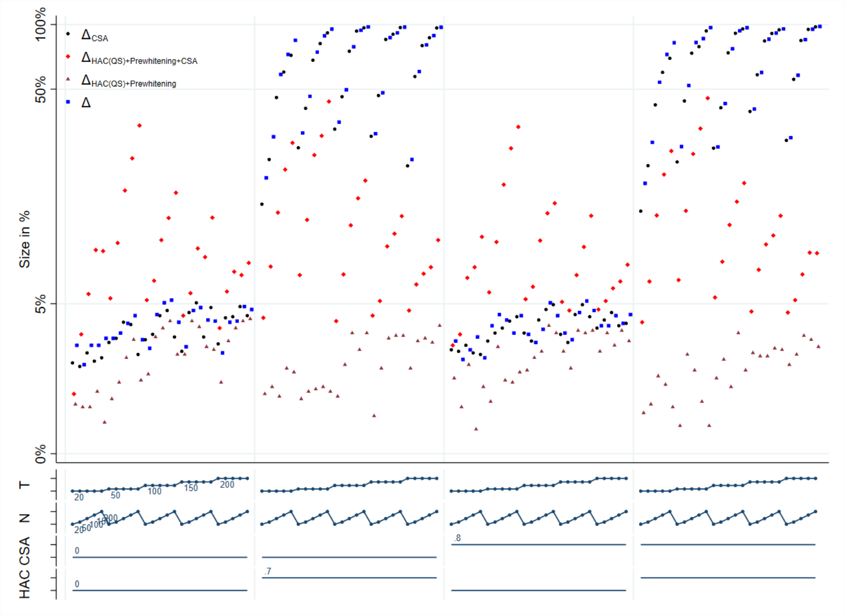
$$x_{i,l,t} = \mu_i \left(1 - \rho_{x,i,l}\right) + \rho_{x,i,l} x_{i,l,t-1} + \left(1 - \rho_{x,i,l}\right)^{\frac{1}{2}} v_{i,l,t}$$

$$u_{i,t} = \rho_{u,i} u_{i,t-1} + \sqrt{1 - \rho_{u,i}^2} \left(\gamma_{u,i} f_t + e_{i,t}\right)$$

- x and u are allowed to independent or autocorrelated and have no cross-sectional dependence and strong cross-sectional dependence.
- Power and Size are compared for standard Delta test, HAC with QS kernel and prewhitening, CSD robust Delta test and a mix of all.
- Graphs generated by `resultplot` (coming soon on SSC by Wursten and Ditzen).

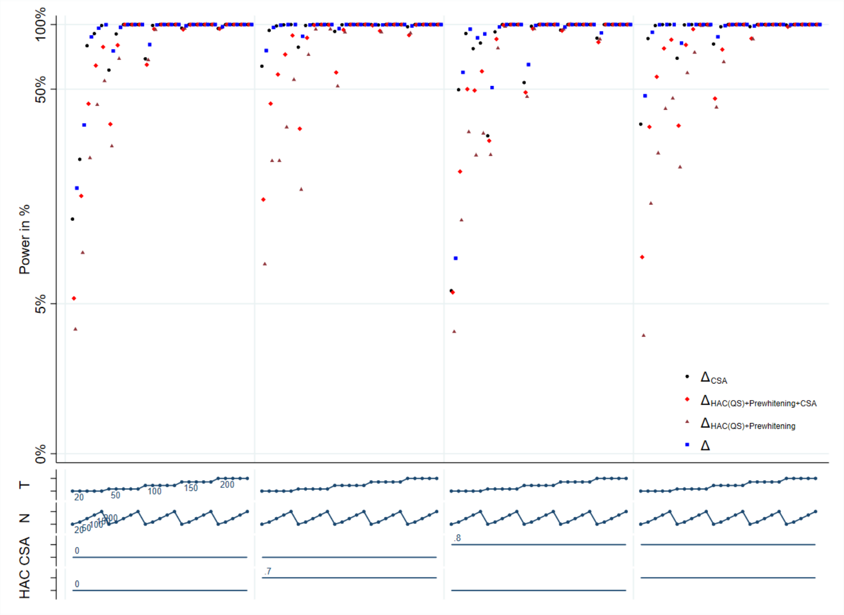
Monte Carlo Results

Size



Monte Carlo Results

Power



Empirical Examples

- Growth model with GDP per capita growth in logarithms, `log_rgdpo` and explanatory variables are human capital, `log_hc`, physical capital, `log_ck`, and population growth added with break even investments of 5%, `log_ngd`.
- Data from Penn World Tables 8.0 (Feenstra et al., 2015).
- 93 countries (N_g) and $T = 48$ years between 1960 and 2007.

