Institute for
New Economic Thinking
AT THE OXFORD MARTIN SCHOOL

INTRODUCTORY MACRO-ECONOMETRICS LECTURE SLIDES

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Lecture 1 guide post

1. Macro-econometrics concerns analyses of data like inflation, unemployment and gross domestic product (GDP).
2. A substantial annual data base for the UK starting in 1860 allows graphs and analyses of its economic history.
3. The last 150 years have witnessed huge changes, especially living standards from technological, legal, medical and financial innovations, with consequential social and demographic shifts.
4. Macroeconomic data exhibit evolution with abrupt changes.
5. Our first focus is on wages and prices, which have risen roughly 700-fold and 100-fold respectively.
6. Purchasing power of wages (real wages) has increased about 7-fold, in line with average productivity.
7. That broadly matches a neo-classical model of firms equating the marginal revenue per worker with their marginal costs, but systematic departures between the data and the theory remain.
8. Trends and sudden shifts are common, so we first develop simple models of trends.

Imagine you are living in 1860–just over 150 years ago.

You work 65 hours a week for 1/5th of modern incomes;
you would be ill from numerous diseases,
and hungry much of the time;
living up to 10 to a room with little sanitation,
your children ill clad and barefoot,
many dying at birth or infancy;
often unemployed–when your family would starve;
on average you would die in your mid-40s.
that was in the then richest country in the world.

How did we get here from there?

What statistical tools are needed to study evidence in a world that has changed so dramatically?

Consider models and methods for investigating macro-economic data: seek to explain the empirical evidence.

Route map

(1) **Macroeconomic data: evolution with abrupt change**
(2) Taming trends and breaks
(3) Dependence between variables and over time
(4) Economic theory and statistical analysis: 2 key ingredients
(5) Modelling UK unemployment
(6) Modelling UK wages
(7) Modelling UK money demand
(8) Modelling UK prices
(9) Conclusion

Huge changes over 1860–2011 (during the recession):
will look at the ***time series*** of the relevant variables.

- Nominal wages (W) rose more than 680 **fold**:
  £1 per week in 1860 becomes £680 by 2011.
  That is a **68,000%** increase in 150 years.
- Prices (P) rose more than 98 fold:
  £1 per item in 1860 costs £98 in 2011–
- **so real wages (W/P) rose almost 7 fold**.
- Industrial composition, technology, wealth distribution, laws, education, social security, pensions, health care, longevity, roles of women, social mores, housing tenure, etc., also all changed vastly.

Change is the norm: both evolution and sudden–
and is central to macroeconomic data analysis.
Financial crisis and 'Great Recession' just latest examples.

UK Wages and prices over 1860–2011

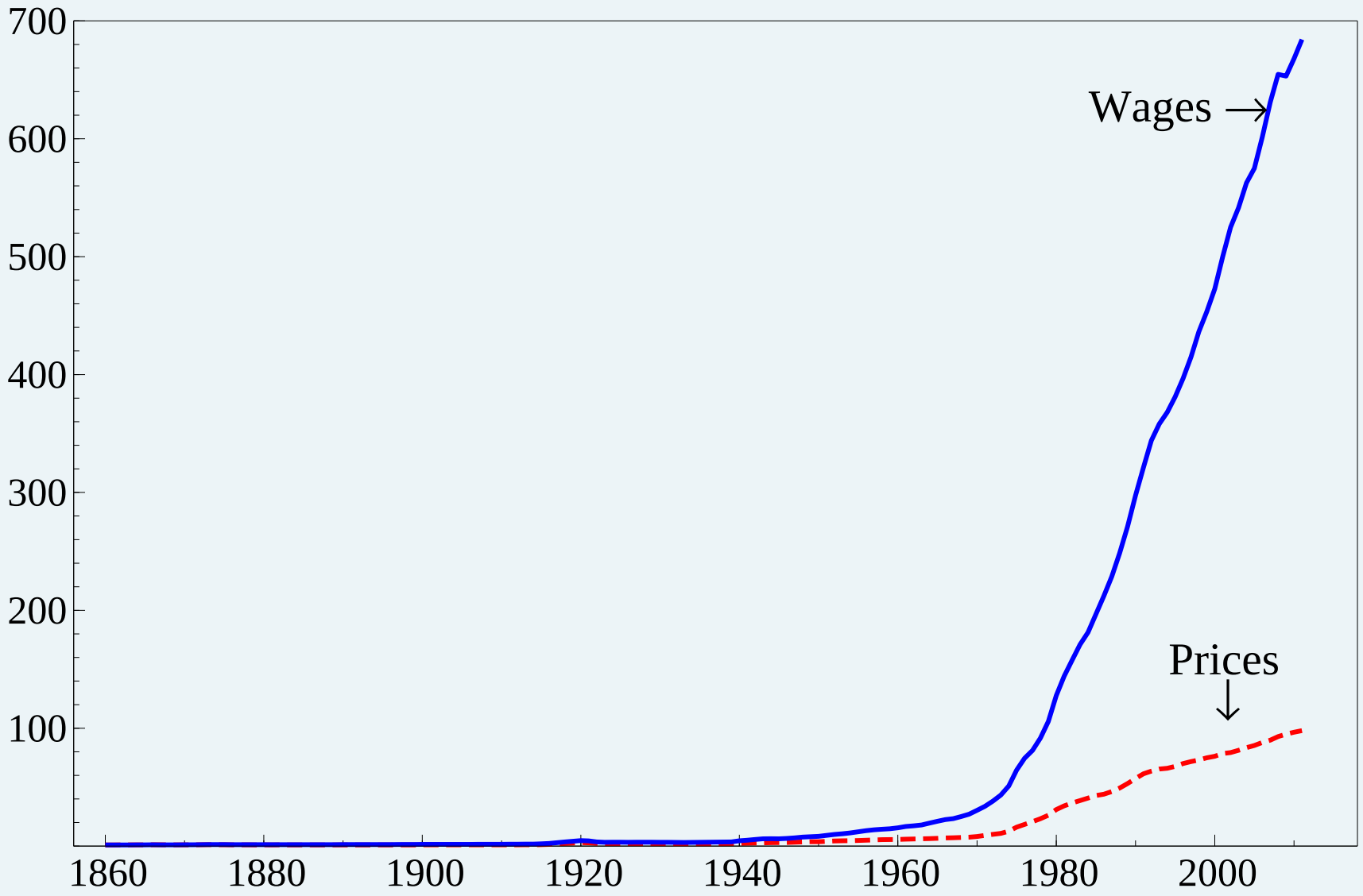


Figure: UK Wages and prices over 1860–2011.

Evidence from the graph

Show time on the horizonal axis, from 1860 to 2011;

values of the variables at each point on the vertical axis.

Measured as **indices**, from bases of unity in 1860.

Denote a generic time-series process by $\{x_t\}$.

Most of increases in $\{W_t\}$ and $\{P_t\}$ come post-1975.

Early period looks unchanging–but an artefact of later increases as graph shows **absolute** changes.

Rising from 1 to 2 is 100% increase–
but is dwarfed by 100 to 120, just a 20% rise.

Solution: as W_t, P_t must both be positive
use ***logarithms*** of wages ($\log(W_t) = w_t$) and prices ($\log(P_t) = p_t$)
to measure *relative* changes.

Figure 8 shows the relative changes in logs.

