Maximum Non-extensive Entropy Block Bootstrap

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Overview

- In this paper, we propose a correct maximum entropy-based resampling scheme — the Maximum Entropy Block Bootstrap (MEBB) — valid for non-stationary data.
- We illustrate the performance of our procedure on the unit root test — Dickey-Fuller test.
- We extend the MEBB by considering the generalized notion of entropy and introduce the Maximum non-extensive Entropy Block Bootstrap which allows for fat tails.
- We provide a Monte Carlo simulation to show the properties of the MnEBB.

Literature Review I

- Since Efron (1979) for *identically and independent distributed* (iid) data, the bootstrap method has been extended to handle more complex data structures.
- **Time-series** data may fail to satisfy the iid-ness

1. The data distribution might change over time
2. The observations are mutually dependent – memory in the process

We focus on the case 2: Mutually dependent observations:

- Künsch (1989) proposes the non-parametric *block* bootstrap technique which involves re-sampling blocks of data rather than individual observations.
- Buhlmann (1997) develops a parametric alternative usually called *sieve* bootstrap which circumvents the dependence structure in the data by first fitting an AR(p) process – where p grows with the sample size T – and then resampling from supposedly iid residuals.
- For other resampling schemes, see Politis, Romano, and Wolf (1999) for an overview.

It works well for the time series when the dependence dies out over time...
...yet, in economics and finance, we frequently study relationships that involve integrated processes, mostly $I(1)$.

Literature Review II

- The problem of interest is in bootstrapping test procedures to assess the unit root hypothesis.
- Two approaches to chose from:

1. Resample the first differences and treat the dependence in differences, see, e.g., Palm, Smeekes, and Urbain (2007) for a review
2. Resample directly from the original non-differenced data

- With respect to the case 2:
  - The major benefit: we can ignore the dependence structure in the first differences; though still uncommon
  - The only valid (in the sense of *consistency* of the bootstrap procedure) method proposed in the statistical literature is the Continuous Path Block Bootstrap of Paparoditis and Politis (2001)
  - Theoretical validation of the block bootstrap consistency is provided by Phillips (2010), who uses a different algorithm to Paparoditis and Politis (2001) – Phillips method is inconsistent under the alternative
  - Alternative proposed by Vinod and López-de Lacalle (2009) – Maximum Entropy Bootstrap – based on maximum entropy principle and perfect rank correlation – significant flaws.

→ **Maximum (Non-)extensive Entropy Block Bootstrap**

Maximum Entropy Bootstrap (MEB)

MEB, introduced by Vinod and López-de Lacalle (2009), is a fully non-parametric bootstrap technique dealing with inference for time series with any persistence.

- The maximum information entropy principle: the probability distribution to find a system in a given state conditional on the prior is such that the information entropy is maximized

Let $f(x)$ be a probability density function to find the system in a state x, then the Shannon entropy, H_S, is defined as:

$$H_S = E\left[-\log\left(f(x)\right)\right], \quad (1)$$

and the probability distribution function such that it satisfies the following optimization problem

$$f = \arg\max_{f'} E\left[-\log\left(f'(x)\right)\right]. \quad (2)$$

1. Finite and bounded support – the *uniform distribution*.
2. Half-infinite support and finite means – the *exponential distribution*.
3. Infinite support and given mean and standard deviation – the *normal distribution*.

The MEB Algorithm

Let us consider $X = x_1, \ldots, x_T \rightarrow$ order statistics as $x_{(t)}$. Support of the order statistics $x_{(t)} \in \left[x_{(0)}, x_{(T+1)}\right]$. We define the midpoints z_t as

$$z_0 = x_{(0)}\ ,\ z_t = \frac{1}{2}\left(x_{(t)} + x_{(t+1)}\right)\ ,\ t \in \{1, \ldots, T-1\}\ ,\ z_T = x_{(T+1)}\ .$$

Using the midpoints, we define T half-open intervals $I_t = (z_{t-1}, z_t]$ around each observation.

The maximum entropy density function is solution to (2) with:

a. The **mass preserving constraint** imposed on the density function states that, on average, $1/T$ of the mass of the density function lies in each of the intervals I_t.

b. The **mean preserving constraint** states that

$$\sum_{t=1}^{T} x_t = \sum_{t=1}^{T} x_{(t)} = \sum_{t=1}^{T} m_t\ ,$$

where m_t is the mean of f over the interval I_t.