Empirical Examples

Delta Test

- Dynamic model and test if any of the slope coefficients are homo- or heterogeneous

```
. xthst d.log_rgdp L.d.log_rgdp log_hc log_ck log_ngd
Testing for slope heterogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
-------------------------------------
         Delta          p-value
          2.957           0.003
 adj.     3.171           0.002
-------------------------------------
Variables partialled out: constant
```

- `xthst` assumes a heterogeneous constant and partials it out.
- The null of slope homogeneity and an estimator allowing for heterogeneous slopes, such as the mean group estimator should be used.

Empirical Examples

Testing a subset

- Assume we want to test if only the lag of the dependent variable is heterogeneous.
- partial() is used to remove all other variables:

```
. xthst d.log_rgdp L.d.log_rgdp log_hc log_ck log_ngd, ///
> partial(log_hc log_ck log_ngd)
Testing for slope heterogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
-------------------------------------
         Delta          p-value
          2.324           0.020
 adj.     2.409           0.016
-------------------------------------
Variables partialled out: log_hc log_ck log_ngd constant
```

Empirical Examples

HAC robust Test

- Option hac can be employed to use the HAC robust standard errors.
- Default is to use *bartlett* kernel with data driven bandwidth.

```
. xthst d.log_rgdp L.d.log_rgdp log_hc log_ck log_ngd, hac
Testing for slope heterogeneity
(Blomquist, Westerlund. 2013. Economic Letters)
H0: slope coefficients are homogenous
-------------------------------------
         Delta           p-value
         12.203            0.000
 adj.    13.086            0.000
-------------------------------------
HAC Kernel: bartlett
with average bandwith 3
Variables partialled out: constant
```

Empirical Examples

Option comparehac

- xthst should be used for model selection, comparison of results next to each other useful.
- Option comparehac compares the standard and HAC robust delta test.
- It also tests for cross-sectional dependence using xtcd2 (Ditzen, 2018).

```
. xthst d.log_rgdp L.d.log_rgdp ///
log_hc log_ck log_ngd , comparehac
Testing for slope heterogeneity
H0: slope coefficients are homogenous
-------------------------------------------------
           Delta           p-value
            2.957            0.003
  adj.      3.171            0.002
-------------------------------------------------
           Delta (HAC)     p-value
           -0.534            0.593
  adj.     -0.573            0.567
-------------------------------------------------
Tests disagree. Autocorrelation might occur.
See helpfile for further info.
HAC Settings:
       Kernel: quadratic spectral (QS)
       with average bandwith 45
Variables partialled out: constant
Cross Sectional dependence in base variables detected:
    D.log_rgdpo LD.log_rgdpo log_hc log_ck log_ngd
See helpfile for xthst and xtcd2 for further info.
```

Conclusion

- Testing for slope homogeneity important for selection of appropriate econometric method.
- `xthst` introduces two such tests in panels with large number of observations over time and cross-sectional units.
- Options involve:
  - HAC robust tests with different bandwidth and kernels
  - Cross-sectional dependence robust
  - Pure autoregressive model
- Empirical examples and results of Monte Carlo given.
- Left for further research:
  - Error correction models.
  - Improve cross-sectional dependence robust test.

References I

Andrews, D. W. K., and J. C. Monahan. 1992. An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator. Econometrica 60(4): 953–966.

Bersvendsen, T., and J. Ditzen. 2020. xthst : Testing for slope homogeneity in Stata. CEERP Working Paper Series (11).

Blomquist, J., and J. Westerlund. 2013. Testing slope homogeneity in large panels with serial correlation. Economics Letters 121(3): 374–378.

———. 2016. Panel bootstrap tests of slope homogeneity. Empirical Economics 50(4): 1359–1381.

