

U.S. Treasury Yields and Foreign Holdings of U.S. Securities: An Interim Report

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Abstract

This paper examines empirically whether foreign holdings of U.S. Treasury securities are relevant to explaining U.S. Treasury yields. Interest in this topic is motivated by the failure of the long-term interest rate to rise during 2004-2005 in response to increases in the short-term rate, a phenomenon that took place against a backdrop of rapidly rising foreign holdings of U.S. securities and declining foreign interest rates. There are not many papers examining this question and the little evidence available offers no agreement on whether holdings of U.S. Treasuries affect U.S. long-term interest rates. What is not clear is what is responsible for it. We requested their data, replicated their results, and found them to be sensitive to minor changes. Our strategy is to re-examine the association by extending the framework used in previous work. Specifically, previous work does not recognize that foreigners can redirect their financial holdings anywhere in the world with effects on world interest rates. We address these limitations using a variety of vantage points: single-equation, Vector Error-correction models, yield-curve models, and DSGE models. The evidence suggests that foreign holdings of U.S. securities help explaining movements in U.S. Treasury yields.

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1 Introduction

This paper reports our preliminary results studying the question of whether movements in foreign holdings of U.S. Treasury securities matter for understanding movements in U.S. Treasury yields. Our interest in this topic is motivated by several considerations. First, foreign demand for U.S. Treasuries has increased substantially in the last three decades. As a share of U.S. Treasuries outstanding, foreign holdings increased from 17 percent in 1981 to 50 percent by 2009 (figure 1) with much of this increase stemming from foreign official holdings. Obvious as it is, this demand-side factor is not considered in most existing models of the U.S. yield curve. Second, the (scant) literature that allows for this demand factor offers no reliable guidance as to whether the foreign holdings affect U.S. interest rates. Warnock and Warnock (2009), using a linear static equation and data from the U.S. Treasury International Capital (TIC) system, find that foreign official purchases of U.S. long-term securities help explain the downward trend of the U.S. 10-year yield (figure 2). Rudebusch et al. (2006) combine affine and latent-factor models of the term structure with data on custodial holdings at the Federal Reserve Bank of New York and find that their models' residuals are not contemporaneously correlated to these net purchases, contradicting Warnock and Warnock.

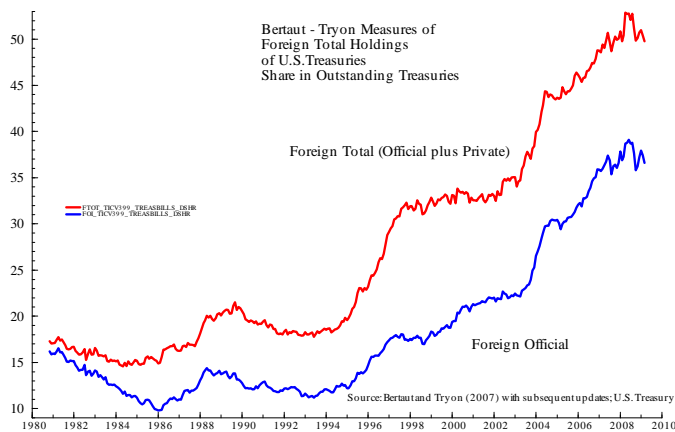


Figure 1

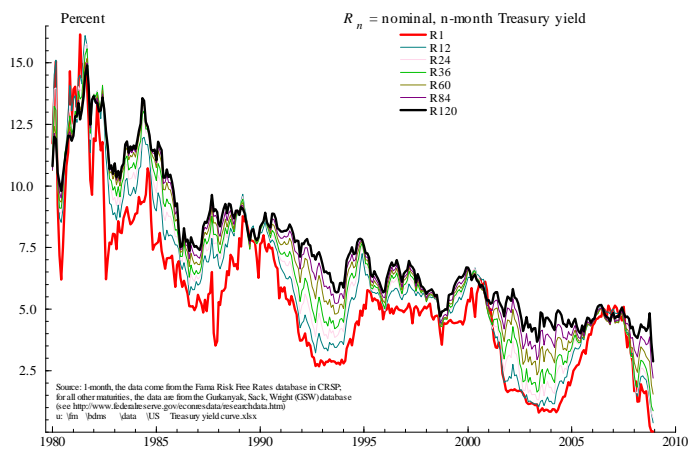


Figure 2: U.S. Treasury Yields, selected maturities

That such a stark difference in predictions matters is clear. What is not clear is what to make of it. Specifically, one cannot tell whether methodological differences among these studies are responsible for the

difference in results because they do not report tests central to judging the reliability of their results. Thus, to understand the source of this disagreement, section 2 replicates their results and we find that they are sensitive to minor changes.

We argue that this lack of robustness stems from neglecting the interdependency between asset prices and asset holdings. First, there is a two-way direction of causation: foreign purchases of Treasuries respond to yields (bond prices), and yields respond to foreign purchases. Second, the effect of foreign purchases of U.S. Treasuries on U.S. interest rates are likely influenced by the close substitutability between U.S. and foreign government bonds. For example, an unexpected surge in demand for 10-year U.S. Treasury bonds would raise their prices, but this in turn would make foreign 10-year bonds cheaper relative to U.S. bonds. As investors take advantage of this bigger gap between U.S. and foreign bond prices by purchasing foreign bonds, foreign bond prices rise too. This imperfect substitutability between U.S. and foreign bonds would explain some of the correlation between U.S. and foreign long term interest rates (figure 3).

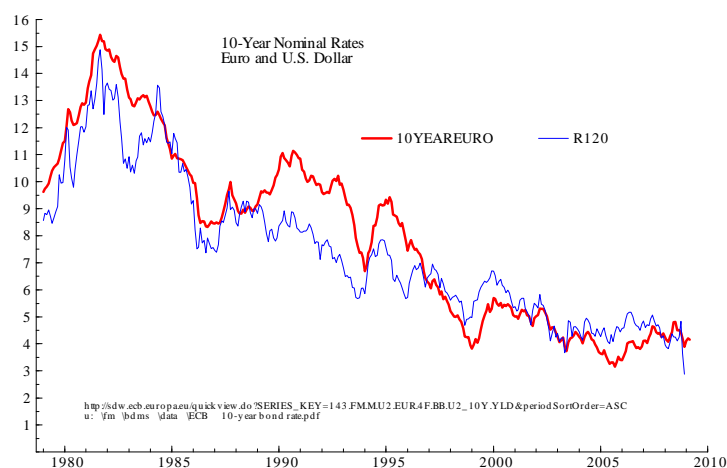


Figure 3

Because there is no simple, or universally accepted, approach to model these interdependencies, the paper uses several models that differ on whether foreign holdings and foreign interest rates are endogenous. This distinction, though artificial, helps quantifying the gains from endogenizing foreign holdings which, in addition, are hard to model empirically. Given our empirical motivation, we focus on the ability of the models to explain the data. Thus we examine the sensitivity of the parameter estimates and out-of-sample predictions to the choice of econometric modeling: common factors, vector error-correction, and DSGE formulations; we have also examined latent factor and affine formulations but the results are too tentative to report. The evidence so far indicates that movements in foreign holdings matter for explaining movements in U.S. Treasury yields but these magnitude of the effect is sensitive to the allowance and modelling of general equilibrium considerations.

2 Existing Evidence

2.1 Replication of Warnock and Warnock (2009)

The model used by Warnock and Warnock (2009) assumes a preferred-habit in which the long-term rate is not connected explicitly to other yields. Their formulation is

$$R_{120,t} = \alpha + \beta \cdot \pi_{t+10}^e + (1 - \beta) \cdot R_{ff,t} + \gamma \cdot Y_t^e + \delta \cdot rp_t + \mu \cdot (\pi_{t+1}^e - \pi_{t+10}^e) + \zeta \cdot deficit_{t-1} + \eta \cdot foreign_t + u_t, \quad (1)$$

where

R_{120} : 10-year U.S. government-bond yield, nominal.

π_{t+10}^e : Expected 10-year ahead inflation rate.

R_{ff} : 3-month euro dollar rate, nominal.

Y^e : Expected growth rate of GDP.

rp : Risk premium: 36-month moving standard deviation of changes in R_{10} .

π_{t+1}^e : Expected 1-year ahead inflation rate.

$deficit$: Structural budget deficit scaled by lagged GDP.

$foreign$: 12-month change in foreign officials' holdings of U.S. long-term securities scaled by nominal GDP.

$u_t \sim IN(0, \sigma^2)$.

Using monthly data from January 1984 to May 2005, Warnock and Warnock use OLS for parameter estimation; they find that an increase in *foreign* lowers R_{120} . We replicated exactly their results using their data: Column 1 of table 1 shows that the point estimates we obtained are identical to the ones they report in their paper (table 1, column 2).¹

We examine the sensitivity of their results to changes in the measure of *foreign* and to changes in the specification. Specifically, we replaced their TIC-based measure of *foreign* with the 12-month change in foreign official holdings of U.S. Treasuries in custody at the FRBNY relative to monthly lagged GDP.² The two series have comparable contours but there are periods when both series differ significantly (see figure below). Column 2 of table 1 reports that the estimates of equation (1) are robust to the measure foreign official purchases of U.S. Treasuries. We also re-estimated the coefficients of equation (1) without imposing the restriction that the coefficients of π_{t+10}^e and $R_{ff,t}$ in equation (1) add up to one. We find that the resulting point estimates differ substantially from those using the parameter restriction (table 1, col. 3). Indeed, the coefficient for *foreign* turns "insignificant" and is numerically close to zero; using a different measure of *foreign* does not change this result. Further, a test of the restriction $\beta + \kappa = 1$ is rejected by the data.

¹Unlike W&W, we did not correct the standard errors because such a correction does not eliminate the message from the diagnostics: the estimating equation is misspecified.

²We used FAME's interpolations of nominal GDP; these interpolations constrain the average of the monthly value to be equal to the recorded quarterly value.

2.2 Replication of Rudebusch, Swanson, and Wu (2006)

Rudebusch, Swanson, and Wu (2006) use two models of the U.S. term structure: the affine model of Bernanke-Reinhart-Sacks (BRS) and the latent-factor model Rudebusch and Wu (RW); by design, these models exclude foreign financial flows. Rudebusch, Swanson, and Wu assess whether this exclusion carries a loss of information by regressing the models' residuals for the 10-year U.S. Treasury yield on several variables one of which is foreign official net purchases, measured by the 12-month change in custodial holdings at FRBNY.

To assess whether foreign official purchases explain these residuals, Rudebusch et al. (2006) use bivariate and multivariate regressions. The bivariate regressions are

$$R_t^{brs} = \alpha + \beta \cdot F_t^{ny} + u_{brs,t} \quad (2)$$

$$R_t^{rw} = \alpha + \beta \cdot F_t^{ny} + u_{rw,t}, \quad (3)$$

where

R_t^{brs} is the gap between the 10-year U.S. Treasury yield and the prediction from the BRS model,

R_t^{rw} is the gap between the 10-year U.S. Treasury yield and the prediction from the RW model,

F_t^{ny} is the 12-month change in foreign official holdings of U.S. Treasuries in custody at the FRBNY relative to Federal Debt held by the public.