MEB = Max. Entropy+Mass Preserving+Mean Preserving+Perfect Rank Correlation

The MEB Algorithm – cont'd

To create a single realization $x_t \to x_t^*$, the MEB is based on the following algorithm:

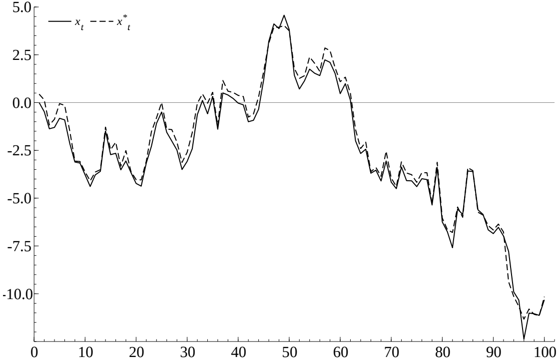
1. Order statistics $x_{(t)}$ based on x_t and define the support of the order statistics $\left[x_{(0)}, x_{(T+1)}\right]$ with $x_{(0)} = x_{(1)} - d_{trm}$ and $x_{(T+1)} = x_{(T)} + d_{trm}$, with $d_{trm} = E_{trim}\left[\left|x_{(t)} - x_{(t-1)}\right|\right]$.
2. We define a $T \times 2$ sorting matrix, S_1, and place the index set $t = \{1, \ldots, T\}$ in the first column and the observed time series x_t in the second column.
3. We sort the matrix S_1 with respect to the second column, x_t, and define the order statistics $x_{(t)}$. We then define the midpoints z_t and the half-open intervals I_t.
4. We draw T uniform pseudo-random numbers $p_s \sim U[0, 1]$, with $s \in \{1, \ldots, T\}$ and assign the range $R_t = \left(\frac{t}{T}, \frac{t+1}{T}\right]$ for $t \in \{0, T-1\}$ wherein each p_s falls.
5. We match each R_t with I_t and using the density function defined above, we draw the new set $\tilde{x}_t^*$.
6. We define a corresponding $T \times 2$ sorting matrix S_2, analogous to S_1. We sort the T elements $\tilde{x}_t^*$ in an increasing order of the magnitude to form the ordering statistics $x_{(t)}^*$.
7. We replace the second column of S_1, the order statistics $x_{(t)}$, by the second column of S_2, the order statistics $x_{(t)}^*$ of the newly generated set. We sort the $x_{(t)}^*$ based on the first column of S_1, and thus recover x_t^*. The set x_t^* represents a resampled set of observations x_t.

The Maximum Entropy Bootstrap to Assess the Unit Root Hypothesis

Let us consider a standard AR(1) process frequently used in the econometric analysis

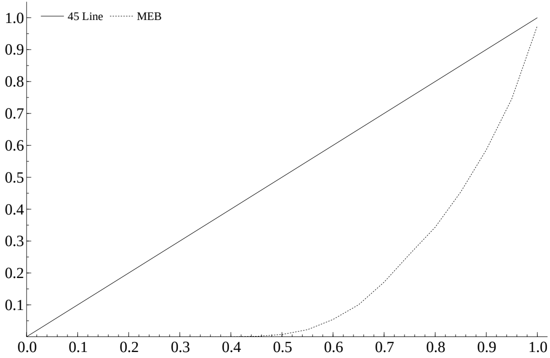
$$y_t = \rho \cdot y_{t-1} + \varepsilon_t \, , \tag{3}$$

where $\varepsilon_t \sim$ i.i.d.$N(0,1)$. In this paper we focus on the unit root case, i.e., $\rho = 1$. We consider 100 points and $y_0 = 0$.



Size of the Test to Reject the Null

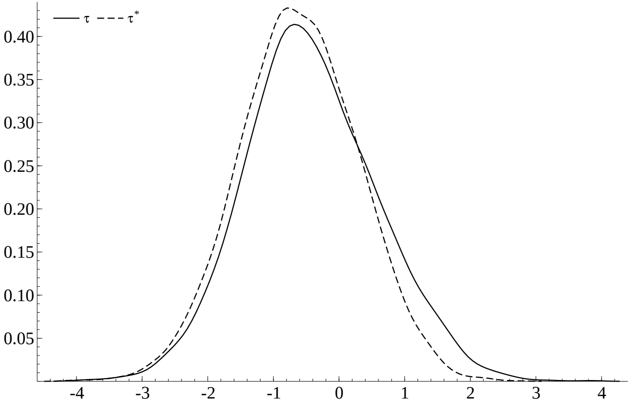
We employ the MEB to assess the rejection frequency of the test with null hypothesis $\rho = 1$ against the alternative of $\rho < 1$: The empirical rejection frequencies to assess the quantile of the bootstrap distribution of the t-statistic.