Log wages and prices over 1860–2011

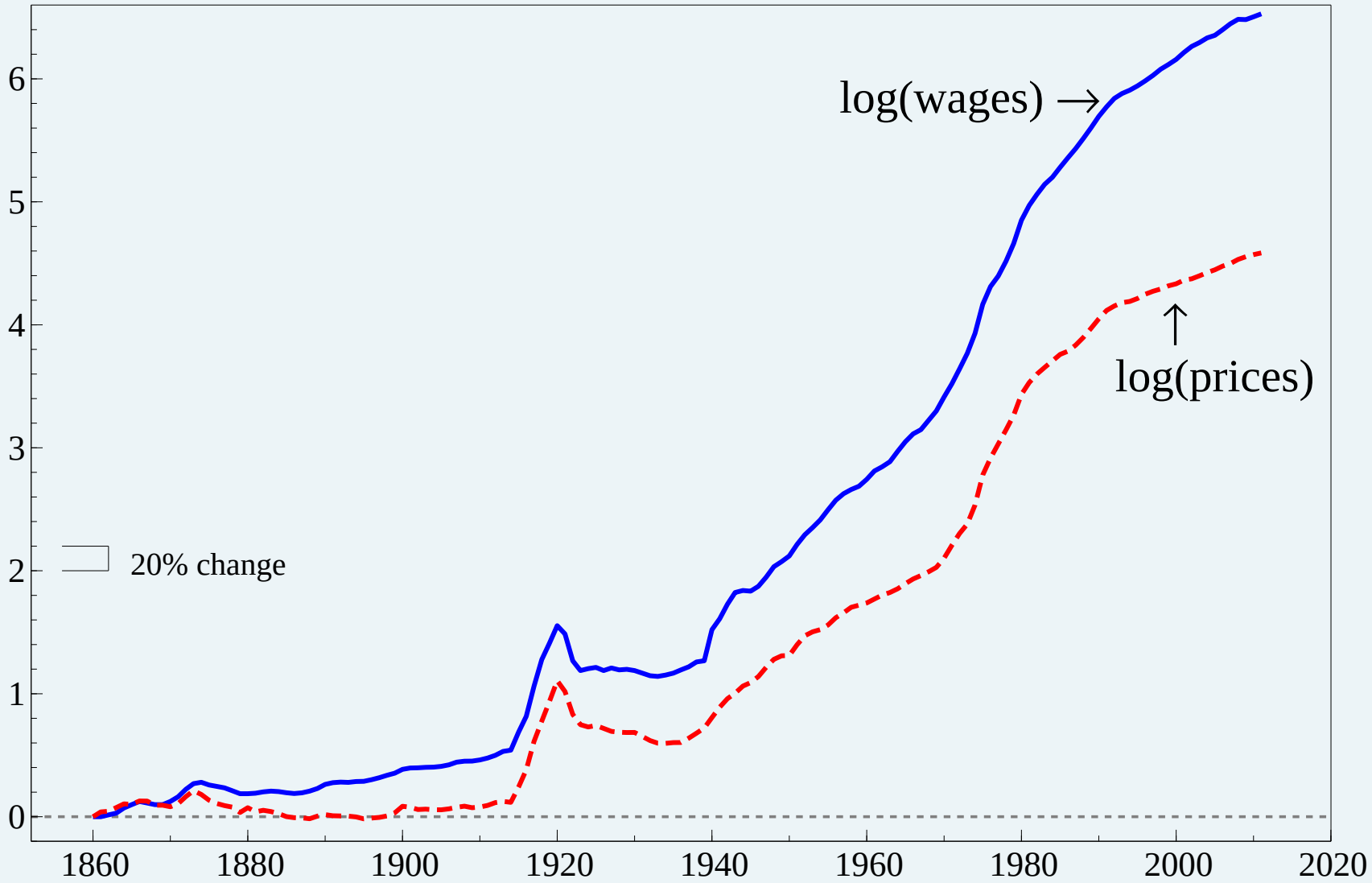


Figure: UK log wages and log prices over 1860–2011.

Properties of logarithmic transformations

Distance between successive 'tick' marks is a 20% change.

Remember a basic result in calculus:

$$\frac{\partial \log x}{\partial x} = \frac{1}{x} \tag{1}$$

so changes in logs are relative to their level.

If $x = 1$, one unit increase in x to 2 is 1/1 which is 100%.
When $x = 100$, 1 unit increase in x to 101 is 1/100, or 1%.
Need to go from $x = 100$ to $x = 200$ for a 100% change.

But equation **(1) only holds for small changes:**
$\log 1 = 0$ to $\log 1.5 = 0.4055$ is just over 40%.

Shops record 1 to 1.5 as 50% whereas 1.5 to 1 is a 33% reduction, so not a symmetric change.

Logs show symmetric changes:
the reverse of $\log 1.5$ to $\log 1$ is a 40% fall as well.

Evidence from the log graph

Early period far from unchanging: quite volatile between 1914 and 1945, with **large falls in interwar period.**

Overall, wages have grown much faster than prices.
But neither shows **constant growth** over the 150 years.

Can fit a trend by regression to describe average growth, as in Figure 11.
But 1 regression line for w_t is hopeless.
Still, let's fit that trend line live for w_t using OxMetrics
The squares of the projections are what least squares minimizes.

Moving a regression line up or down, or altering its slope, will produce a larger sum of squared projections.

Hard work to calculate by hand–trivial on a computer.

Log wages and log prices with one trend line for log wages

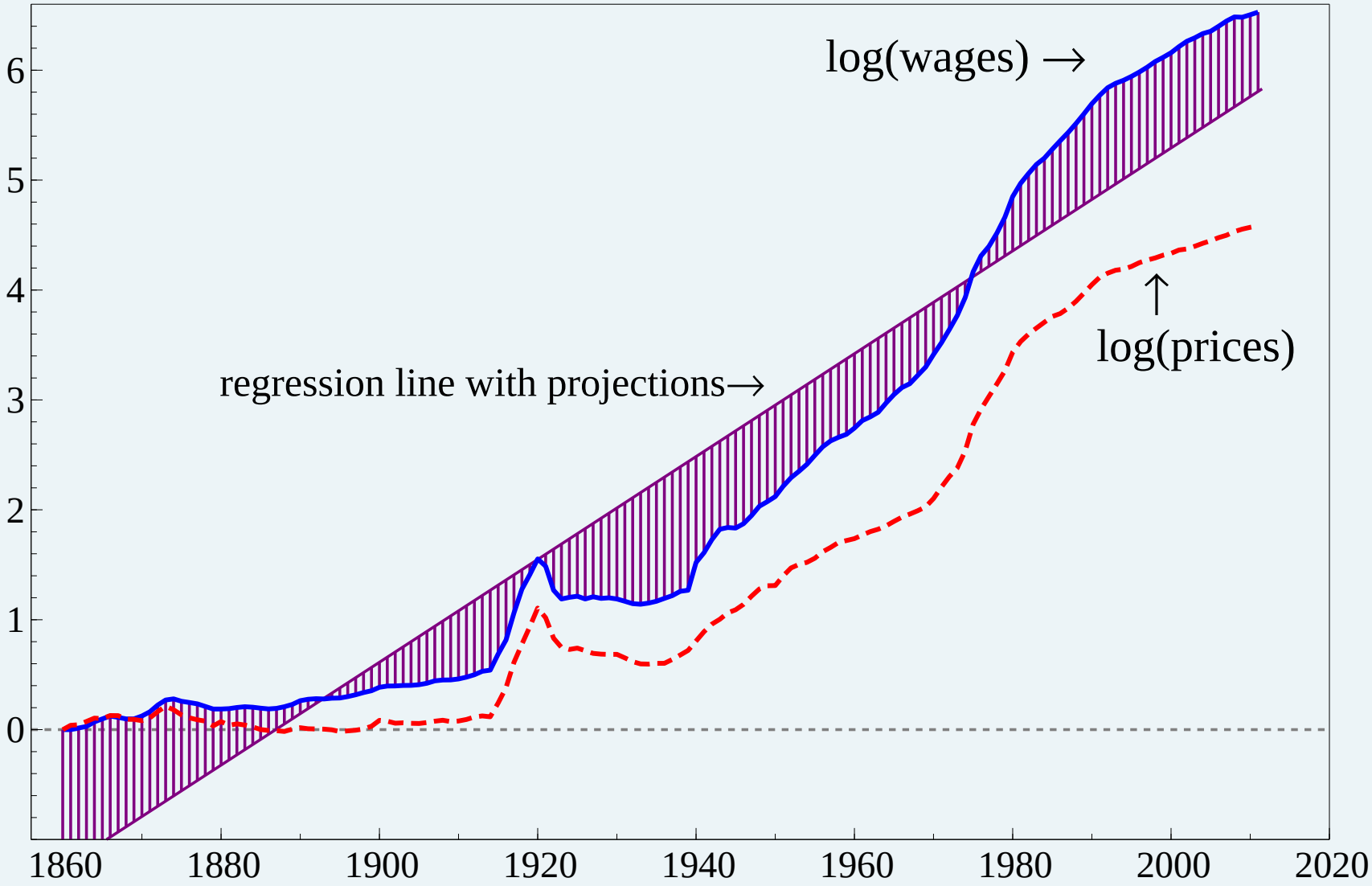


Figure: Log wages and log prices with one trend line for w.

Evidence from the log graph

As one regression line did not describe the evidence, divide the sample into 6 periods of about 25 years and fit a separate regression to each 'epoch'.

Again do that live using OxMetrics

Least squares applies to any data–
let me act Picasso and autograph my pretty picture, then fit a regression to my signature!

Can do lots of fun things in modern econometrics.....

Doing same to prices, get similar, but not identical patterns of slopes.

Why so much change? How can we model change?

How would you know that 6 regressions, rather than 1, or 12, were required?

What else is needed to explain such changes?