The multivariate regressions are

$$R_t^{brs} = \alpha + \beta \cdot F_t^{ny} + \gamma \cdot \mathbf{Z}'_t + u_{brs,t} \quad (4)$$

$$R_t^{rw} = \alpha + \beta \cdot F_t^{ny} + \gamma \cdot \mathbf{Z}'_t + u_{rw,t} \quad (5)$$

where \mathbf{Z}'_t is a vector of additional explanatory variables:

Z1: implied volatility on longer-term Treasury securities,

Z2: implied volatility on six-month ahead eurodollar deposits

Z3: implied volatility of the S&P 500,

Z4: realized volatility of GDP growth,

Z5: Realized volatility of monthly core CPI.

The exact definitions of these variables appear in tables 6 and 7 of Rudebusch et al. (2006).

Table 2 compares the results of Rudebusch, Swanson, and Wu (tables 6 and 7 of their paper) to the results obtained here. For the BRS residuals (columns 1-4), the point estimates are quite close but not identical to the estimates of Rudebusch, Swanson, and Wu. For the RW residuals (columns 5-8), the point estimates for the bivariate case (cols. 5-6) are quite different but equally statistically insignificant. However, the point estimates for foreign official purchases in the multivariate case (cols 7-8) are very different: 38 versus -40; the remaining coefficients are quite similar. After consulting with Swanson, we arrived at the conclusion that the difference in results owes to us getting data from a different vintage.

Having replicated their results, we examine their sensitivity to changes in the specification and to changes in the measure of F_t^{ny} . We replaced their formulations with an autoregressive distributed lag of order 3 and re-estimated their parameters. Table 3 shows the results for the BRS residuals and table 4 shows the results for the RW residuals. The evidence reveals that the results reported by Rudebusch-Swanson-Wu are robust to including lags in their specifications (cols. 3 and 4 of tables 3 and 4): the sum of coefficients for foreign official purchases is zero in every instance.

We also replaced F_t^{ny} with the Bertaut-Tryon measure for the 12-month change in foreign official holdings of U.S. long-term U.S. treasuries (notes and bonds) as well as agency bonds; these changes are scaled them by the total outstanding federal debt of the U.S. government. The results shown in cols. 5-8 of tables 3 and 4 indicate that using the Bertaut-Tryon’s measure of foreign official purchases explains the residuals regardless of whether the specifications have lags or not. This finding confirms the importance of measurement of foreign inflows for explaining U.S. interest rates.

Summary The replication results indicate that relaxing the estimation of assumptions of Warnock and Warnock yields a model that supports the findings of Rudebusch et al and that relaxing the estimation assumptions of Rudebusch et al. yields a model that is consistent with the results of Warnock and Warnock. We interpret this finding as suggesting that, right now, the literature offers no reliable guidance as to whether the foreign official purchases affect U.S. interest rates.

3 Partial Equilibrium Models

We report below the specifications and results from three partial-equilibrium models that abstract from no-arbitrage considerations; we have developed and estimated latent-factor models and affine (no-arbitrage) models but the results are too tentative to report here.

3.1 Specifications

3.1.1 Preferred Habitat

This framework focuses on explaining the 10-year Treasury yield. To this end, we re-formulated the W&W model as autoregressive distributed lag of order 1

$$R_{120,t} = \alpha + \beta(L) \cdot R_{ff,t} + \varepsilon(L) \cdot R_{10,t}^{\text{€}} + \gamma(L) \cdot Y_t^e + \mu(L) \cdot \pi_{t+1}^e + \eta(L) \cdot F_t + \varphi \cdot R_{120,t-1} + u_t, \quad u_t \sim IN(0, \sigma^2). \quad (6)$$

where $R_{10}^{\text{€}}$ is the 10-year euro nominal interest rate and F is foreign holdings of U.S. securities.

3.1.2 Johansen's Method and the Yield Curve

This method argues that changes in Treasury yields owe to short-run dynamics and an adjustment to the long-run yield curve. This view is implemented as

$$\begin{pmatrix} \Delta R_{120,t} \\ \Delta R_{84,t} \\ \Delta R_{60,t} \\ \Delta R_{36,t} \\ \Delta R_{24,t} \\ \Delta R_{12,t} \\ \Delta R_{1,t} \\ \Delta R_{ff,t} \end{pmatrix} \equiv \Delta \mathbf{R}_t = \underbrace{\sum_{k=1}^2 \underbrace{\Gamma_k}_{8 \times 8} \cdot \Delta \mathbf{R}_{t-j}}_{\text{short run dynamics}} + \underbrace{\underbrace{\Pi}_{8 \times 13} \cdot \underbrace{\mathbf{X}_{t-1}}_{13 \times 1}}_{\text{adjustment to long run}} + \mathbf{v}_t, \quad (7)$$

where $\mathbf{X}'_t = \left(\mathbf{R}_t \ R_{10,t}^e \ F_t \ \pi_{t+1}^e \ Y_t^e \ 1 \right)$, and $\mathbf{v}_t \sim IN(0, \Omega)$. This method recognizes the interdependencies among Treasury yields, differentiates between short-run dynamics and long-run adjustment, and avoids simultaneity biases.³

We tested the rank of Π using the Trace and Max tests, both with and without correction for degrees of freedom; the results indicate that one cannot reject the hypothesis that the rank of Π is at most seven, meaning that there are seven cointegration vectors. Given this result, we express Π as

$$\Pi = \underbrace{\boldsymbol{\alpha}}_{8 \times 7} \cdot \underbrace{\boldsymbol{\beta}'}_{7 \times 13}, \quad (8)$$

where

$$\boldsymbol{\beta}' = \begin{pmatrix} 1 & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} & \beta_{1,6} & \beta_{1,7} & \beta_{1,8} & \cdots & \beta_{1,13} \\ \beta_{2,1} & 1 & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} & \beta_{2,6} & \beta_{2,7} & \beta_{2,8} & \cdots & \beta_{2,13} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \beta_{7,1} & \beta_{7,2} & \beta_{7,3} & \beta_{7,4} & \beta_{7,5} & \beta_{7,6} & 1 & \beta_{7,8} & \cdots & \beta_{7,13} \end{pmatrix} = \begin{pmatrix} \beta'_1 \\ \beta'_2 \\ \vdots \\ \beta'_7 \end{pmatrix}$$

and

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{87} \\ \alpha_{21} & \cdots & \alpha_{87} \\ \vdots & \vdots & \vdots \\ \alpha_{81} & \cdots & \alpha_{87} \end{pmatrix}.$$

Because there are more α 's and β 's than the number of elements of Π , we need to address the issue of identification. We were not able to get satisfactory results with various schemes, either from a statistical standpoint or from an economic viewpoint. Thus we pursued the scheme offered by the Term-premium hypothesis, equation (10) above. The scheme assumes that which implies that the yield curve is

$$R_{n,t} = \beta_{n,8} \cdot R_{ff,t} + \ell_n, \quad (9)$$

where $\beta_{n,8}$ is the pass-through coefficient of the federal funds rate to the n-month Treasury yield and ℓ_n is the liquidity premium for yields of n-months.

³For choosing the number of lags, we began with six lags and then tested for fewer lags.

To allow a role for foreign considerations in influencing U.S. Treasury yields, we assumed that

$$\ell_{nt} = \lambda_{n0} + \lambda_{n1} \cdot R_{10,t}^{\epsilon} + \lambda_{n2} \cdot F_t + \lambda_{n3} \cdot \pi_{t+1}^e + \lambda_{n4} \cdot Y_{t+1}^e.$$

Intuitively, changes in R_{10}^{ϵ} might affect the term premia to the extent that U.S. and foreign bonds are imperfect substitutes for each other. Similarly, the inclusion of F recognizes that foreign investors might have different degrees of substitutability among Treasury securities than U.S. investors. If $\lambda_{n1} = \lambda_{n2} = 0$, then foreign considerations are not relevant for the U.S. term structure.

Given these assumptions, the resulting term structure is

$$R_{n,t} = R_{ff,t} + \lambda_{n0} + \lambda_{n1} \cdot R_{10,t}^{\epsilon} + \lambda_{n2} \cdot F_t + \lambda_{n3} \cdot \pi_{t+1}^e + \lambda_{n4} \cdot \pi_{t+1}^e. \quad (10)$$

We assumed that the above equation holds in the long-run and not minute by minute. What is needed is a method to estimate the λ 's and to separate short-run dynamics from adjustment to the long-run yield curve.

$$\begin{pmatrix} \beta'_1 \\ \beta'_2 \\ \vdots \\ \beta'_7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -\beta_{1,8} & -\lambda_{120,0} & -\lambda_{120,1} & \cdots & -\lambda_{120,4} \\ 0 & 1 & 0 & 0 & 0 & 0 & -\beta_{2,8} & -\lambda_{84,0} & -\lambda_{84,1} & \cdots & -\lambda_{84,4} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 1 & -\beta_{7,8} & -\lambda_{12,0} & -\lambda_{12,1} & \cdots & -\lambda_{12,4} \end{pmatrix}. \quad (11)$$

Though we are imposing, instead of testing, the term hypothesis, we have not imposed any restrictions about the parameters of the term functions. Even so, this scheme does not generate enough parameter restrictions to identify the β 's and the α 's. Thus we generated additional identifying restrictions on α by assuming that $R_{ff,t}$ is weakly exogenous ($\alpha_{8j} = 0$ for all j) and that each R_n reacts to one cointegration vector, instead of seven. With these assumptions, we expressed α as

$$\alpha = \begin{pmatrix} \alpha_{11} & 0 & 0 & \cdots & 0 \\ 0 & \alpha_{22} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha_{77} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (12)$$

3.1.3 Koopman's Method and the Yield Curve

We model Treasury yields as being driven by two common factors and exogenous variables. Following Koopman, we use a recursive model for U.S. yields:

$$R_{1,t} = R_{1,t-1} + \sum_{j=1}^K \phi_{1,j} \cdot X_{j,t} + \eta_{1,t}, \quad \eta_{1,t} \sim N(0, \sigma_{\eta,1}) \quad (13)$$

$$R_{12,t} = R_{12,t-1} + \sum_{j=1}^K \phi_{12,j} \cdot X_{j,t} + \eta_{12,t}, \quad \eta_{12,t} \sim N(0, \sigma_{\eta,2}) \quad (14)$$

$$R_{n,t} = (\mu_n + \lambda_{n1} \cdot R_{1,t} + \lambda_{n2} \cdot R_{12,t}) + \sum_{j=1}^K \phi_{nj} \cdot X_{j,t} + \varepsilon_{nt}, \quad \varepsilon_{nt} \sim N(0, \sigma_{\varepsilon,n}); \quad n > 12 \quad (15)$$

where

R_n is the n -month nominal Treasury yield

$\phi_{n,j}$ coefficient of the j th exogenous variable in the equation for R_n

X_j j th exogenous variable, $j = 1, \dots, K$

μ_n is an intercept to be estimated

$\lambda_{n,1}, \lambda_{n,2}$ constant factor loadings, to be estimated

$E(\eta_{1t} \cdot \eta_{2t}) = \sigma_{\eta,12}$ and $E(\varepsilon_{nt} \cdot \varepsilon_{ft}) = 0$.