We perform a 1000 replications of the true data generating process and for each replication, we create 299 bootstrap samples. We consider $T = 100$ and initial point to be set at $y_0 = 0$.

Why the Maximum Entropy Bootstrap Fails?

- **Reason to Fail 1:** The distribution of the MEB statistic is on average more dispersed than that of the statistic itself.
- **Assessment 1:** The figure reports the distribution of the original *t*-statistic and one corresponding bootstrapped replication under $H_0 : \rho = 1$, using 1000 replications under the null.



Why the Maximum Entropy Bootstrap Fails?

- **Reason to Fail 2:** For each replication, the bootstrap statistics are positively correlated with the true statistic and the bootstrap does not provide an independent draw of a sample.
- **Assessment 2:** We estimate the correlation between the *t*-statistics of the data generating process, τ_i, and the *t*-statistics of the MEB, τ_i^*, using the OLS:

$$\tau_i^* = -\underset{(0.00326)}{0.2452} + \underset{(0.00299)}{0.8728}\,\tau_i \qquad i = 1, \ldots, 1000\,. \tag{4}$$

The adjusted $R^2 = 0.9445$ suggests that the *t*-statistic of the MEB is too close to the data generating process.

- The strong positive correlation between the two *t*-statistics suggests that the MEB is not a consistent method. Further, the failure is in the lack of variation in the MEB sample. The MEB in fact mimics the data generating process too closely

MEB = Max. Entropy+Mass Preserving+Mean Preserving+PerfectRankCorrelation

Maximum Entropy Block Bootstrap

We introduce the Maximum Entropy Block Bootstrap (MEBB), which preserves the perfect rank correlation *locally* and is free of tail trimmings.

The MEBB Algorithm

Step A

We choose the block length $\ell < T$ and let $i_0, i_1, \ldots, i_{k-1}$ i.i.d. uniform random numbers on the set $[1, 2, \ldots, T - \ell]$ where $k = \lfloor T/\ell \rfloor$, (number of blocks).

Step B

For each i_j, with $j = 0, \ldots, k-1$, we get the subset of the original time series $X^{(j)} = \left\{ x_{i_j}, x_{i_j+1}, \ldots, x_{i_j+\ell-1} \right\}$ and apply the Paparoditis and Politis (2001) demeaning.

Step C

We apply the MEB algorithm corresponding to Steps 1-7 in Section 2 for each subset $X^{(j)}$ separately and generate $X^{*(j)} = \left\{ x^*_{i_j}, x^*_{i_j+1}, \ldots, x^*_{i_j+\ell-1} \right\}$.

Step D

We recover the bootstrapped sample path by sewing the $X^{*(1)}, \ldots, X^{*(k)}$ such that $x^*_{i_{j+1}} - x^*_{i_j+\ell-1}$ is set in a way to correspond to the difference $x_{i_{j+1}} - x_{i_j+\ell-1}$.

Step E

If the length of the bootstrapped sample path is exceeding T, we take the first T values.

cont'd

- We omit the trimming: Step 1 in the MEB algorithm is modified as follows:

Step 1*
We create an order statistics $x_{(t)}$ based on the empirical data set x_t and define the support of the empirical data to be $[-\infty, \infty]$. The constrained solution for the maximum entropy distribution on the half-open interval $[0, \infty)$ is given by $f = \lambda e^{-\lambda x}$ with mass at $1/\lambda$. Therefore, for the intervals $I_1 \equiv (-\infty, z_1]$ and $I_T \equiv [z_T, \infty)$, respectively, is given by

$$f_{I_1}(x) = \beta_1 \lambda_1 e^{-\lambda_1 (z_1 - x)}, \quad x \in I_1, \quad m_1 = \frac{3x_{(1)}}{4} + \frac{x_{(2)}}{4} \tag{5}$$

$$f_{I_T}(x) = \beta_T \lambda_T e^{-\lambda_T (x - z_T)}, \quad x \in I_T, \quad m_T = \frac{x_{(T-1)}}{4} + \frac{3x_{(T)}}{4}, \tag{6}$$

