A pairwise approach to model and forecast a large set of disaggregates with common trends

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- **General Objective:** Model and forecast all the components of a macro or business aggregate.
- **Main contribution**: cases with large number of components, when multivariate approaches are not feasible.

Motivation:

1. **Disaggregating is relevant**
   - **Own interest** for economic and business decision making.
   - **Diagnosis and fore. the agg:** increasing agreement that deeper knowledge of the components may lead to better understanding of the agg. (Espasa et.al, 2002; GG, 2004; HH, 2005, 2011; EA, 2007; EMB, 2013)
   - **Comparative analysis** (e.g relative prices)
2. **PW strategy** (EMB, 2013) as a procedure to deal with the **Estimation uncertainty** *vs* **informational losses trade-off**, and the problem of non-pervasive common factors in DFM.

Outline

How the *pairwise* procedure (restricted to common trends) works...

1. Perform **Johansen cointegration tests** between all the $N(N-1)/2$ **pairs** (4950 for $N = 100$).
2. Find the largest subset in which every series is cointegrated with all the others (maximum *clique* - *fully cointegrated*).
3. Continue looking for the second largest and so on...
4. For ease of exposition just focus on the largest.

Asymptotic properties...

- The Pairwise strategy inherits the asy. props.of the Johansen test
- However; two specific features deserve special attention:
  1. Partial model estimation
  2. **Multiple testing;** are we inflating the false rejection probability? **NO**

Theorem (for pairs inside the FC set):

Given a set of n_1 pairwise cointegrated series, the prob. of finding the same result in all the $n_1(n_1-1)/2$ Johansen's tests $\to 1$ as $T \to \infty$

- $\Rightarrow$ $FWER = \alpha$. Even if the hypothesis of interest is the *universal* one ($FWER$), **p-values should not be corrected.**
- For the other pairs $\Rightarrow$ bounds for error probabilities already 'tolerable'

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Multiple testing: analytical results...

In summary:

Size corrections are not needed; multiple testing does not occur (FC series), or bounds provide already tolerable error probabilities"

Simulation experiments:

- Confirm the results for the pairs in the FC set
- Bounds are quite loose
- The PW strategy dominates a DFM alternative in terms of *Gauge* and *Potency*

A note about the forecasting equations

The effects of alternative normalizations for β

- The CI VAR remains the same if we change $\alpha\beta'$ by $\alpha H^{-1} H \beta'$. Let:

$$\alpha = \begin{bmatrix} 0 & 0 & 0 \\ -0.2 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & -0.2 \end{bmatrix}, \quad \beta' = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}, \quad W = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

- Define H in order to express CI rels as deviations form the aggregate

$$\begin{pmatrix} \textit{equivalence} \quad \textit{condition}; \\ \alpha^* = \tilde{\alpha}(I_r - \beta^*_{nf} W_{nf})^{-1} \end{pmatrix}, \quad \alpha H^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ -0.4 & -0.2 & -0.2 \\ -0.2 & -0.4 & -0.2 \\ -0.2 & -0.2 & -0.4 \end{bmatrix},$$

- **Nº of relevant CI rels.** in each equation (except for the first one) **increased** form 1 to 3.

The normalization may not be innocuous for the forecasting stage

- N fore single eq models including CI rels. and other regressors:

$$\Delta x_{i,t} = c + \alpha_{i1} CR_{1_i,t-1} + \ldots + \alpha_{ir} CR_{r_i,t-1} + other\ regressors + \epsilon_{i,t}$$

- N° of relevant CI rels. may change depending on β's normalization.
- $\Rightarrow$ Possible **'free' reduction in the number of parameters** $\Rightarrow$ 'free' estimation uncertainty reduction $\Rightarrow$ **forecasting accuracy improvement.**
- **Strategy**: try different normalizations and select over CI rels and the other regressors using *Autometrics*

Forecasting: exercise design

Regressors are selected using *Autometrics* in the following initial GUMs:

- Cointegrated series:

$$\Delta x_{i,t} = c + \alpha_{i1} CR_{1_i,t-1} + \ldots + \alpha_{ir} CR_{r_i,t-1} + \sum_{k_1=1}^{K_1} \phi_{k_1} \Delta x_{i,t-k_1} + \ldots$$
$$\sum_{k_2=1}^{K_2} \theta_{k_2} \Delta SubAgg_{i,t-k_2} + \sum_{k_3=1}^{K_3} \delta_{k_3} \Delta CPI_{t-k_3} + \sum_{i=1}^{11} \gamma_i S_{i,t} + \epsilon_{i,t}$$

- Rest of the series:

  Restricted such that the forecast for the sub-aggregate 'rest' is the one coming form a uni-equational model.

- In both cases forecasts are produced with and without IIS.

Forecasting results: RMSE ($\Delta_{12} log(CPI)$)

- Recursive forecasting 2011.1 - 2013.12, from $H = 1$ to $H = 12$ steps ahead.
- Baseline: univariate model for the CPI.
- Form Diebold-Mariano: PW dominates the baseline in horizons 4 to 8.
- Form Capistrán: considering jointly horizons 1 to 6 PW dominates the baseline (this is not the case when using all horizons, but few observations)

Conclusions

- The pairwise procedure allows to discover blocks of series that share a unique common trend in a large set of series.
- Statistical properties were studied analytically, and confirmed by Monte Carlo.
- A small samples correction procedure was designed and its properties studied by simulations.
- Comparison with DFM: PW clearly dominates when n_1 is not large.
- Forecasting accuracy may be improved by changing β's normalization.
- PW strategy may improve CI test power
- A strategy for dealing with outliers and breaks was designed.
- Application US CPI: The PW procedure improves forecasting accuracy wrt the baseline at least for some horizons.

Thanks for your attention!