This approach has several advantages. First, yields are endogenous and related to each other even if exogenous variables were irrelevant for them ($\phi_{nj} = 0$). Second, the modeling embodies the finding that nominal yields are integrated of order one. Specifically, equations (21) and (22) assume that the *changes* in $R_{1,t}$ and $R_{12,t}$ respond to exogenous variables. These exogenous variables need also to be cointegrated among themselves with one cointegration vector at most.⁴ Further, the equation for R_n can be re-arranged as

$$R_{n,t} - (\mu_n + \lambda_{n1} \cdot R_{1,t} + \lambda_{n2} \cdot R_{12,t}) = \sum_{j=1}^K \phi_{nj} \cdot X_{jt} + \varepsilon_{nt}; \quad n > 12 \quad (16)$$

where, following Koopman, the left-hand side represents deviations from the cointegrating relation between R_n and the linear combination of $R_{1,t}$ and $R_{12,t}$; these deviations respond to changes in exogenous variables. We assume five exogenous variables: R_{ff} , R_{10}^e , F , Y^e , and π_{t+1}^e .

3.2 Implementation

- Estimation
 - Sample: monthly from August 1987 to June 2007
 - Single-equation with OLS
 - Vector error-correction with Johansen’s FIML
 - Observed Common Factors with Koopman’s FIML
- Measures of foreign holdings, expressed as percentage of potential nominal GDP:
 - *FOI*: Bertaut and Tryon measure (BT) for official investors’ holdings of U.S. Treasuries (Bills plus bonds plus notes plus indexed bonds)
 - *FTOT*: BT measure for the aggregate of foreign official and foreign private investors’ holdings of U.S. Treasuries
 - *FRBNY*: Custodial Holdings at the New York Fed
 - 12-month changes in each of these three measures: $\Delta_{12}FRBNY$, $\Delta_{12}FOI$, $\Delta_{12}FTOT$
- Evaluation:

⁴Results, not shown here, cannot reject hypothesis of one cointegration vector among exogenous variables.

- Tests of properties of the residuals: normality, serial independence, homoskedasticity
- Out-of-sample Forecasts: Dynamic simulation from July 2007 to December 2008 for all 18 models along with forecasts for time-series models: AR(2), Random Walk, VAR(2), Common Factor with no exogenous variables.

3.3 Results

Table 5 shows the long-run coefficients along with the results evaluating the properties of the residuals and out-of-sample forecast accuracy; the focus is on the 10-year Treasury yield.

1. Column 1 shows that the effect of an increase of 100 basis points in the federal funds rate is sensitive to the modeling of the choice of model curve: 20 basis points for the Johansen model and no effect for the other formulations.
2. Column 2 shows that an increase of 100 basis points in the 10-year euro rate raises the 10-year U.S. Treasury yield regardless of model specification; the effect ranges from 11 basis points to 60 basis points, depending on the measure of foreign holdings. For a given measure of foreign holdings, however, the estimated effect is substantially greater for our term-structure models than for our preferred habitat model.
3. Column 3 shows that the effect of an increase of 100 basis points in foreign holdings on the 10-year U.S. Treasury yield depends critically on the choices of model and measures of foreign holdings. This sensitivity replicates the tension found in the replication section: minor changes to the model design translate into large changes in model estimates.
4. Columns 4-6 show that the choice of model has also implications for the properties of the residuals. Only the preferred habitat and Johansen formulations have residuals that are serially independent and homoskedastic; normality is normally rejected.
5. Column 7 shows the root mean squared forecast error from dynamic (s-step) simulations from July 2007 to December 2008; note that the second forecast horizon includes the Lehman Brothers failure.
6. Column 9 shows the ratio of the model's RSMFE to the lowest RMSFE from the four time-series models.
7. Out of the 18 models, only four preferred-habitat models are congruent: the residuals in all remaining models are not white noise. Out of these four models, the ones with the lowest RSMFE use the level of foreign holdings as measured by Bertaut and Tryon (2007). For these models, an increase in foreign holdings of 100 basis points lowers the 10-year U.S. Treasury by a bit over 20 basis points.

4 General Equilibrium Model

To design the general equilibrium model, we exploit the main result from table 5: that the best forecasting and congruent model of the 10-year U.S. Treasury uses the preferred habitat model with Bertaut and Tryon measures of foreign holdings.

4.1 Specification

- Two countries with symmetric relations but different parameters
- Short-term interest rate depends on inflation and output:

$$\begin{aligned} R_{1,t} &= \rho_i \cdot R_{1,t-1} + (1 - \rho_i) \cdot [\psi_\pi \cdot \pi_t + \psi_y \cdot Y_t] + \varepsilon_{i1,t} \\ R_{1,t}^* &= \rho_i^* \cdot R_{1,t-1}^* + (1 - \rho_i^*) \cdot [\psi_\pi^* \cdot \pi_t^* + \psi_y^* \cdot Y_t^*] + \varepsilon_{i1,t}^* \end{aligned}$$

- Inflation depends on expected inflation, past inflation, and output:

$$\begin{aligned} \pi_t &= \mu_\pi \cdot E_t \pi_{t+1} + (1 - \mu_\pi) \cdot [\alpha_{\pi 1} \cdot \pi_{t-1} + \alpha_{\pi 2} \cdot \pi_{t-2}] + \alpha_y \cdot Y_{t-1} + \varepsilon_{\pi,t} \\ \pi_t^* &= \mu_{\pi^*} \cdot E_t \pi_{t+1}^* + (1 - \mu_{\pi^*}) \cdot [\alpha_{\pi^* 1} \cdot \pi_{t-1}^* + \alpha_{\pi^* 2} \cdot \pi_{t-2}^*] + \alpha_y^* \cdot Y_{t-1}^* + \varepsilon_{\pi,t}^* \end{aligned}$$

- Output depends on expected future output, past output, and the ex-ante real interest rate:

$$\begin{aligned} Y_t &= \beta_y \cdot E_t Y_{t+1} + (1 - \beta_y) \cdot [\beta_{y1} \cdot Y_{t-1} + \beta_{y2} \cdot Y_{t-2}] + \beta_r \cdot [R_{1,t} - E_t \pi_{t+1}] + \varepsilon_{y,t} \\ Y_t^* &= \beta_{y^*} \cdot E_t Y_{t+1}^* + (1 - \beta_{y^*}) \cdot [\beta_{y1^*} \cdot Y_{t-1} + \beta_{y2^*} \cdot Y_{t-2}] + \beta_{r^*} \cdot [R_{1,t}^* - E_t \pi_{t+1}^*] + \varepsilon_{y,t}^* \end{aligned}$$

- 10-year Bond Rate depends on the short-term rate, the expected inflation, foreign holdings of bonds, and a risk premium:

$$\begin{aligned} R_{120,t} &= R_{1t} + E_t \pi_{t+1} + \psi_{Fbonds} \cdot F_t^{bonds} + rp_t \\ rp_t &= \rho_{rp} \cdot rp_{t-1} + \varepsilon_{rp,t} \\ R_{120,t}^* &= R_{1t}^* + E_t \pi_{t+1}^* + \psi_{FOther} \cdot F_t^{other} + rp_t^* \\ rp_t^* &= \rho_{rp^*} \cdot rp_{t-1}^* + \varepsilon_{rp^*,t} \end{aligned}$$

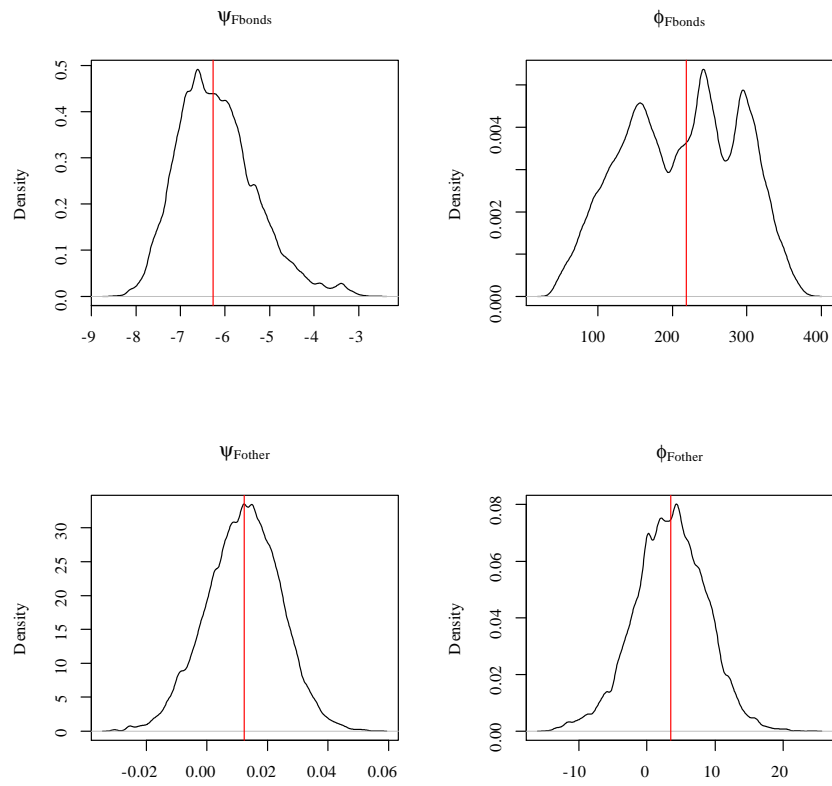
- Foreign holdings of U.S. Treasury bonds depend on the re-allocation of wealth and real-interest rate differentials:

$$\begin{aligned} F_t^{bonds} &= \gamma_{Fbonds} \cdot F_{t-1}^{bonds} + (1 - \gamma_{Fbonds}) \cdot [\lambda_{Fbonds} \cdot \Delta F_{res,t} + \phi_{Fbonds} (r_{120,t} - r_{120,t}^*)] + \varepsilon_{Fbonds,t} \\ F_{res,t} &= \rho_{Fres} \cdot F_{res,t-1} + \varepsilon_{Fres,t} \\ r_{120,t} &= R_{120,t} - E_t \pi_{t+1} \\ r_{120,t}^* &= R_{120,t}^* - E_t \pi_{t+1}^* \\ F_t^{other} &= \gamma_{FOther} \cdot F_{t-1}^{other} + (1 - \gamma_{FOther}) \cdot [\lambda_{FOther} \cdot \Delta F_{res,t} + \phi_{FOther} (r_{120,t} - r_{120,t}^*)] + \varepsilon_{FOther,t} \end{aligned}$$