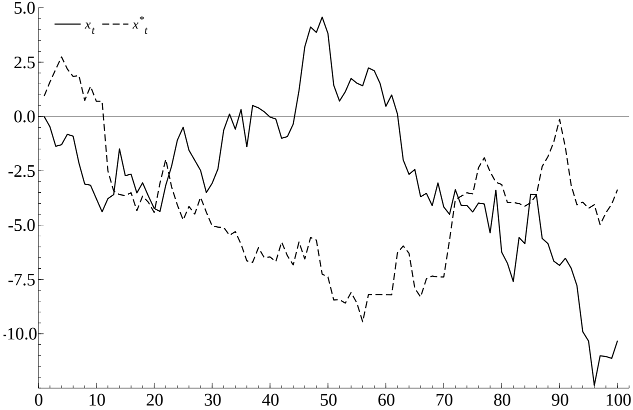
where the parameters λ_1 and λ_T are set such that the mean preserving constraints imposed on m_1 and m_T are satisfied.

- Finally, Steps 2-7 of the MEBB algorithm for a given subset X^* are the same as for the MEB.

Such a modified algorithm — MEBB — preserves the perfect rank correlation locally and uses the proper form of the tails in the maximum entropy distribution function.

A Numerical Illustration

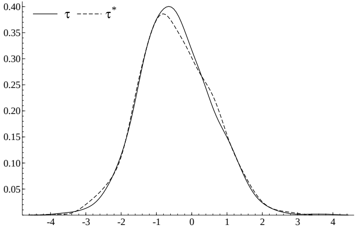
We report a bootstrap sample path based on the new MEBB algorithm.



The sample path x_t and the replicated path x_t^* by the MEBB. The true data generating process is given as $y_t = y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$, $y_0 = 0$, and $T = 100$.

Does the Correction Work?

Reason to Fail 1: The right panel depicts the distribution of the original t-statistic and one corresponding bootstrapped replication under $H_0: \rho = 1$.



Reason to Fail 2: Indeed, a regression similar to the one above gives

$$\tau_i^* = \underset{(0.0354)}{-0.4541} - \underset{(0.0323)}{0.005663}\,\tau_i \qquad i = 1, \dots, 1000 \quad , \tag{7}$$

with the adjusted $R^2 = 0.00097$.

No problem detected!

Monte Carlo Simulation Study

Focus: bootstrapping the t-statistic distribution for the AR(1) coefficient.
Benchmark: the standard residuals-based bootstrap (RB) and continuous path block bootstrap(CPBB)
The data generating process:

$$y_{m,t} = \rho_0 y_{m,t-1} + \eta_{m,t}$$
$$y_{m,0} = 0 \quad t = 1, \ldots, T\,,$$

with m denoting a realizations of the sample path, and where we let $\rho_0 = 1$ for different sample lengths $T = \{50, 100, 300\}$ in order to assess the size and $\rho_0 = \{0.99, 0.95, 0.90, 0.80, 0.70, 0.60, 0.5\}$ to assess power. Moreover, we generate the $\{\eta_{m,t}\}$ series allowing for "progressive" fat-tails: $\mathcal{N}(0,1)$, $\mathcal{T}(5)$, $\mathcal{T}(3)$.
Test statistic: We fit an AR(1) model and compute the t-statistic

$$t_m = \frac{\hat{\rho}_m - 1}{\hat{\sigma}_{\eta_m}(\sum_{t=1}^{T} y_{m,t-1}^2)^{-1/2}}, \quad \text{for testing} \quad \begin{cases} H_0: & \rho = 1 \\ H_1: & |\rho| < 1 \end{cases} \tag{8}$$

where $\hat{\rho}_m = \left(\sum_{t=1}^{T} y_{m,t-1}^2\right)^{-1} \left(\sum_{t=1}^{T} y_{m,t-1} y_{m,t}\right)$ is the least squares estimator of the autoregressive coefficient and $\hat{\sigma}_{\eta_m} = (T-1)^{-1/2}(\sum_{t=1}^{T} \hat{\eta}_{m,t}^2)^{1/2}$ is the residuals standard deviation.