Log wages and log prices both with six trend lines

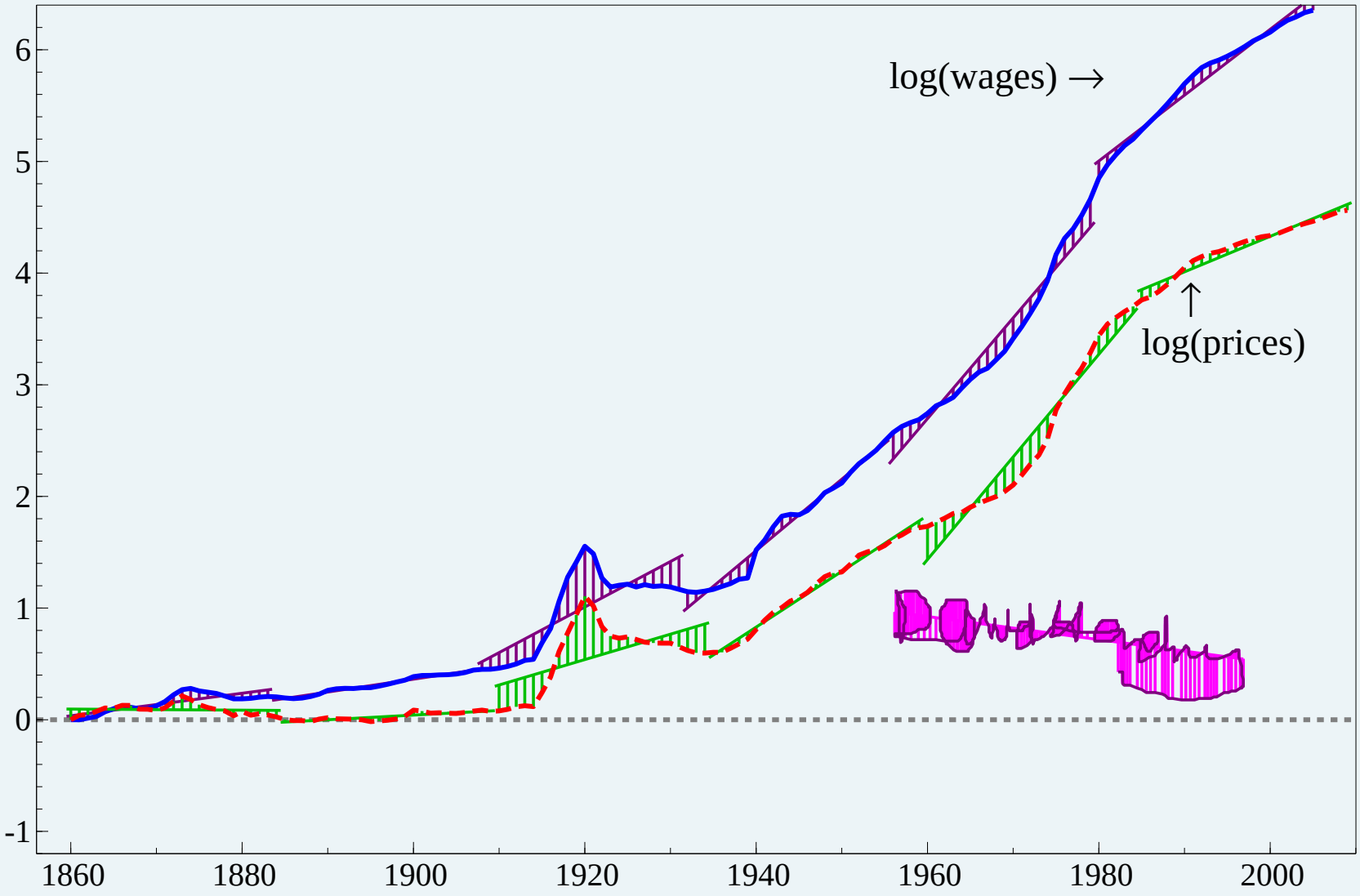


Figure: Log wages and log prices both with six trend lines, signed.

Major UK historical events

Characterize major historical events by 4 sub-groups:
[A] dramatic shifts; [B] key financial innovations; [C] important societal changes and technology advances; and [D] policy regime shifts.

[A] **Dramatic shifts include**: World War I (WWI); 1920–21 flu' epidemic; 1926 general strike; 1930's Great Depression; World War II (WWII); 1970's oil crises; 2008–2012 financial crisis.

[B] **Key financial innovations** & changes in credit rationing: personal cheques (1810s), telegraph (1850s), credit cards (1950s), ATMs (1960s); deregulating banks, etc., (1980s).

[C] **Societal and technological changes**: demography, health, longevity; laws, social mores. Huge technology advances: electricity, cars, flight, nuclear, computers, communications, mobiles.
Cotton, coal, steel, shipbuilding, all almost vanished.

[D] **Many policy regime shifts**: gold standard till Bretton Woods (1945), floating exchange rates (1973); Keynesian, Monetarist, inflation-targeting policies; EU then Euro; etc.

So what happened to real wages, $\log(W_t/P_t) = (w_t - p_t)$**?**

Log real wages over 1860–2011

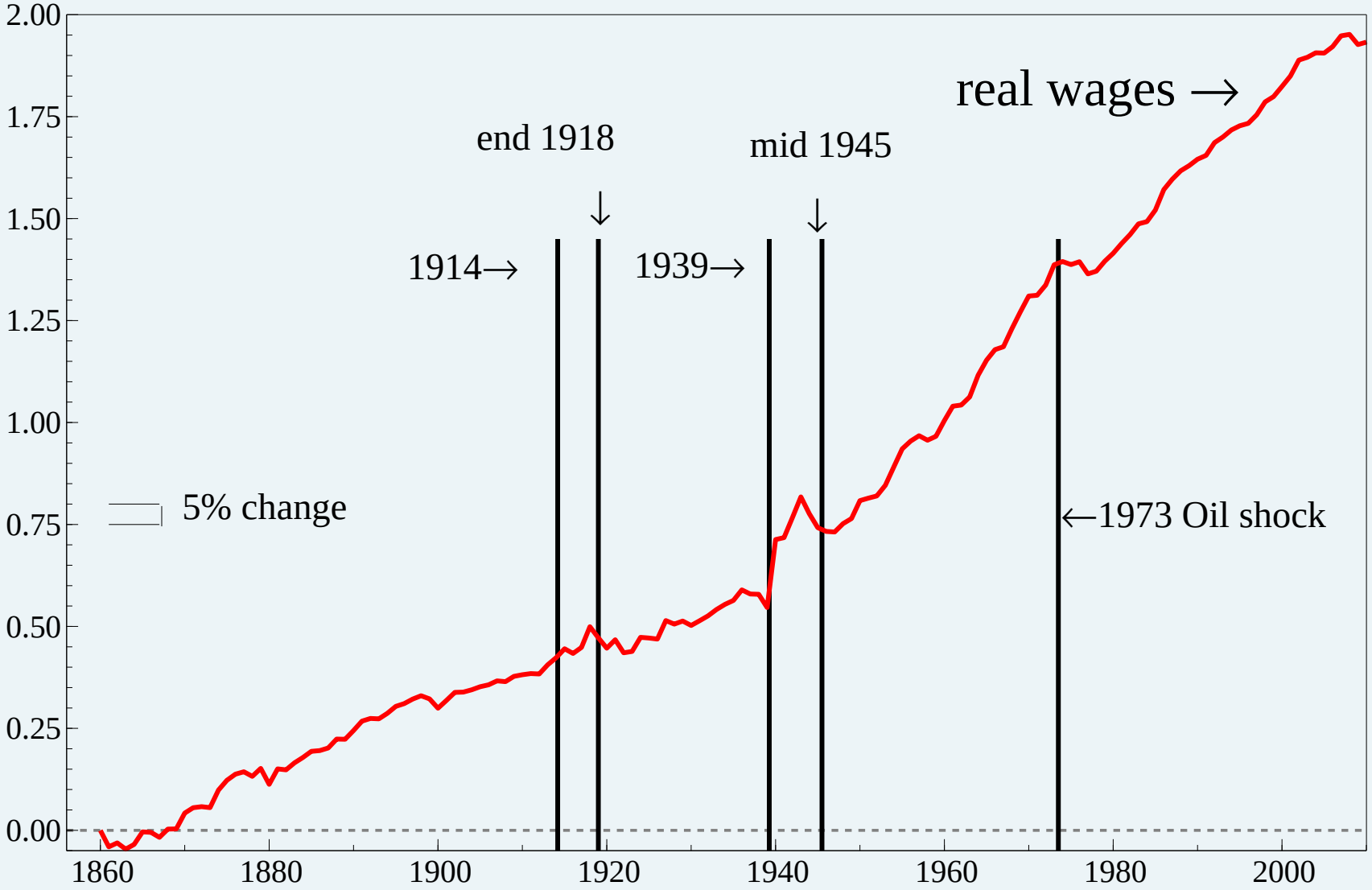


Figure: Log real wages, $w - p$, over 1860–2011.

Substantial growth:
move between each small tick mark is now about 5%.
Cumulates to more than 5-fold increase over period.

Impacts of two world wars clearly visible:
sharp rise then falls back.

Growth rate not constant, with many falls.
Single regression does not describe the history:
systematic departures from $(w - p)$:
deviations from the regression line are not random
–only change sign **twice** in 150 years.

Use 4 sub-periods for different 'epochs'–reveals changes in growth rate.

Again, we can do that live, using *OxMetrics*

But why have real wages grown at all?