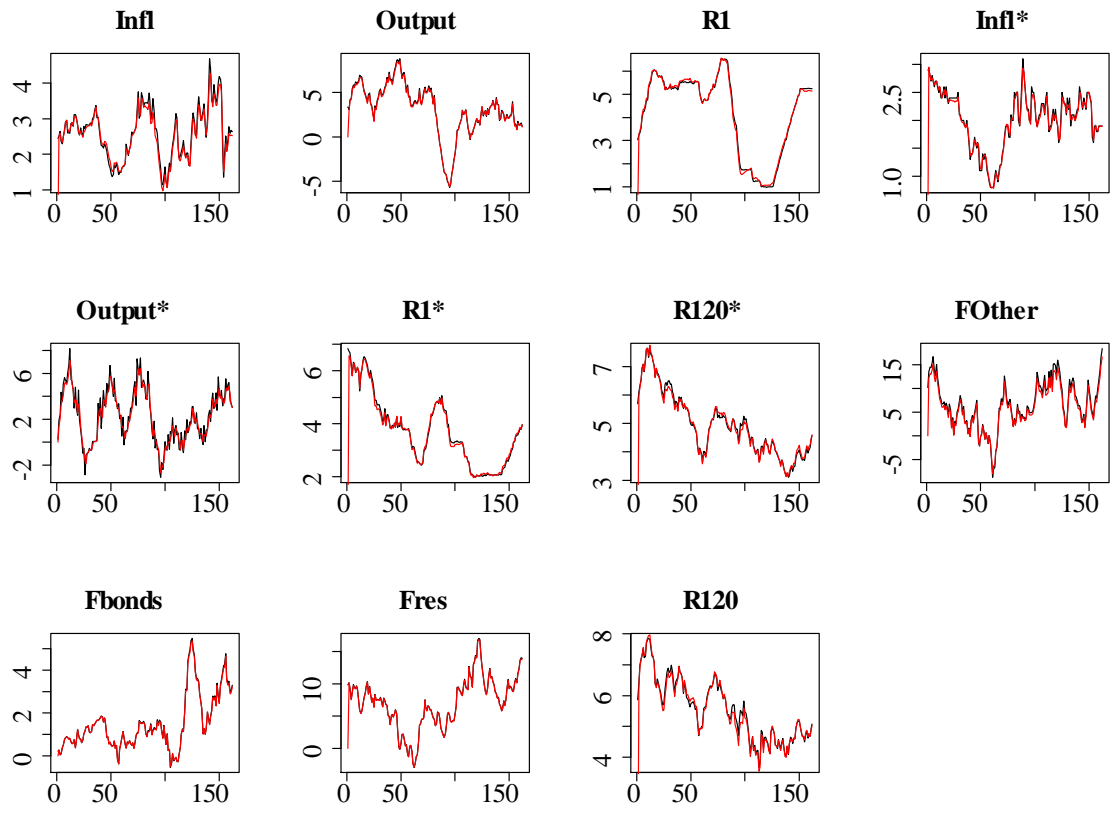
- Parameters of interest

- ψ_{FOther} effect of FOther on foreign long rate (-)
- ψ_{Fbonds} effect of FBonds on U.S. long rate (-)
- ϕ_{FOther} effect of U.S.-foreign long-term interest rate differential on FOther (-)
- ϕ_{Fbonds} effect of U.S.-foreign long-term interest rate differential on FBonds (+)

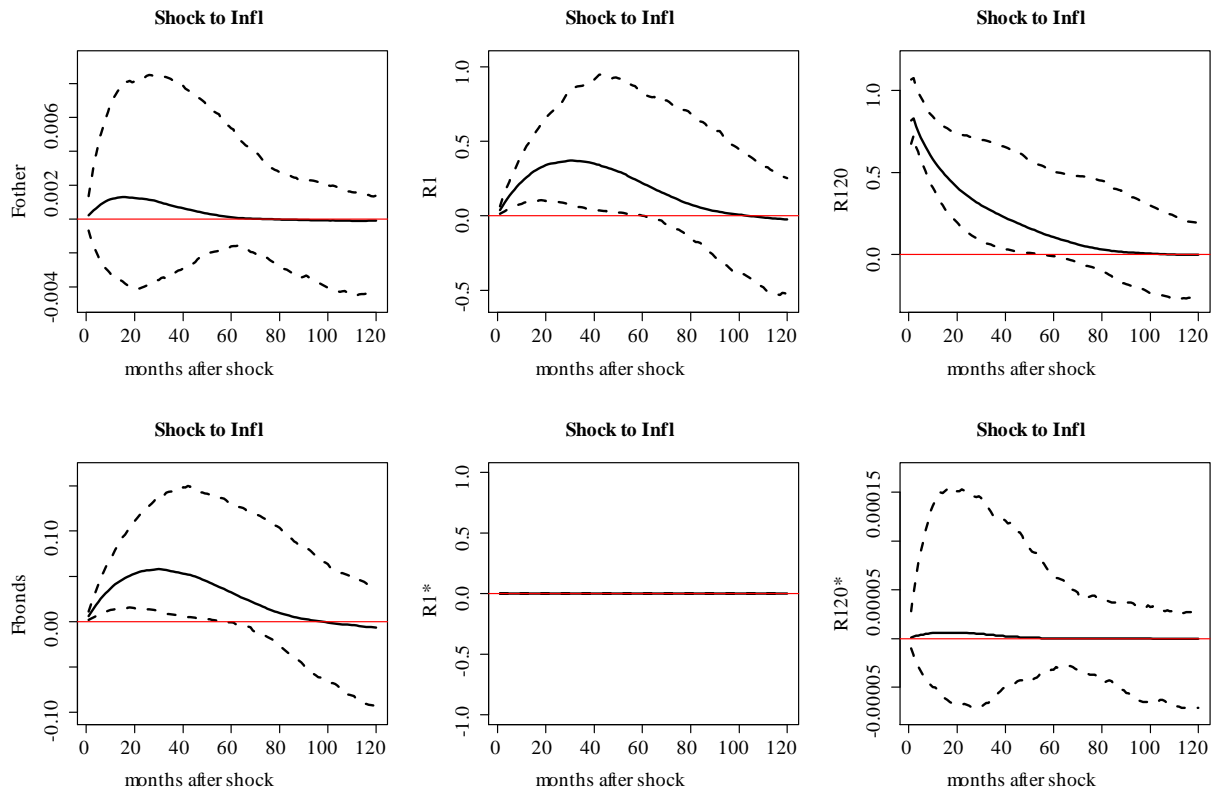
4.2 Results



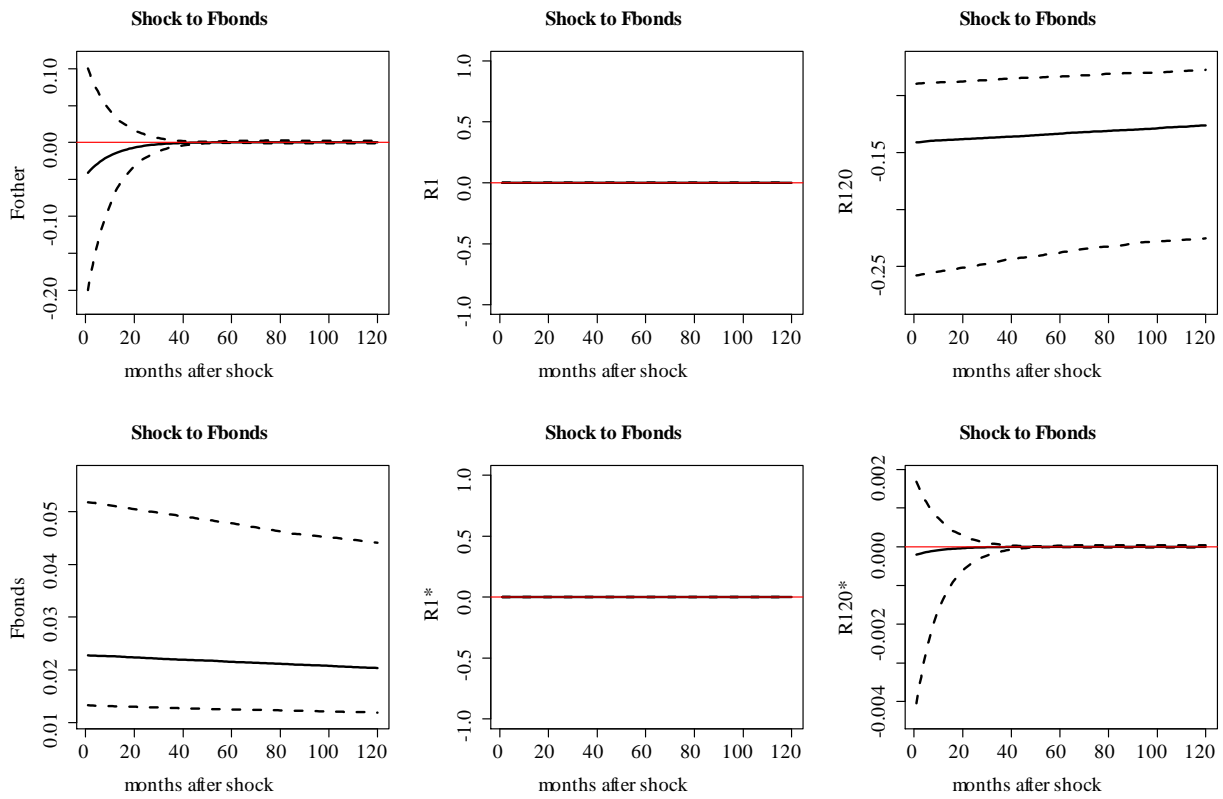
Markov-Chain Monte Carlo Estimates



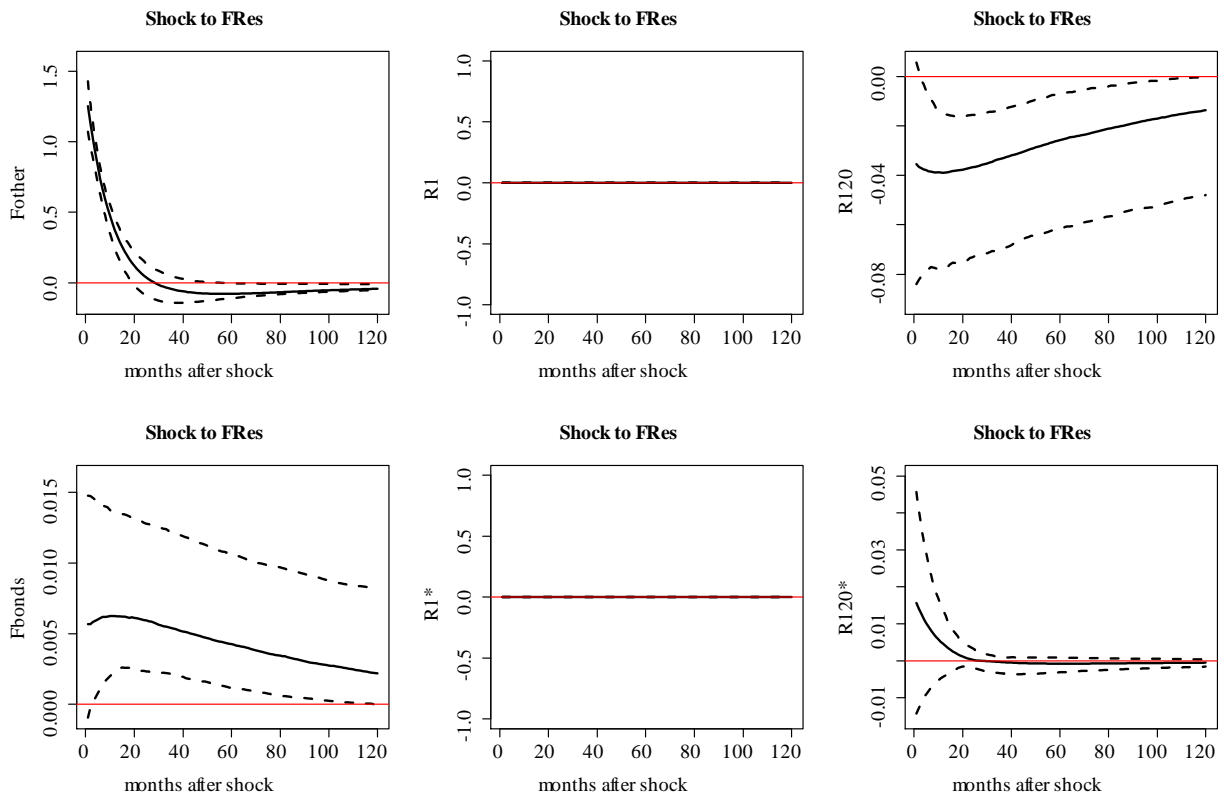
Fitted Values with Kalman Filter Estimates