Benchmark Bootstrap Methods

Residuals Bootstrap

$$y^*_{b,m,0} = y_{m,0}$$
$$y^*_{b,m,t} = \rho_0 y^*_{b,m,t-1} + \eta^*_{b,m,t} \quad t = 1, \dots, T$$

where $\eta^*_{b,m,t}$ are drawn from the centered residuals $\left\{\hat{\eta}_{m,t} - \frac{1}{T}\sum_{t=1}^{T}\hat{\eta}_{m,t}\right\}_{t=1}^{T}$ obtained from the residuals of the regression of $y_{b,m,t}$ on its first lag.

Continuous Path Block Bootstrap

Following Paparoditis and Politis (2001):

1. Compute the centered residuals

$$\hat{u}_{m,t} = y_{m,t} - y_{m,t-1} - \frac{1}{T-1}\sum_{t=1}^{T}(y_{m,t} - y_{m,t-1}) \text{ with } \tilde{y}_{m,t} = \begin{cases} y_{m,1} & t = 1 \\ y_{m,1} + \sum_{j=2}^{t}\hat{u}_{m,j} & t = 2, \dots, T. \end{cases}$$

2. Choose the block length $\ell < T$ and let $i_0, i_1, \dots, i_{k-1}$ i.i.d. uniform random numbers on the set $[1, 2, \dots, T - \ell]$, where $k = \lfloor T/\ell \rfloor$ (number of blocks).
3. Build the bootstrapped series of length $l = k \cdot \ell$ as

$$y^*_{b,m,j} y^*_{b,m,rs+j} = y_{m,1} + [\tilde{y}_{i_0+j-1} - \tilde{y}_{i_0}] \text{ first block }, = y^*_{b,m,r\ell} + [\tilde{y}_{i_m+j} - \tilde{y}_{i_r}]\,(r+1)^{\text{th}} \text{ block}$$

for $j = 1, \dots, \ell$ and $r = 1, \dots, k-1$.

Simulation Results

We then use $\{y^*_{b,m,t}\}_{t=1}^T$ to compute the bootstrapped counterpart of (8),

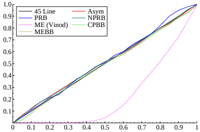
$$t^*_{m,b} = \frac{\hat{\rho}^*_{m,b} - \rho_0}{\hat{\sigma}^*_{\eta_{m,b}} (\sum_{t=1}^T {y^*_{m,b,t-1}}^2)^{-1/2}} \qquad b = 1, \ldots, B \tag{9}$$

and we select the α-quantile $t^*_m(\alpha)$ of the distribution of the bootstrapped statistic (at the m^{th} iteration) such that $B^{-1}\sum_{b=1}^B I(t^*_{m,b} \leq t^*_m(\alpha)) \approx \alpha$. The empirical rejection frequencies are computed as

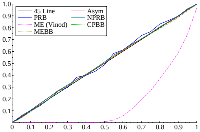
$$\frac{1}{M}\sum_{m=1}^M I(t_m \leq t^*_m(\alpha)), \tag{10}$$

being a one-sided test with rejection to the left.
To compare the size of the different approaches, we compute (10) for $\alpha \in [0.01,$ $0.025, 0.05, 0.10, 0.20, \ldots, 1]$. and we plot the calculated values against α.

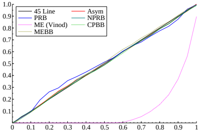
The Size



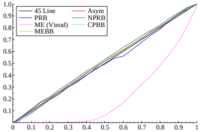
$T = 50\ \mathcal{N}(0,1)$



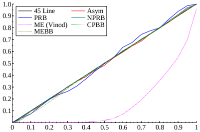
$T = 100\ \mathcal{N}(0,1)$



$T = 300\ \mathcal{N}(0,1)$



$T = 50\ \mathcal{T}(3)$



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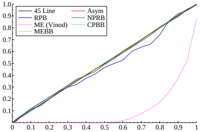

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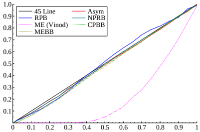

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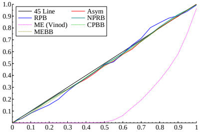

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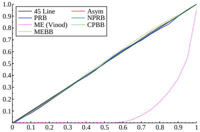
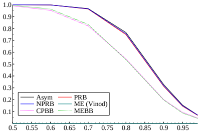
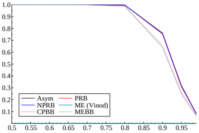