Log real wages with one trend line

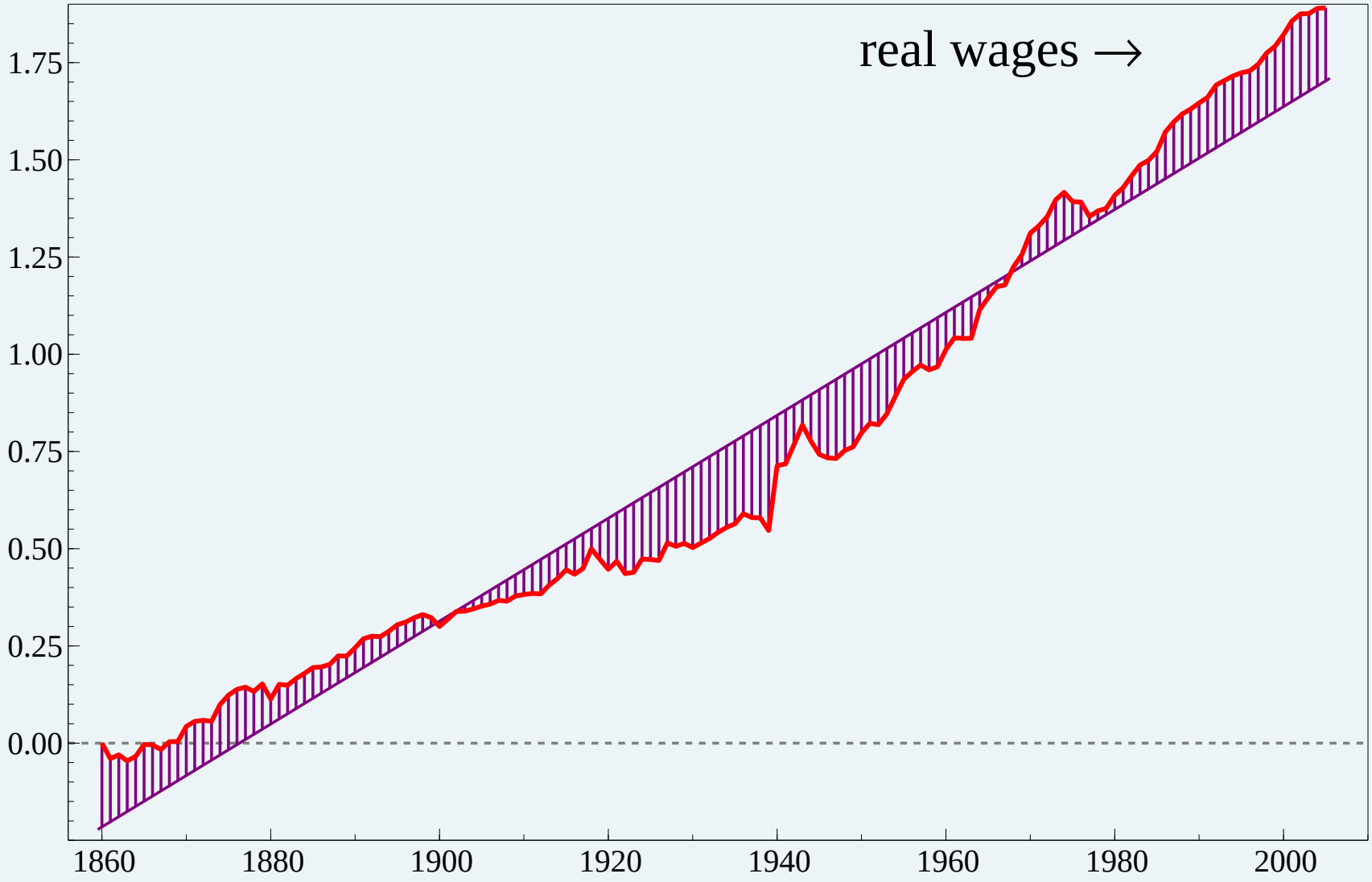


Figure: Log real wages with one trend line.

Log real wages with four trend lines

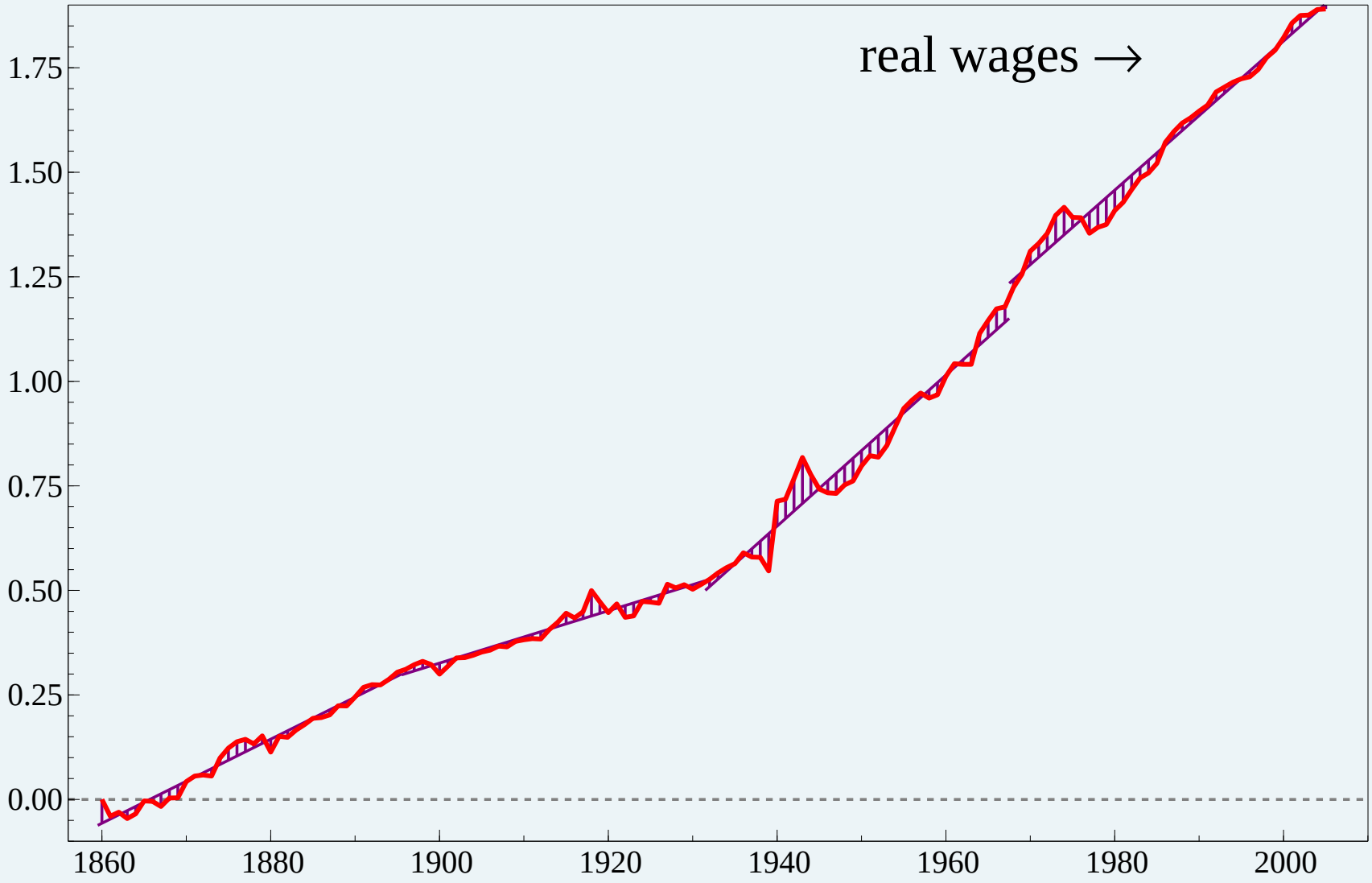


Figure: Log real wages with four trend lines.

Many theories of wages from Malthusian subsistence;
Walrasian labour-market clearing;
Keynesian nominal rigidities or norms;
micro-founded search and matching with roles for imperfect information and adverse selection;
efficiency wages etc.

Consider neo-classical theory of wage determination:
profit-maximizing competitive firms hire till real marginal cost of labour (additional real wage cost) equilibrates to real marginal revenue product of labour, i.e., MC=MR:

$$\frac{\partial(WL/P)}{\partial L} = \frac{\partial(PG/P)}{\partial L} \tag{2}$$

where G is real GDP, WL is the nominal wage bill, and L is employment.
Will consider determinants of MR then of MC.

If **constant-returns to scale** at aggregate level:
marginal proportional to average, MR $\propto$ AR,
so in logs, have average product per worker of $g - l$.

Assume Cobb–Douglas production function where $A(t) = \exp(at)$ is 'disembodied' technical progress, as most technical progress embodied in capital K_t and labour, so:

$$G_t = \exp(at) L_t^{\alpha} K_t^{(1-\alpha)} \quad 0 < \alpha < 1 \tag{3}$$

Limited data on hours, which fell greatly, or holidays and 'human capital' (skills and knowledge) which rose considerably. So in logs using $g - l$ for real AR, from (3):

$$(g - l)_t = at + (1 - \alpha)(k - l)_t \tag{4}$$

Figure 21 shows 'production function' linking $g - l$ and $k - l$.

Figure 21 shows 3-dimensional relationship over time.