Chudik, A., and M. H. Pesaran. 2015. Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors. Journal of Econometrics 188(2): 393–420.

Ditzen, J. 2018. Estimating dynamic common-correlated effects in Stata. The Stata Journal 18(3): 585 – 617.

References II

Feenstra, R. C., R. Inklaar, and M. Timmer. 2015. The Next Generation of the Penn World Table. The American Economic Review 105(10): 3150–82. URL www.ggdc.net/pwt.

Newey, W. K., and K. D. West. 1994. Automatic Lag Selection in Covariance Matrix Estimation. Review of Economic Studies 61(4): 631–653.

Pesaran, H., R. Smith, and K. S. Im. 1996. Dynamic Linear Models for Heterogenous Panels, 145–195. Dordrecht: Springer Netherlands.

Pesaran, M., and R. Smith. 1995. Estimating long-run relationships from dynamic heterogeneous panels. Journal of Econometrics 68(1): 79–113.

Pesaran, M. H. 2006. Estimation and inference in large heterogeneous panels with a multifactor error structure. Econometrica 74(4): 967–1012.

References III

Pesaran, M. H., and T. Yamagata. 2008. Testing slope homogeneity in large panels. Journal of Econometrics 142(1): 50–93.

Swamy, P. A. V. B. 1970. Efficient Inference in a Random Coefficient Regression Model. Econometrica 38(2): 311–323.

Options ▸ back

- `noconstant` suppresses the individual heterogeneous constant, μ_i.
- `partial(`*varlist_p*`)` requests exogenous regressors in *varlist_p* to be partialled out. The constant is automatically partialled out, if included in the model. Regressors in *varlist* will be included in $\mathbf{z}_{it}$ and are assumed to have heterogeneous slopes.
- `ar` allows for an AR(p) model. The degree of freedom of $\tilde{\sigma}^2$ is adjusted. May not be combined with `hac`.
- `hac` implements the HAC consistent test by Blomquist and Westerlund (2013). If `kernel` and `bw` are not specified, kernel is set to *bartlett* the data driven bandwidth selection is used. May not be combined with `ar`.
- `kernel(`*kernel*`)` specifies the kernel function used in calculating the HAC consistent test statistic. Available kernels are *bartlett*, *qs* (quadratic spectral) and *truncated*. Is only required in combination with `hac`.

Options back I

- `bw(#)` sets the bandwith equal to # for the HAC consistent test statistic, where # is an integer greater than zero. Is only required in combination with `hac`. Default is the data driven bandwidth selection.
- `whitening` performs pre-whitening to reduce small-sample bias in HAC estimation. Is only required in combination with `hac`.
- `crosssectional(`*varlist_cr* `[,cr_lags(`*numlist*`)])` defines the variables to be added as cross-sectional averages to approximate strong cross-sectional dependence. Variables in *varlist_cr* are partialled out. `cr_lags(`*numlist*`)` sets the number of lags of the cross-sectional averages. If not defined, but `crosssectional()` contains a *varlist*, then only contemporaneous cross sectional averages are added but no lags. `cr_lags(0)` is the equivalent. The number of lags can be variable specific, where the order is the same as defined in `cr()`. For example if `cr(y x)` and only contemporaneous cross-sectional averages of `y` but 2 lags of `x` are added, then `cr_lags(0 2)`.

Options back II

- `nooutput` omits output.
- `comparehac` compares the standard delta test to the HAC robust version. First the standard delta test is run, then the HAC robust version. Results for both tests are displayed. If the tests disagree a message is posted. In addition the base of all variables are tested for cross-sectional dependence using `xtcd2` (Ditzen, 2018). If cross-sectional dependence is found, a message is posted. The options `crosssectional()`, `partial()` and `noconstant` are hold constant across both tests. All HAC related options only apply to the HAC robust run. This option is only for testing purposes and should not replace further testing.

Stored Values

▸ back

Scalars	
`r(bw)`	bandwith
Macros	
`r(cross-sectional)`	variables of which cross-section averages are added
`r(partial)`	variables partialled out
`r(kernel)`	used kernel
Matrices	
`r(delta)`	delta and adjusted delta
`r(delta_p)`	p-values of the above