$T = 300\ \mathcal{T}(5)$

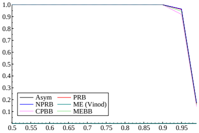
The Power



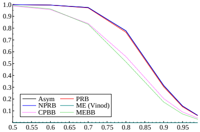
$T = 50\ \mathcal{N}(0,1)$



$T = 100\ \mathcal{N}(0,1)$



$T = 300\ \mathcal{N}(0,1)$



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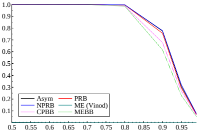

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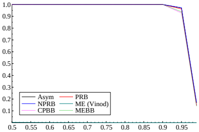

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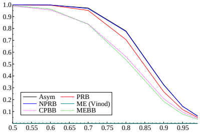

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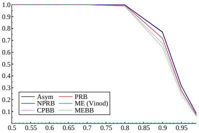

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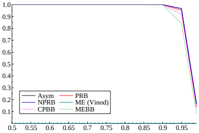


$T = 300\ \mathcal{T}(5)$

The Maximum *non-extensive* Entropy Block Bootstrap

The key concept of our framework is the generalized Tsallis (1988) entropy:

$$H_q = -\frac{1}{1-q}\left(1-\sum_{i=1}^{N}(p_i)^q\right) \rightarrow H_q = -\frac{1}{1-q}\left(1-\int dx\,(p(x))^q\right),$$

where the parameter q governs the non-extensiveness of the system. The Tsallis entropy converges to the Shannon entropy in the limit when $q \rightarrow 1$.
For a given q, the density function f_q is given as

$$f_q(x) = \frac{[1-\beta(1-q)x]^{1/(1-q)}}{Z_q},\ Z_q = \int dx\,[1-\beta(1-q)x]^{1/(1-q)}. \qquad (11)$$

Some Properties:

- The non-extensiveness: $H_q(A+B) = H_q(A) + H_q(B) + (1-q)H_q(A)H_q(B)$.
- The limit of this distribution function is exponential function for $q \rightarrow 1$.
- For $q \in (1,2)$: power law behavior & fat tails.
- For $q < 1$: non-standard behavior – f is infinite over the semi-definite interval and thus it requires a normalization by an infinite normalization factor.
- As $q \rightarrow 0$, we get uniform distribution, or, $f_{q\rightarrow 0}(x) = c/\infty$, where c does not depend on x. For $q > 5/3$, we get a distribution with non-existing second moment, or $E\left[x^2\right] = \infty$.
- At $q = 2$, the first moment cease to exist.
- We consider $q \in [1, 5/3)$.

The MnEBB Algorithm

Considering non-extensive entropy:

$$f_q(x) = \alpha_q (1 - \beta (1-q) x)^{\frac{1}{1-q}}, \quad x \in I_1 \quad m_1 = \frac{3x_{(1)}}{4} + \frac{x_{(2)}}{4} \tag{12}$$

$$\alpha_q : \quad \int_{I_1} x dx f_q(x) = m_1 \tag{13}$$

$$f_q(x) = \frac{1}{z_k - z_{k-1}}, \quad x \in I_k|_{k=2,\dots,T-1} \quad m_k = \frac{x_{(k-1)}}{4} + \frac{x_{(k)}}{2} + \frac{x_{(k+1)}}{4} \tag{14}$$

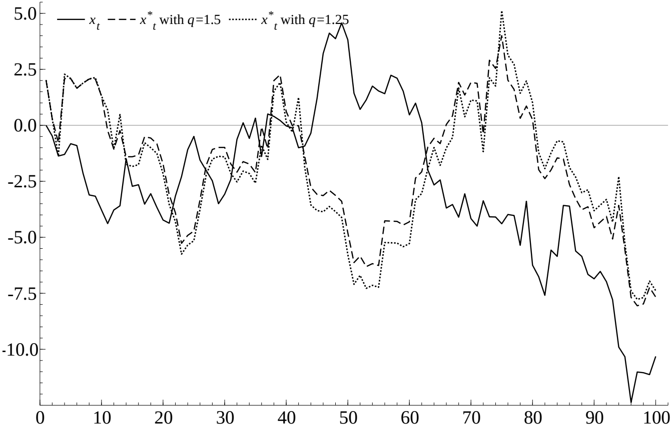
$$f_q(x) = \omega_q (1 - \beta (1-q) x)^{\frac{1}{1-q}}, \quad x \in I_T \quad m_T = \frac{x_{(T-1)}}{4} + \frac{3x_{(T)}}{4} \tag{15}$$