‘Production function’ over 1860–2011

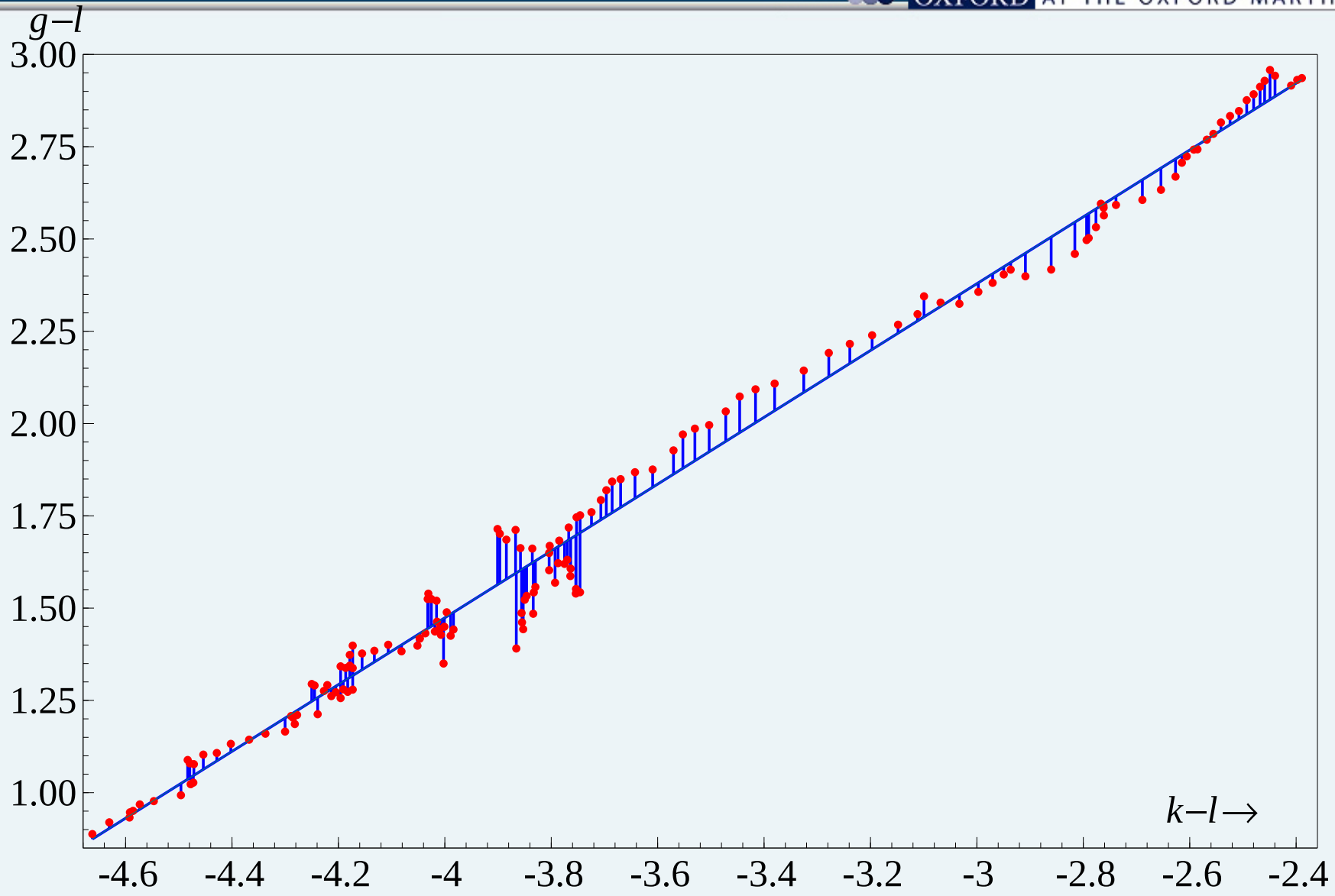
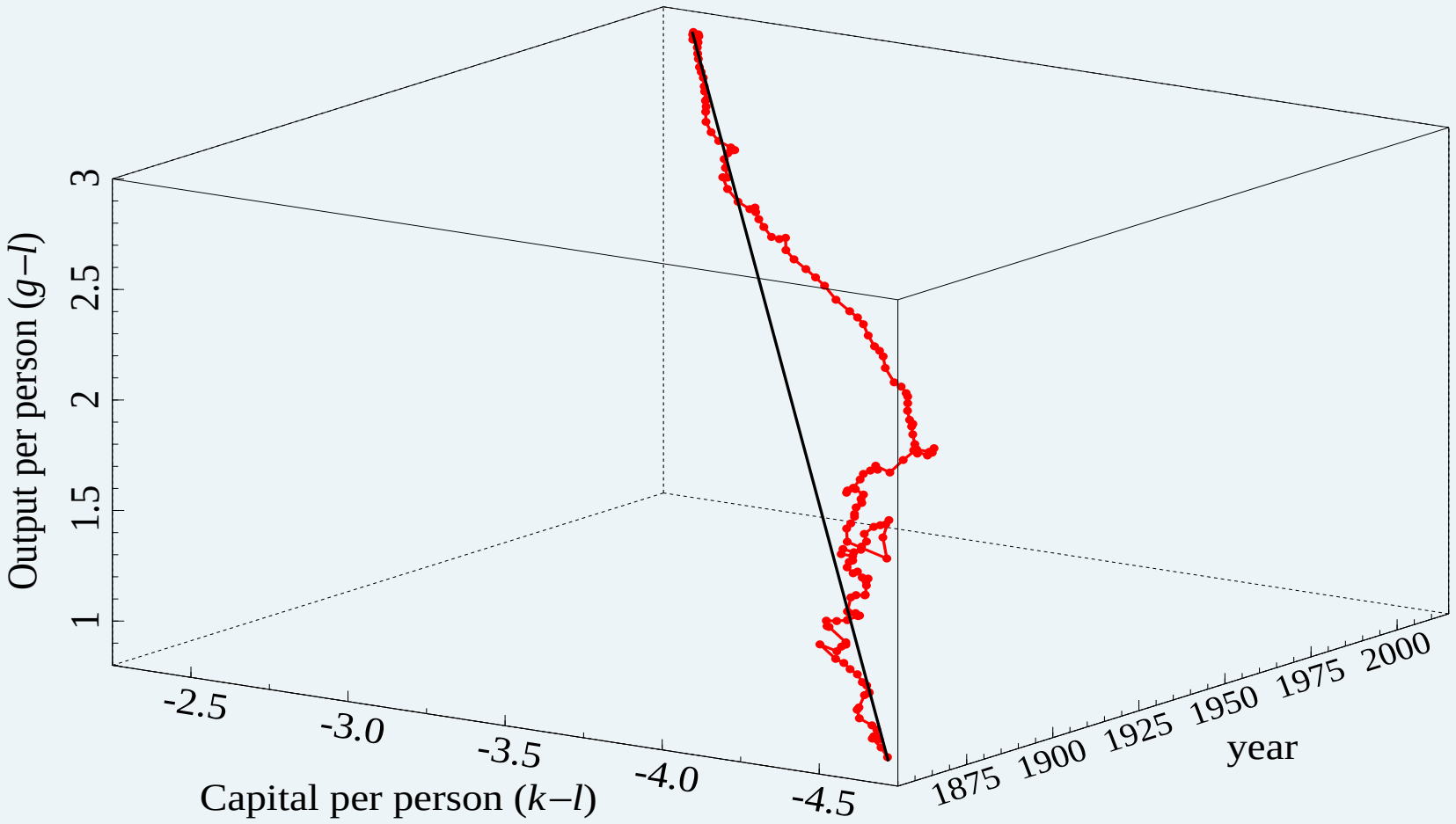


Figure: ‘Production function’ for $g - l$ from $k - l$ over 1860–2011.

‘Cobb–Douglas’ production function



Close to linear apart from 1920–1940, with different rates of technical progress pre-1918 and post-1945.

Consistent with long-run constant-returns to scale.

In (2), it is often assumed that:

$$\frac{\partial(WL/P)}{\partial L} = W/P.$$

Requires that changes in L do not affect W/P, so firms are 'price takers' in labour market–unlikely in macroeconomy.
In some theories, unemployment directly affects W/P.

AC comprises W and taxes τ (but no data on fringe benefits).

Long-run constant-price relationship in logs for $AC = AR$:

$$w - p = g - l - \tau \tag{5}$$

Then, $(w - p - g + l)$ is wage share, or real unit labour costs, so should only change with τ.

If so, trades unions (TUs), strikes (S), unemployment benefits (replacement ratios), National Insurance Contributions (NICs) etc., mainly impact on unemployment rate, U_r, rather than $(w - p)$–which is a testable hypothesis: see Figure 24.

Figure 25 plots log real wages and average productivity.