Impulse Response Function to Inflation

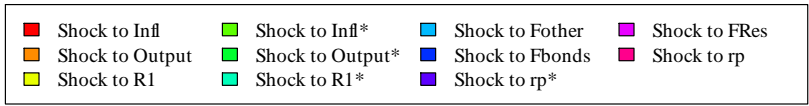
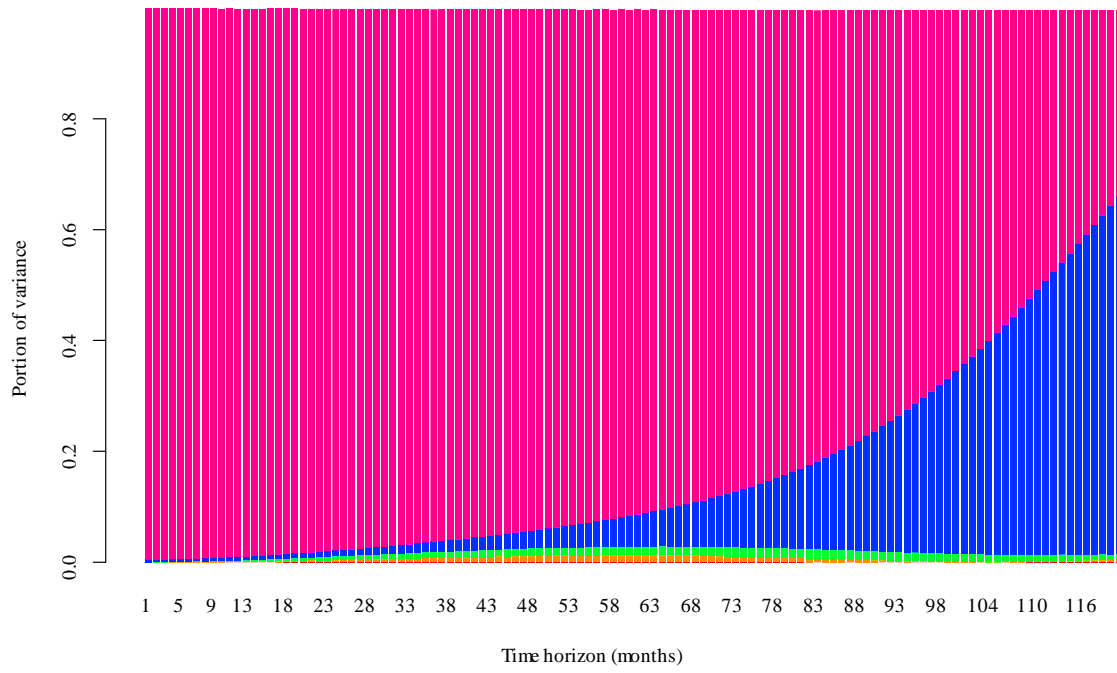


Impulse Response Functions: Shock to Fbonds

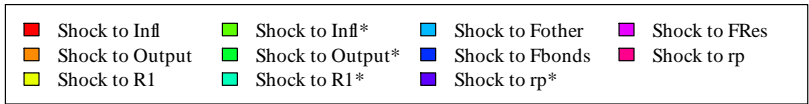
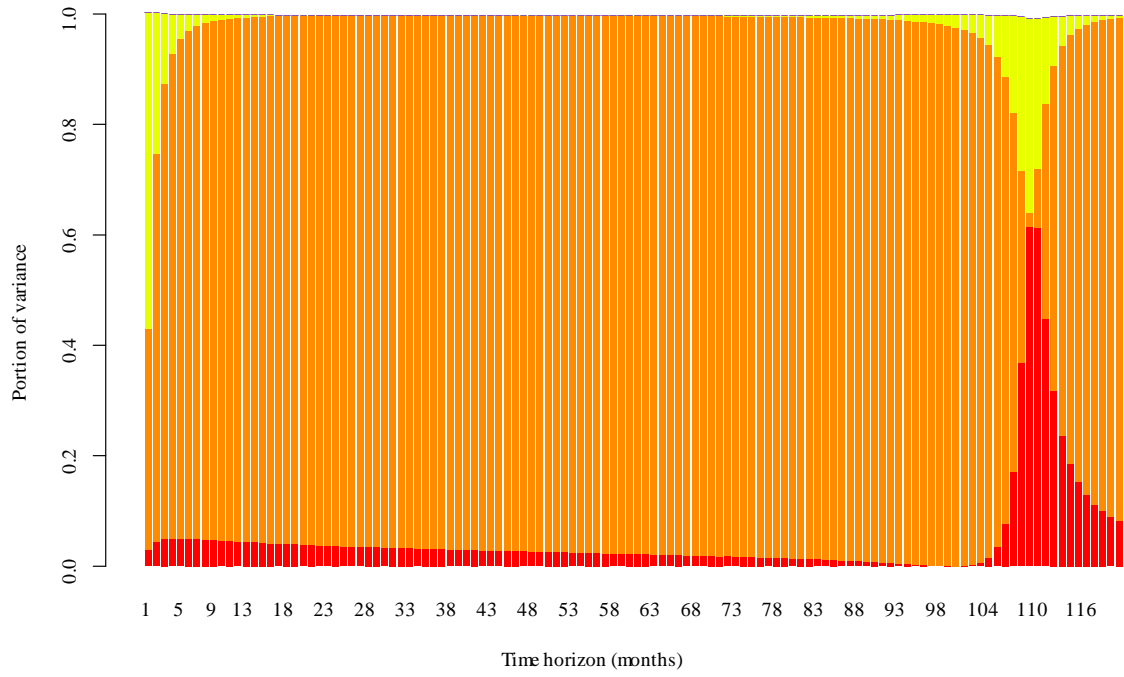


Impulse Response Functions: Shock to FRes

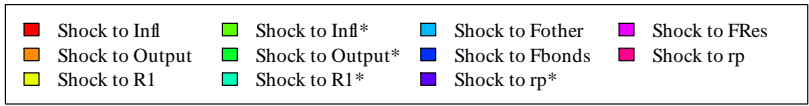
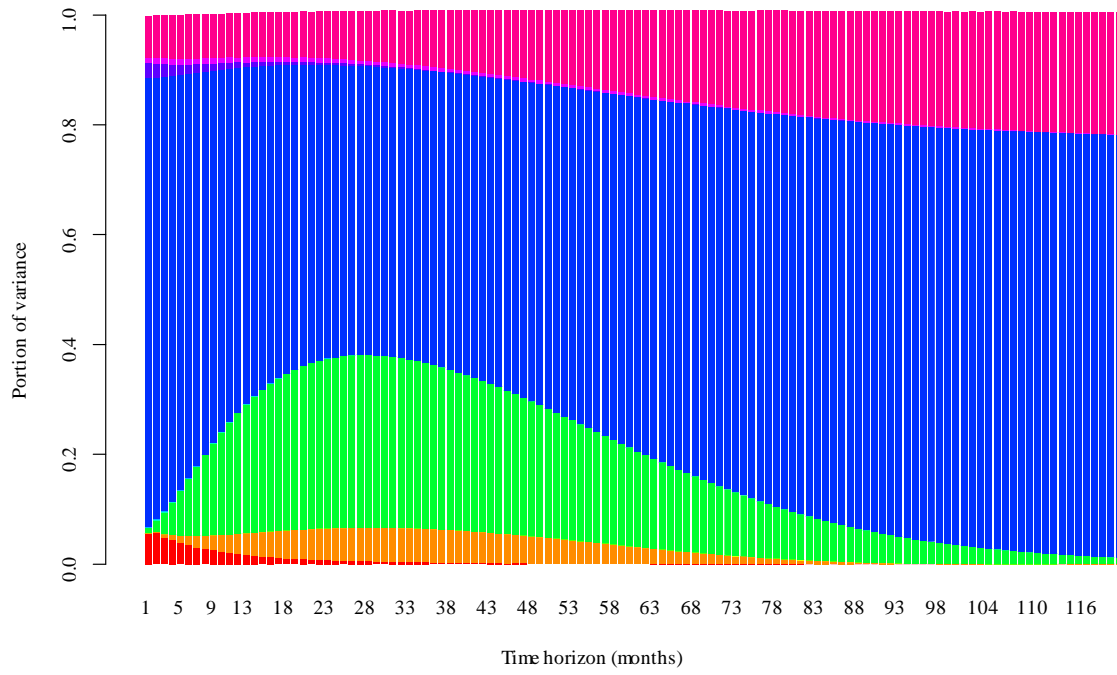
Forecast variance decomp. for Fbonds