$$\omega_q : \quad \int_{I_T} x dx f_q(x) = m_T \tag{16}$$

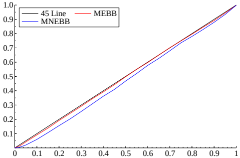
- The remaining structure of the MnEBB algorithm is just as defined for the MEBB.
- The choice of $q > 1$ suggests using the distribution with fatter tails than implied by the standard entropy.

Illustration of the MnEBB

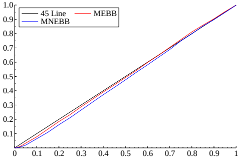
The figure reports the sample path x_t and the replicated path x_t^* by the MnEBB with $q = 1.25$ and $q = 1.5$.



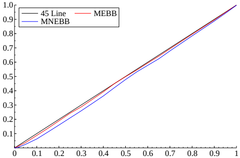
Simulation Results – The Size



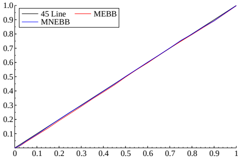
$T = 100\ \mathcal{N}(0,1)\ q = 1.25$



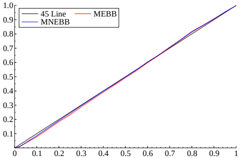
$T = 100\ \mathcal{T}(3)\ q = 1.25$



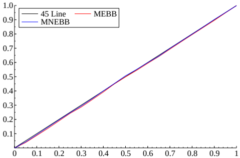
$T = 100\ \mathcal{T}(5)\ q = 1.25$



$T = 100\ \mathcal{N}(0,1)\ q = 1.5$

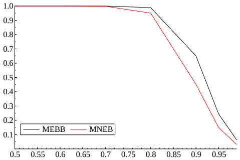


$T = 100\ \mathcal{T}(3)\ q = 1.5$

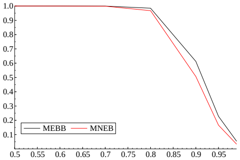


$T = 100\ \mathcal{T}(5)\ q = 1.5$

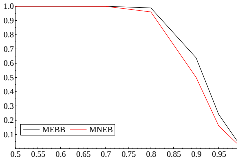
Simulation Results – The Power



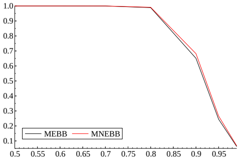
$T = 100$ $\mathcal{N}(0,1)$ $q = 1.25$



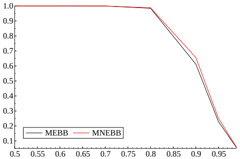
$T = 100$ $\mathcal{T}(3)$ $q = 1.25$



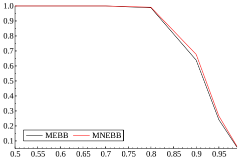
$T = 100$ $\mathcal{T}(5)$ $q = 1.25$



$T = 100$ $\mathcal{N}(0,1)$ $q = 1.5$



$T = 100$ $\mathcal{T}(3)$ $q = 1.5$



$T = 100$ $\mathcal{T}(5)$ $q = 1.5$

Conclusion and Future Work

- We proposed the Maximum Entropy Block Bootstrap, a fully non-parametric bootstrap procedure, to sample directly the time series with a general persistence structure.
- Our procedure: the maximum entropy principle and preserving locally the rank correlation between the true data generating process and the bootstrap draws.
- The unit root test suggests that our procedure performs well.
- We generalized the MEBB to the non-extensive entropy and introduce the Maximum non-extensive Entropy Bootstrap, which allows for inclusion of fat tails and power-law behavior.
- This generalized procedure outperforms the Maximum Entropy Bootstrap for large values of the non-extensiveness even when the underlying data generating process is the normal distribution.
- Future developments.
    - To derive the limiting theory of the MEBB and MnEBB methods proposed in this paper.
    - To extend the proposed procedure to alternative non-stationary frameworks such as co-integration analysis.
    - To connect the non-extensiveness with the Hurt exponent.

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