Strikes, NICs, TUs, replacement ratios

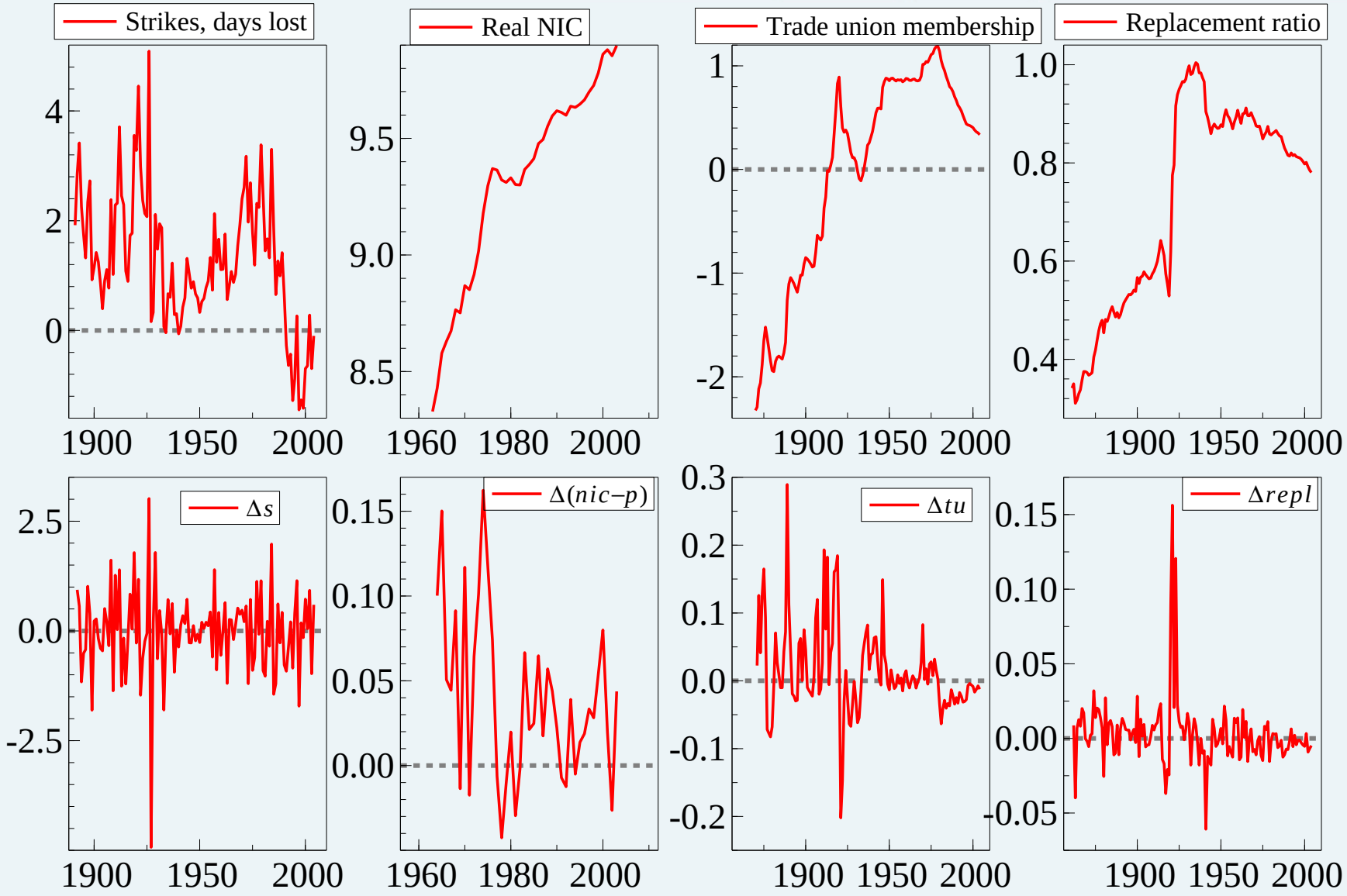


Figure: Strikes, NICs, TUs, replacement ratios and their changes.

Log real wages and productivity over 1860–2011

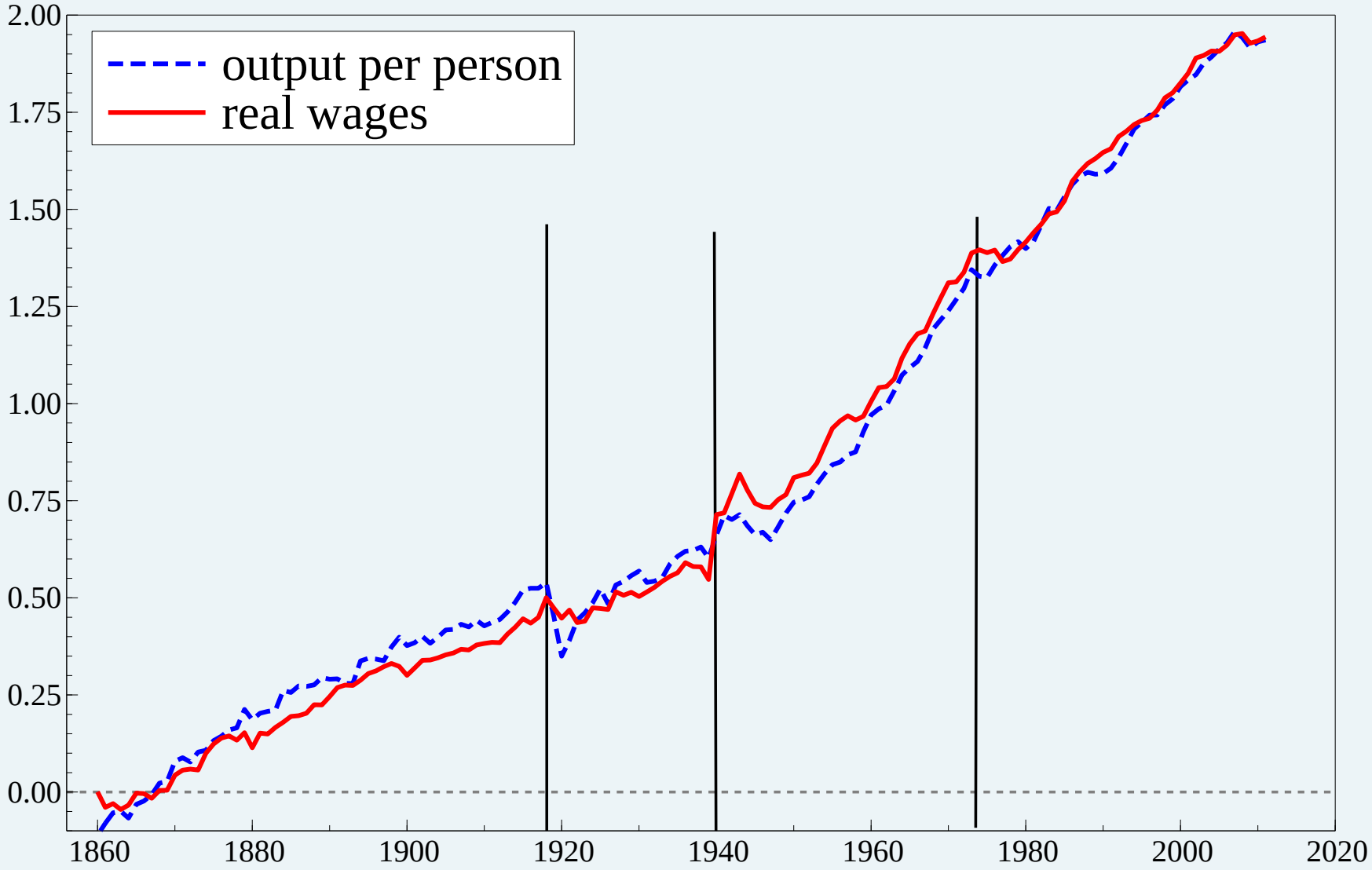


Figure: Log real wages and productivity over 1860–2011.

Evidence about their relationship

Similar long-run trends; and even match over our 4 epochs.

Most of the rise in real wages is due to increased output per worker.

So need to explain increases in $g - l$, so in $k - l$....
which we will not have time to do on this course.

Standard econometric problem–need to extend the system being analyzed to explain 'input variables'.

But also **persistent deviations** ('gaps between the lines'):
$w - p$ is below $g - l$ over 1865–1920; again 1925–1940;
then above 1940–1980; finishing close from 1985 on.

Another standard econometric problem–need other variables to explain systematic mis-match.

Return to that issue after considering **third econometric problem**–how to handle trends.

Most graphs of variables so far have revealed trends: need tools for handling the many sources of trends, some *deterministic*, some *stochastic*.

Begin with *deterministic* trends:

$$x_t = \alpha + \beta t \text{ for } t = 1, 2, 3, \ldots, \tag{6}$$

Then x_t grows at the rate β, as one period later:

$$\begin{aligned} x_{t+1} &= \alpha + \beta(t+1) \\ &= (\alpha + \beta t) + \beta \\ &= x_t + \beta \end{aligned} \tag{7}$$

Subtracting x_t from both sides of (7):

$$x_{t+1} - x_t = \Delta x_{t+1} = \beta \tag{8}$$

where Δx_{t+1} is the change in x_{t+1}:
the **difference Δx_t is constant at β, and is trend free**.
Figure 28 plots the changes in average productivity.