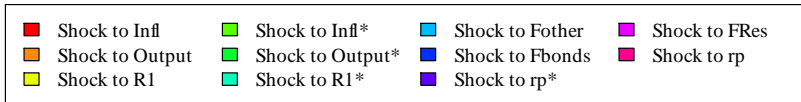
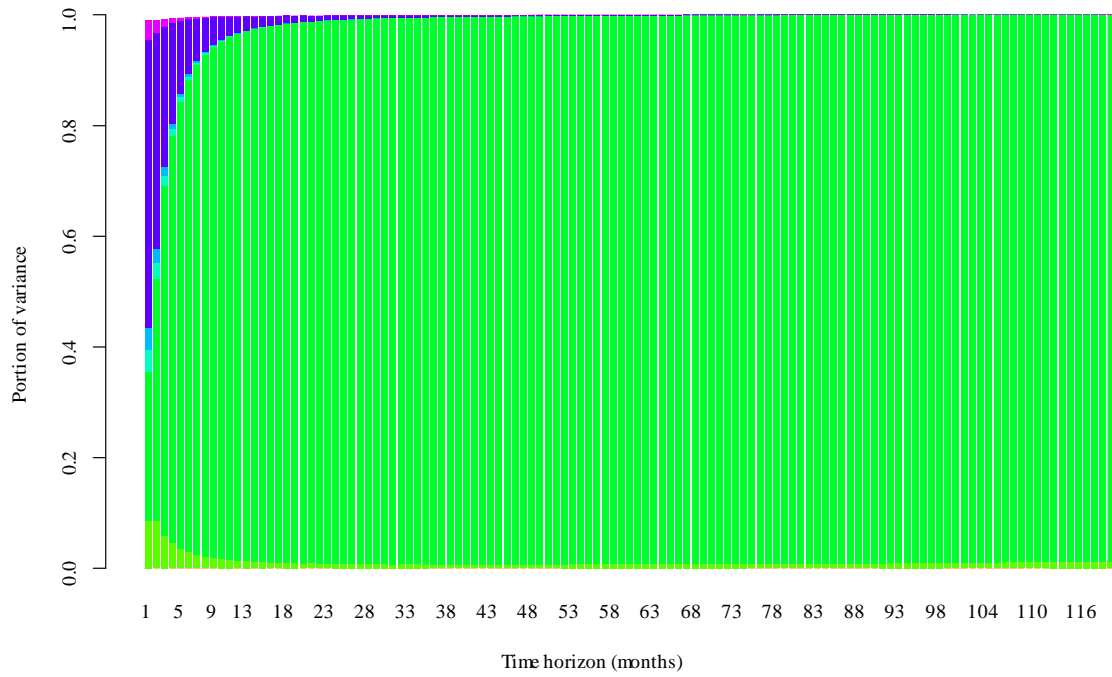
Forecast variance decomp. for R1



Forecast variance decomp. for R120



Forecast variance decomp. for R120*



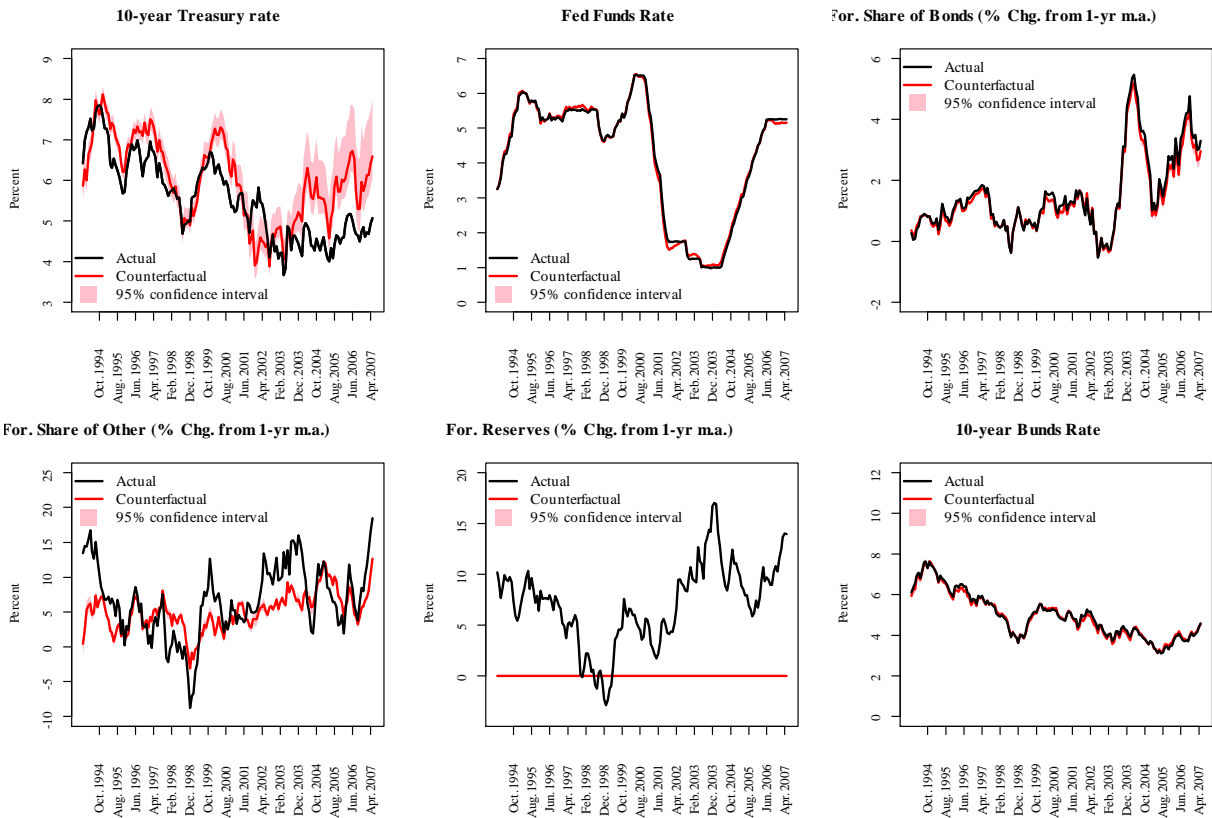
Counterfactual Analysis

What would have happened to U.S. interest rates if foreign official reserves had not grown at all during the period, and if there had been no exogenous shocks to Fbonds and FRes?

Recover time-series of shocks from Kalman filter

Set FRES and Fbonds shocks to zero.

Compare actual versus model-implied paths for state variables.

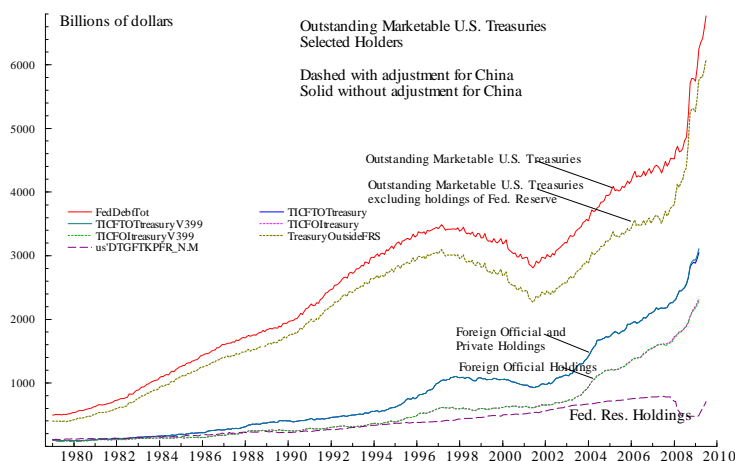


5 Data for Partial Equilibrium Models

5.1 Measuring Foreign Holdings of U.S. Treasuries for Empirical Work

5.1.1 Marketable U.S. Treasuries

- The raw data appear in TREASURY BULLETIN, MONTHLY STATEMENT OF THE PUBLIC DEBT <http://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm>
- Following Treasury's definition, we computed U.S. marketable Treasuries as the sum of the face value of Bonds, Bills, Notes, and Inflation-indexed securities.
- Data for Fed's holdings of U.S. Treasuries are available from The Board of Governors's H.4.1 Release.
- The figure below reports the two measures of marketable U.S. Treasuries; the figure also shows holdings of these securities by the Federal Reserve and by foreigners



5.1.2 Foreign Holdings of U.S. Treasuries

There are several issues associated with the measurement of foreign holdings of U.S. securities:

- Who is the relevant holder of F : Foreign officials or foreign official and private?
- How can one obtain benchmark consistent data?
- What is the data source: Treasury's International Capital System (TIC) or FRBNY?
- What should be the relevant scaling variable: Outstanding Treasuries and Potential GDP?

Benchmark Consistent Positions We are using the Bertaut-Tryon measures of benchmark consistent positions. Greatly simplified, their method reconciles changes in recorded annual positions with the recorded flows and valuation changes. Greatly simplified, the position at time $t = 1$ is

$$P_1 = P_0 + F_1 + V_1$$

where P stands for the recorded position, F stands for flow, and V stands for valuation change; we are ignoring the role of measurement error. The position in subsequent dates is obtained by forward recursion:

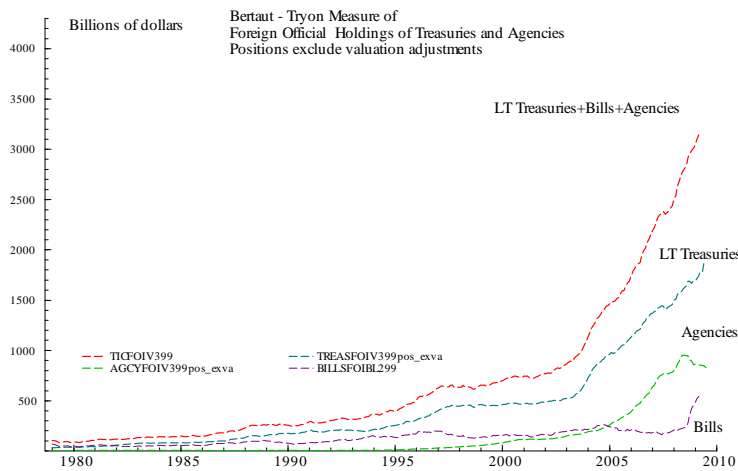
$$\begin{aligned}
 P_2 &= P_1 + F_2 + V_2 \\
 &= P_0 + (F_1 + F_2) + (V_1 + V_2) \\
 &\vdots \\
 P_t &= P_0 + \sum_{j=1}^t F_j + \sum_{j=1}^t V_j.
 \end{aligned}$$

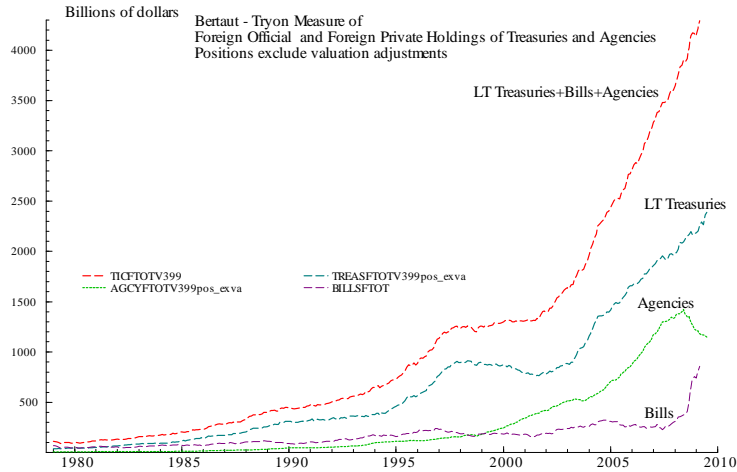
So the recorded position at time t is the sum of an initial condition, cumulated flows, and cumulated valuation changes. To measure the position excluding valuation adjustments we subtract the cumulated valuation adjustments from the recorded position:

$$P_t^{\beta\tau} = P_t - \sum_{j=1}^t V_j = P_0 + \sum_{j=1}^t F_j,$$

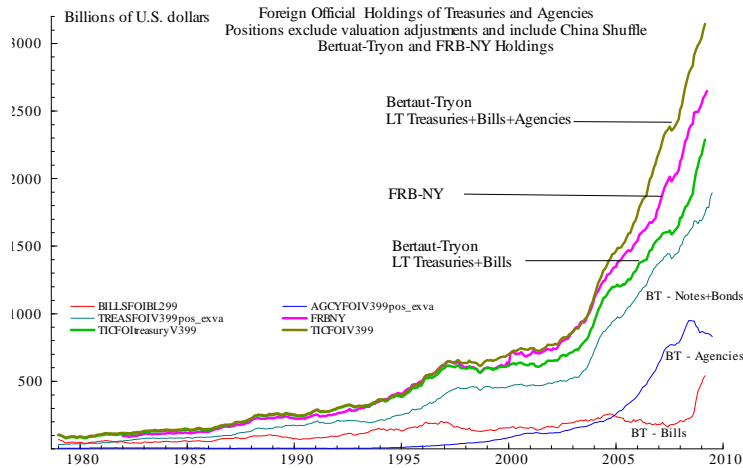
which equals the initial position plus cumulated flows.