Changes in average productivity over 1860–2011

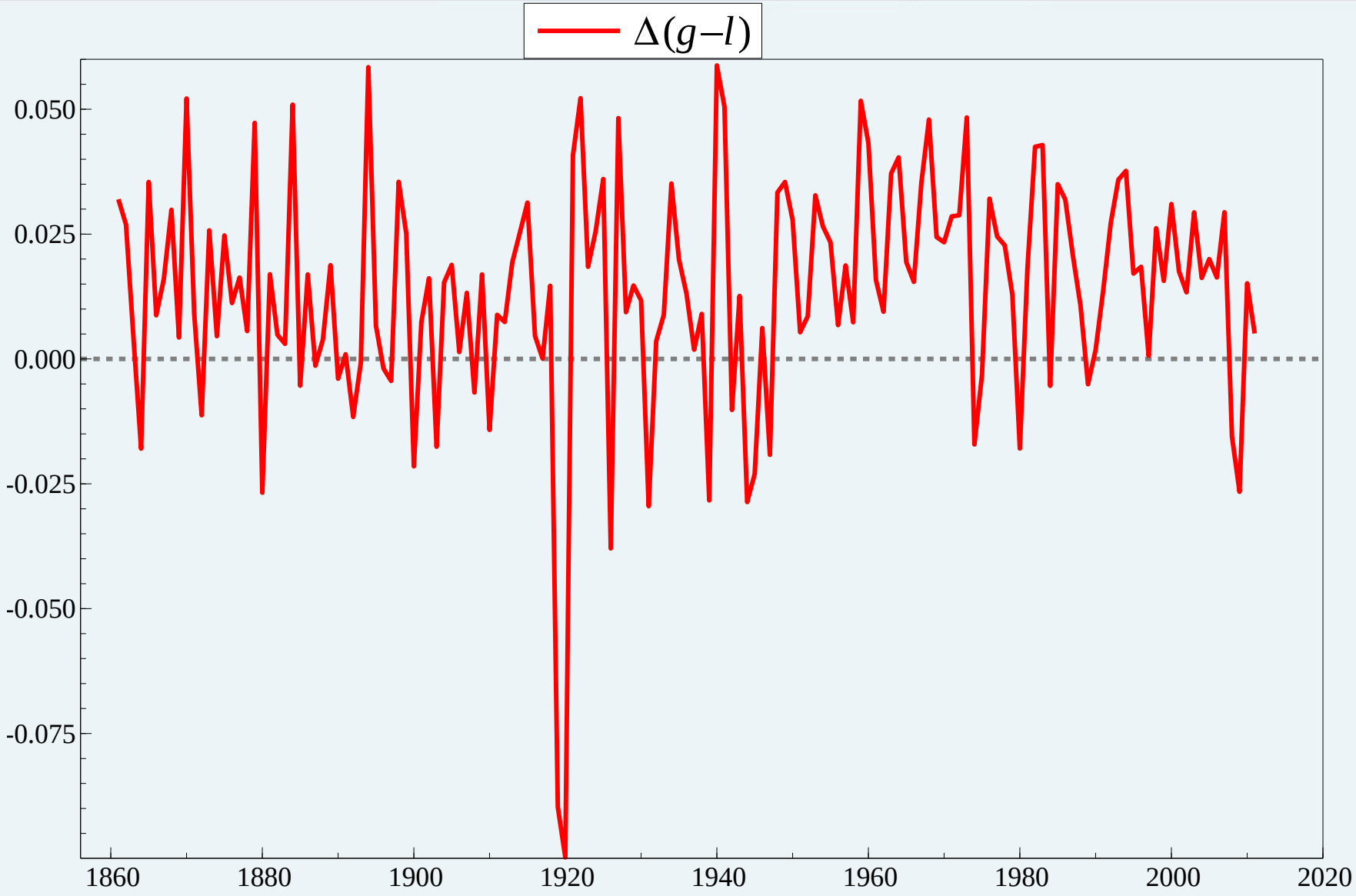


Figure: Changes in average productivity, $\Delta(g-l)_t$, over 1860–2011.

Return to equation in (7), re-written as:

$$x_t = x_{t-1} + \beta + \epsilon_t \tag{9}$$

where x_{t-1} is the previous value, or **lag**, of x_t.

In (9), ϵ_t is a random 'shock', which we can assume to be:

$$\epsilon_t \sim \mathsf{IN}\left[0, \sigma_\epsilon^2\right] \tag{10}$$

denoting an independently distributed normal random variable with a mean, or expectation, of zero, so $\mathsf{E}[\epsilon_t] = 0$ and a variance of $\mathsf{E}[\epsilon_t^2] = \sigma_\epsilon^2$, illustrated in Figure 30.

From (9), $\Delta x_t = \beta + \epsilon_t$, so the change Δx_t is now random around β.

$\{x_t\}$ is called a 'random walk', with 'drift' at the rate β:
equity prices are often treated as random walks with $\beta = 0$ because the change is then unpredictable.

Normal distribution

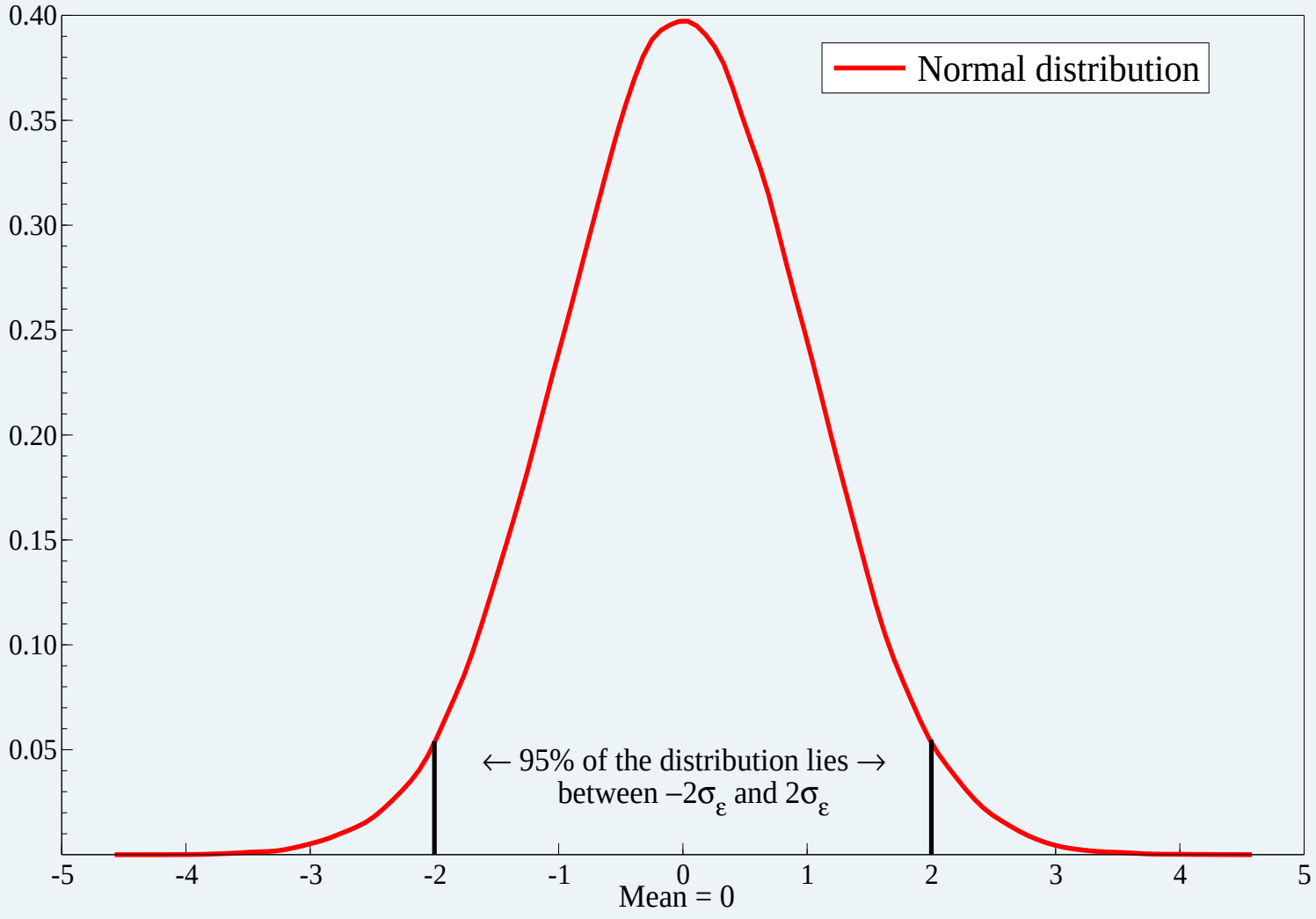


Figure: Normal distribution with mean $E[\epsilon_t] = 0$ and variance $E[\epsilon_t^2] = \sigma_\epsilon^2 = 1$.

At each point in time, x_t has cumulated all the past ϵs as:

$$\begin{aligned}
x_t &= x_{t-1} + \beta + \epsilon_t && \text{[by definition]} \\
&= (x_{t-2} + \beta + \epsilon_{t-1}) + \beta + \epsilon_t && \text{[one period earlier]} \\
&= x_{t-2} + 2\beta + \epsilon_t + \epsilon_{t-1} && \text{[rearranging]} \\
&= \vdots && \text{[repeatedly lagging]} \\
&= x_0 + t\beta + \epsilon_t + \epsilon_{t-1} + \cdots + \epsilon_1 && \text{[at time zero]} \\
&= \alpha + \beta t + \textstyle\sum_{r=1}^{t} \epsilon_r && \text{[changing notation]} \\
&= \alpha + \beta t + u_t && \text{[looking like (6)]} \qquad (11)
\end{aligned}$$

where $\alpha = x_0$. Then x_t 'wanders widely' from **the sum of all past errors**, $\sum_{r=1}^{t} \epsilon_r = u_t$, as well as trending at the rate β.