Bertaut-Tryon Measures of Foreign Holdings

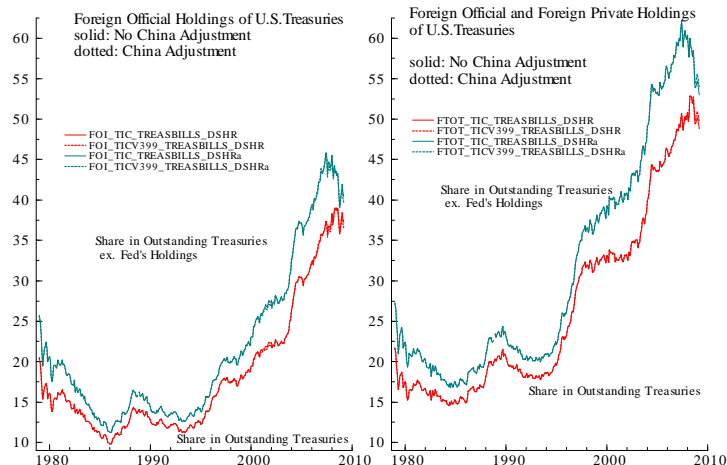




Foreign Official Custodial Holdings at FRBNY Data for all financial holdings of foreign official and international institutions held in custody at FRBNY; the primary source is the Board of Governors’s H.4.1 Release The graph below compares the FRBNY measure against Bertaut-Tryon estimates for both official and official and private.

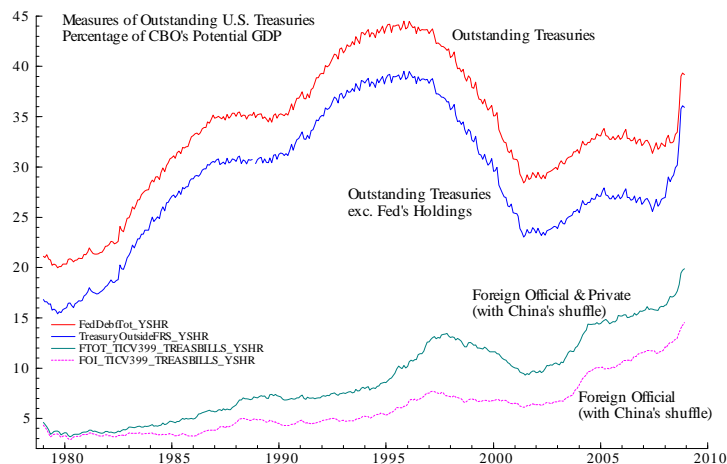


Scaling Foreign Holdings by Outstanding Marketable Treasuries The ratio of foreign holdings of U.S. Treasuries to outstanding Treasuries may be interpreted as the share of U.S. Treasuries held by foreigners. The figure below shows this ratio using foreign official and foreign total and scaling by D with and without including holdings by the Federal Reserve.



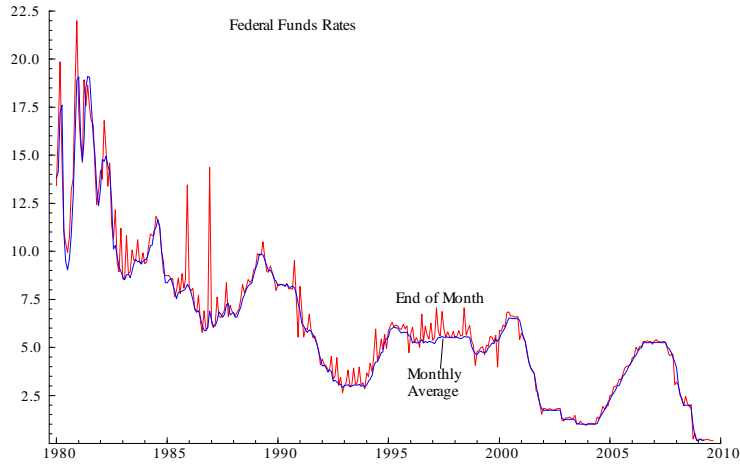
Scaling Foreign Holdings by Potential Nominal GDP However, an increase in f may arise from either an increase in F for a given D , or from a decrease in D for a given F . In either case, there is a decrease in R but the underlying force is different. Thus one might want to differentiate the effect of movements in F from the effect of movements in D . To differentiate the effects of movements in F from the effects of movements in D , we scaled F by potential income. In this formulation, the effects of movements in F on R cannot be confused with the effects of movements in D . Just as important, note that a change in F does not imply a change in D : changes in F alter the ownership composition of D but not its level.

The figure below shows the evolution of marketable outstanding U.S. Treasuries, as a percent of potential disposable income; the figure also shows holdings by foreign residents and by the Federal Reserve, both scaled by Y :

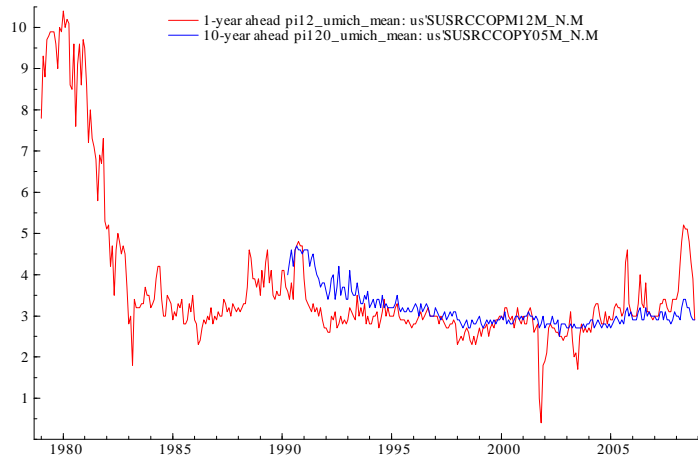


5.2 Macro Variables

Federal Funds Rates The measure corresponds to the effective federal funds rate, which is a weighted average of the reported rates at which different amounts of the day's trading through New York brokers occurs; the primary data source is Board of Governors of the Federal Reserve System H.15 Release.



Expected Inflation ($\pi_{t+10}^e, \pi_{t+1}^e$) Primary source: Survey data from the University of Michigan. Figure shows the one-year and the ten-year inflation expectations (fame mnemonics)



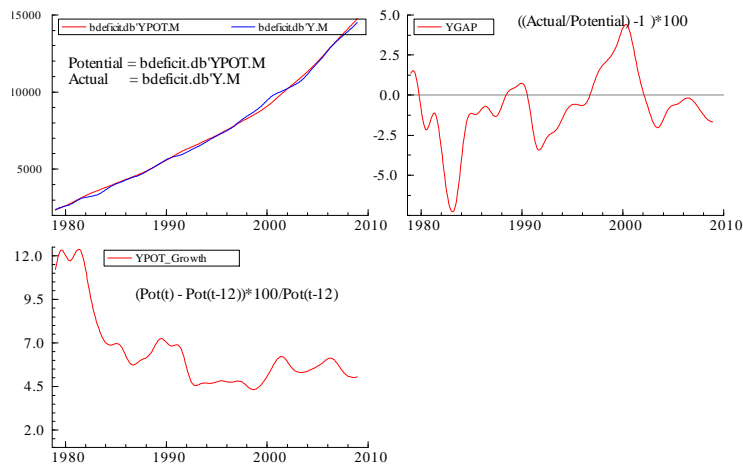
expect various horizons.pdf

Expected Growth (Y^e) Primary source: table 3, page 5 of the Blue Chip Economic Indicators; we computed the average of these series so as to get the expected growth rate *over* the next four quarters.



Nominal GDP Cyclically Adjusted (Y) Primary source: Congressional Budget Office - Historical Budget Data; Table F-11, column S; dated March 20, 2009.

CBO estimates potential real GDP which is then used to generate the cyclically adjusted nominal GDP. The data are annual; monthly observations are generated using Fame's cubic interpolation algorithm.



6 Data for General Equilibrium Model

Monthly data from 1/1994 - 6/2007 (162 obs.)

π_t and π_t^* : U.S. and Euro Area inflation: 12-month change in CPI index

Y_t and Y_t^* : U.S. and Euro Area output: 12-month change in Industrial Production index

$R_{1,t}^*$: U.S. short rate: Fed Funds rate

$R_{120,t}$: U.S. long rate: 10-year T-bond rate

$R_{1,t}^*$: Euro Area short rate: EONIA

$R_{120,t}^*$: Euro Area long rate: 10-year Bund rate

F_t^{bonds} : Foreign share of T-bonds: Foreign holdings of Treasury bonds (excluding valuation changes) as a share of outstanding Treasury bonds. Percent change in share from its 1-year moving average.

$Fres_t$: Total foreign reserves: From IFS database, excludes reserves of the United States (but includes foreign reserves held in the form of U.S. Treasury securities). Percent change from its 1-year moving average.

F_t^{other} : Total other reserves: Foreign official reserves other than foreign official holdings of Treasury bills, bonds, and notes. All measured at market value. Percent change from its 1-year moving average.

References

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- [3] Diebold, F., G. Rudebusch, and S. Arouba (2006), "The Macroeconomy and the Yield Curve," *Journal of Econometrics*, 131, 309-338.
- [4] Koopman, S. (2008) "Fitting the Term-Structure in STAMP 8.10," *OxMetrics News*, September, 8.
- [5] Koopman, S., A. Harvey, J. Doornik, and N. Shephard, (2007), *STAMP 8*, Timberlake: London
- [6] Nelson, C. and A. Siegel (1987), "A Parsimonious Modelling of Yield Curves," *Journal of Business*, 60, 473-489.
- [7] Rudebusch, G., E. Swanson, and T. Wu, 2006, "The Bond Yield 'Conundrum' from a Macro-Finance Perspective," Special Issue of the Bank of Japan's *Monetary and Economic Studies*, 24, 83-128.
- [8] Warnock, F. and V. Warnock, 2006, "International Capital Flows and U.S. Interest Rates," *Journal of International Money and Finance*, forthcoming.

Table 1: Estimates of W&W: Replication and Sensitivity-OLS, Jan. 1984-May 2005*

	foreign= WW data '(1)	foreign= FRBNY '(2)	foreign= WW data '(3)	foreign= FRBNY '(4)
Constant	1.174	1.054	-1.749	-1.660
SE	0.219	0.214	0.376	0.348
RFF	0.371	0.353	0.375	0.371
SEE	0.036	0.038	0.032	0.033
$\pi^e(t+10)$	0.629	0.647	1.642	1.621
SE	0.036	0.038	0.117	0.111
$\pi^e(t+1)-\pi^e(t+10)$	0.486	0.460	-0.324	-0.289
SE	0.209	0.210	0.203	0.199
$Y^e(t+1)$	0.137	0.187	0.312	0.312
SE	0.079	0.082	0.071	0.072
gov't budget surplus**	-0.134	-0.138	0.124	0.117
SE	0.031	0.031	0.039	0.039
risk premium (rp)	4.673	4.737	0.758	0.836
SE	0.796	0.800	0.817	0.811
foreign	-0.399	-0.309	0.023	-0.012
SE	0.084	0.070	0.087	0.069
Adj R2	0.754	0.751	0.938	0.938
SER	0.608	0.612	0.529	0.529
Tests of Assumptions***	0.868	0.678	0.992	0.996
Normality	0.143	0.111	0.030	0.047
Serial Independence	0.000	0.000	0.000	0.000
Homoskedasticity	0.000	0.000	0.000	0.000

* Estimated standard errors WITHOUT correction for serial dependence and heteroskedasticity

** W&W report estimates with the sign reversed

*** If entry is less than 0.05, then reject null hypothesis at the 5% level

Table 2: Replication of the Estimation Results from Tables 6 and 7 of Rudebusch, Swanson, and Wu (2006) -- 1990.m5-2005.m12

	BRS Residuals				RW Residuals			
	Bivariate		Multivariate		Bivariate		Multivariate	
	Original	Replication	Original	Replication	Original	Replication	Original	Replication
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Foreign Official Purchases	149.00	156.19	147.00	128.39	47.00	6.74	38.00	-40.02
t-stat	0.89	0.90	0.76	0.64	0.58	0.08	0.04	-0.37
Volatility of GDP Growth	--	--	15.40	10.58	--	--	3.90	3.52
t-stat			3.10	2.05			1.45	1.27
Volatility of LT Tr. Securities	--	--	1.20	1.34	--	--	0.49	0.48
t-stat			5.47	5.84			4.11	3.93
S&P 500 volatility	--	--	-0.33	0.01	--	--	-0.50	-0.50
t-stat			-0.63	0.01			-1.73	-1.67
Euro-dollar Volatility	--	--	-0.23	-0.25	--	--	-0.17	-0.15
t-stat			-1.35	-1.41			-1.83	-1.60
Core PCE Volatility	--	--	360.00	304.08	--	--	214.00	202.66
t-stat			2.18	1.76			2.39	2.19
Constant	nr	-1.55	nr	-167.45	nr	1.15	nr	-51.98
t-stat		-0.41		-7.50		0.63		-4.35
R^2	0.00	0.00	0.30	0.30	0.00	0.00	0.14	0.13
SER	nr	39.78	nr	33.89	nr	19.15	nr	18.14

nr: not reported

Table 3: Sensitivity of BRS Residuals Sum of Coefficients

	Foreign = ForPurch_Fdebtshr				Foreign = CHG12_FOI_AGTR_Fdebtshr			
	No lags		With Lags		No lags		With Lags	
	'(1)	'(2)	'(3)	'(4)	'(5)	'(6)	'(7)	'(8)
Constant	-1.55	-167.45	0.70	-75.43	9.07	-131.07	4.01	-51.47
SE	3.80	22.33	2.69	24.58	3.91	27.11	2.79	25.76
BRSresiduals	--	--	0.73	0.61	--	--	0.69	0.62
SE			0.06	0.07			0.06	0.07
Foreign	156.19	128.39	-71.18	-118.77	-715.46	-532.01	-418.70	-337.60
SE	173.45	201.48	124.20	167.01	229.16	279.56	168.20	223.30
GDP_Growth_Volat	--	10.58	--	1.98	--	11.16	--	4.23
SE		5.17		4.35		5.13		4.18
MOVE	--	1.34	--	0.78	--	1.40	--	0.63
SE		0.23		0.26		0.21		0.23
S&PVolatility	--	0.01	--	-0.61	--	-0.43	--	-0.63
SE		0.55		0.53		0.49		0.43
EuroVolatility	--	-0.25	--	-0.32	--	-0.33	--	-0.33
SE		0.18		0.17		0.17		0.16
CoreVolatility	--	304.08	--	311.56	--	80.69	--	209.60
SE		172.60		148.48		192.00		156.70
Adj R2	0.00	0.27	0.52	0.61	0.04	0.29	0.54	0.61
SER	39.78	33.89	27.76	25.09	38.86	33.60	27.04	24.93
Tests of Assumptions***								
Normality	0.78	0.57	0.33	0.63	0.62	0.73	0.41	0.50
Serial Independence	0.00	0.00	0.57	0.75	0.00	0.00	0.67	0.96
Homoskedasticity	0.00	0.00	0.80	0.77	0.00	0.00	0.50	0.85

* Estimated standard errors WITHOUT correction for serial dependence and heteroskedasticity

** W&W report estimates with the sign reversed

*** If entry is less than 0.05, then reject null hypothesis at the 5% level

Table 4: Sensitivity of RW Residuals Sum of Coefficients

	Foreign = ForPurch_Fdebtshr				Foreign = CHG12_FOI_AGTR_FDebtshr			
	No lags		With Lags		No lags		With Lags	
	'(1)	'(2)	'(3)	'(4)	'(5)	'(6)	'(7)	'(8)
Constant	1.15	-51.98	0.53	-6.53	9.28	7.51	2.12	-2.35
SE	1.83	11.95	0.93	8.15	1.73	12.73	1.04	8.66
RWresiduals	--	--	0.87	0.87	--	--	0.83	0.83
SE			0.04	0.04			0.04	0.05
Foreign	6.74	-40.02	-38.50	-9.39	-683.64	-1003.52	-175.55	-160.80
SE	83.51	107.81	42.81	60.63	101.21	131.25	64.05	90.42
GDP_Growth_Volat	--	3.52	--	0.05	--	4.42	--	0.34
SE		2.77		1.56		2.41		1.48
MOVE	--	0.48	--	-0.01	--	0.46	--	0.05
SE		0.12		0.09		0.10		0.08
S&PVolatility	--	-0.50	--	0.05	--	-0.91	--	-0.09
SE		0.30		0.20		0.23		0.16
EuroVolatility	--	-0.15	--	0.05	--	-0.24	--	0.02
SE		0.09		0.06		0.08		0.06
CoreVolatility	--	202.66	--	26.31	--	-148.00	--	-12.28
SE		92.36		53.78		90.14		56.30
Adj R2	-0.01	0.10	0.75	0.77	0.19	0.32	0.76	0.78
SER	19.15	18.14	9.62	9.12	17.16	15.77	9.45	8.91
Tests of Assumptions***								
Normality	0.01	0.00	0.08	0.01	0.34	0.01	0.08	0.02
Serial Independence	0.00	0.00	0.22	0.54	0.00	0.00	0.40	0.37
Homoskedasticity	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.02

* Estimated standard errors WITHOUT correction for serial dependence and heteroskedasticity

** W&W report estimates with the sign reversed

*** If entry is less than 0.05, then reject null hypothesis at the 5% level

Table 5: Summary of Estimation Results for U.S. 10-year Treasury Yield

Specification	Measure of F	Ceteris Paribus -- Direct -- Long Run Effects						Properties of Residuals			Forecasts	
		Rff		R10euro		F		Serial Independ.	Normality	Constant Variance	RMSFE* basis points	RSMFE relative to RSMFE(AR(2))
		Estimate	std. error	Estimate	std. error	Estimate	std. error					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)					
Single Equation OLS	FRBNY	0.020	0.139	0.174	0.140	-0.216	0.113	Accept	Reject	Accept	66	0.60
	Δ_{12} FRBNY	-0.059	0.229	0.363	0.119	0.560	0.504	Accept	Accept	Accept	179	1.64
	Foreign Official	0.059	0.144	0.185	0.146	-0.262	0.149	Accept	Accept	Accept	92	0.84
	Δ_{12} Foreign Official	0.023	0.186	0.368	0.105	0.507	0.379	Accept	Reject	Accept	143	1.31
	Foreign Total	0.108	0.128	0.111	0.152	-0.224	0.105	Accept	Accept	Accept	91	0.83
	Δ_{12} Foreign Total	0.010	0.201	0.347	0.114	0.154	0.239	Accept	Reject	Accept	139	1.28
Johansen FIML	FRBNY	0.204	0.072	0.272	0.079	-0.156	0.063	Accept	Reject	Accept	79	0.72
	Δ_{12} FRBNY	0.214	0.086	0.426	0.051	0.014	0.175	Accept	Reject	Accept	134	1.23
	Foreign Official	0.210	0.074	0.313	0.077	-0.153	0.081	Accept	Reject	Accept	96	0.88
	Δ_{12} Foreign Official	0.237	0.085	0.427	0.050	0.154	0.163	Accept	Reject	Accept	134	1.22
	Foreign Total	0.216	0.068	0.243	0.081	-0.162	0.059	Accept	Reject	Accept	98	0.90
	Δ_{12} Foreign Total	0.201	0.089	0.427	0.050	-0.031	0.106	Accept	Reject	Accept	130	1.19
Koopman FIML	FRBNY	0.002	0.068	0.475	0.070	-0.157	0.093	Reject	Accept	Reject	38	0.35
	Δ_{12} FRBNY	-0.015	0.076	0.597	0.076	-0.173	0.096	Reject	Accept	Reject	91	0.84
	Foreign Official	-0.005	0.069	0.497	0.072	-0.048	0.120	Reject	Accept	Reject	63	0.58
	Δ_{12} Foreign Official	0.006	0.076	0.596	0.075	-0.040	0.109	Reject	Accept	Reject	102	0.93
	Foreign Total	-0.020	0.069	0.446	0.072	0.016	0.075	Reject	Accept	Reject	80	0.73
	Δ_{12} Foreign Total	0.005	0.076	0.598	0.075	0.008	0.070	Reject	Accept	Reject	105	0.96
Time Series	AR(2)	--	--	--	--	--	--	Accept	Accept	Accept	109	1.00
	Random Walk+drift	--	--	--	--	--	--	Reject	Accept	Reject	266	2.44
	VAR(2)	--	--	--	--	--	--	Accept	Reject	Accept	110	1.01
	Common Factors without exogenous variables	--	--	--	--	--	--	Reject	Reject	Reject	600	5.50

* S-step ahead simulations from July 2007 to December 2008