Stationary and non-stationary processes

Time series is stationary if the distribution remains the same over time. Mean and variance of that distribution must stay constant.
If the distribution, or any of its moments change over time, then the process is non-stationary.
Figure 33 shows histograms and approximate densities of w_t, p_t, g_t and l_t, shaded pre-WWII, unshaded after
(densities are just smoothed versions of the histograms).
There are large shifts in the distributions, with both the means and the variances changing between the two periods, as well as revealing apparent non-normality, where the closest normal distributions are shown as solid lines.
Figure 34 records the histograms and approximating densities (dashed and dotted lines) of the first differences of the 4 variables, Δw_t, Δp_t, Δg_t and Δl_t, which still change between the two periods, but by much smaller amounts and are also closer to normality within each period.
Nevertheless, it cannot be assumed that differences are necessarily stationary.

Histograms and densities show non-stationary processes

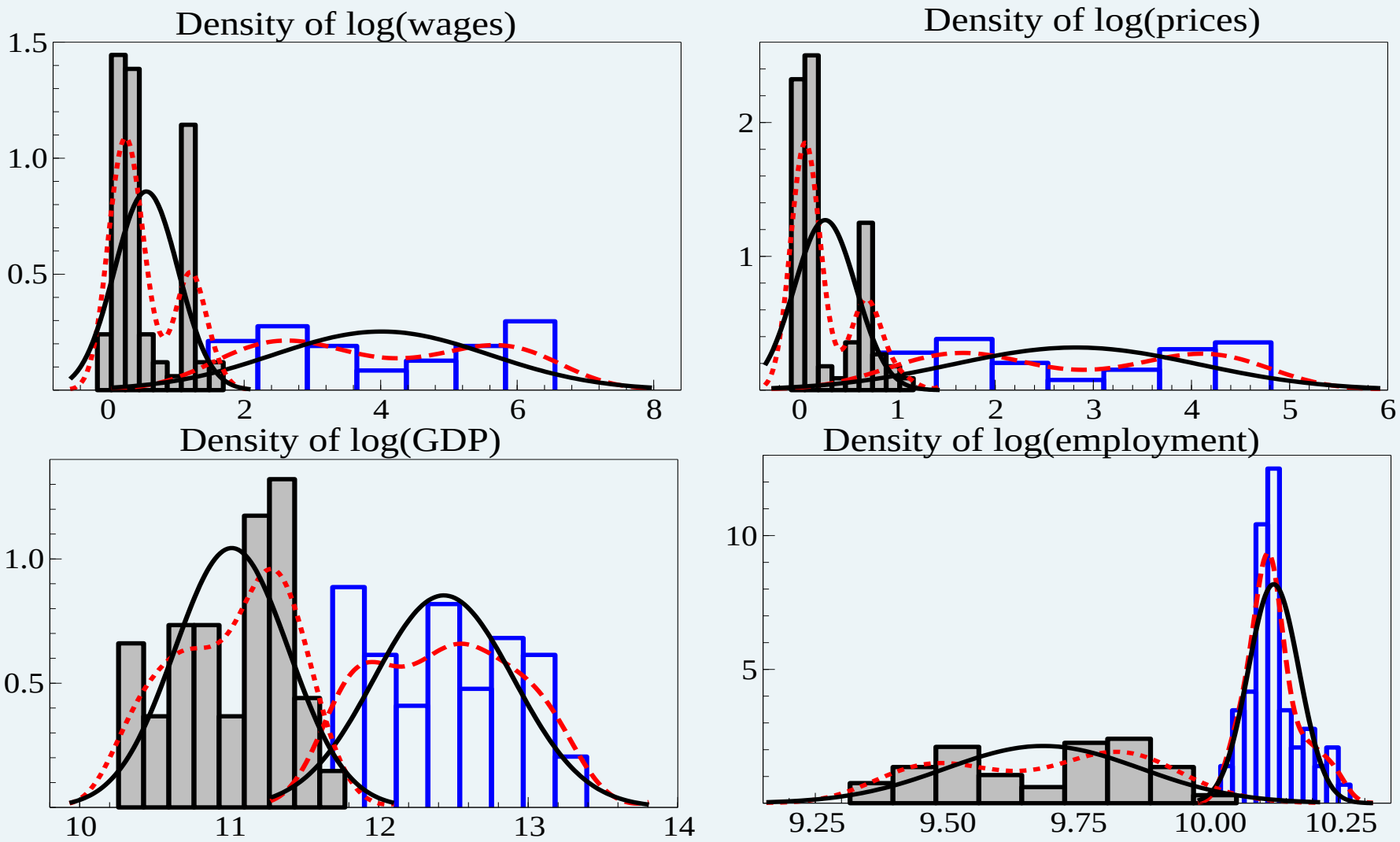


Figure: Histograms and densities of w_t, p_t, g_t and l_t, shaded pre-WWII.

Histograms and densities of changes show less non-stationarity

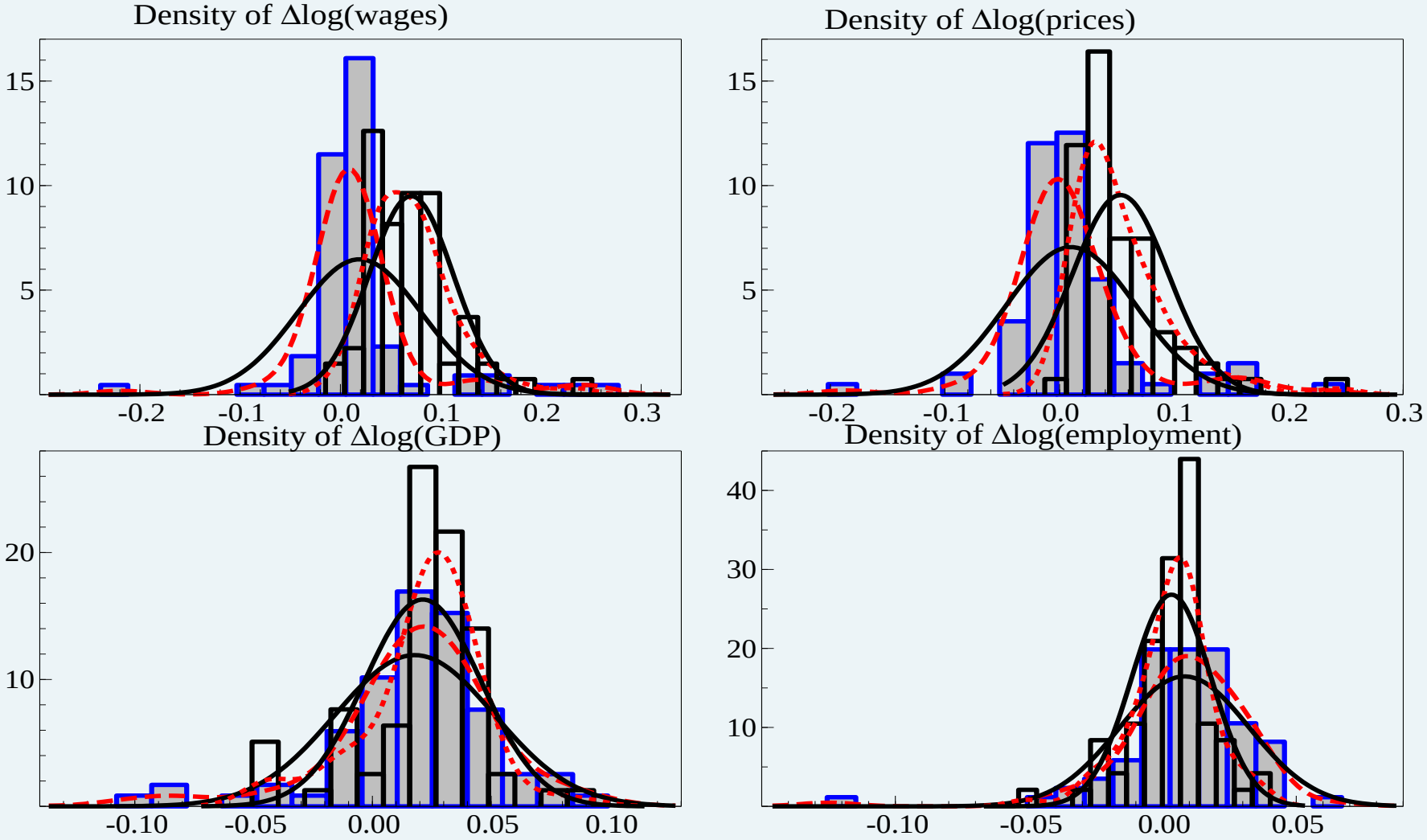


Figure: Histograms and densities of Δw_t, Δp_t, Δg_t and Δl_t, shaded pre-WWII.

Four other variables of interest

Broad money (measured by M4) relative to nominal GDP ($m - g - p$ in logs, analyzed below);
nominal national debt relative to nominal GDP ($N/(PG)$, not in logs);
long- and short-term interest rates (R_L and R_S respectively);
purchasing power of £ sterling against an index of world currencies (weighted by UK export shares, and denoted ppp, in logs).

Figure 36 shows their time series.
All have wandered widely since 1860, but have not trended strongly, although the levels of M and N have increased by almost 200,000% and 170,000% respectively.
$m - g - p$ rose dramatically after 1980 ('monetary control'?)
Most movements in $N/(PG)$ have been non-economic: presently it is far below the early 19th Century levels at which the UK funded the Industrial Revolution and become a major world power;
or post WWII when GDP growth was high and inflation low.
R_L acts like an average of R_S except during major downturns.
And ppp has ended near its original value.

Four time series of interest

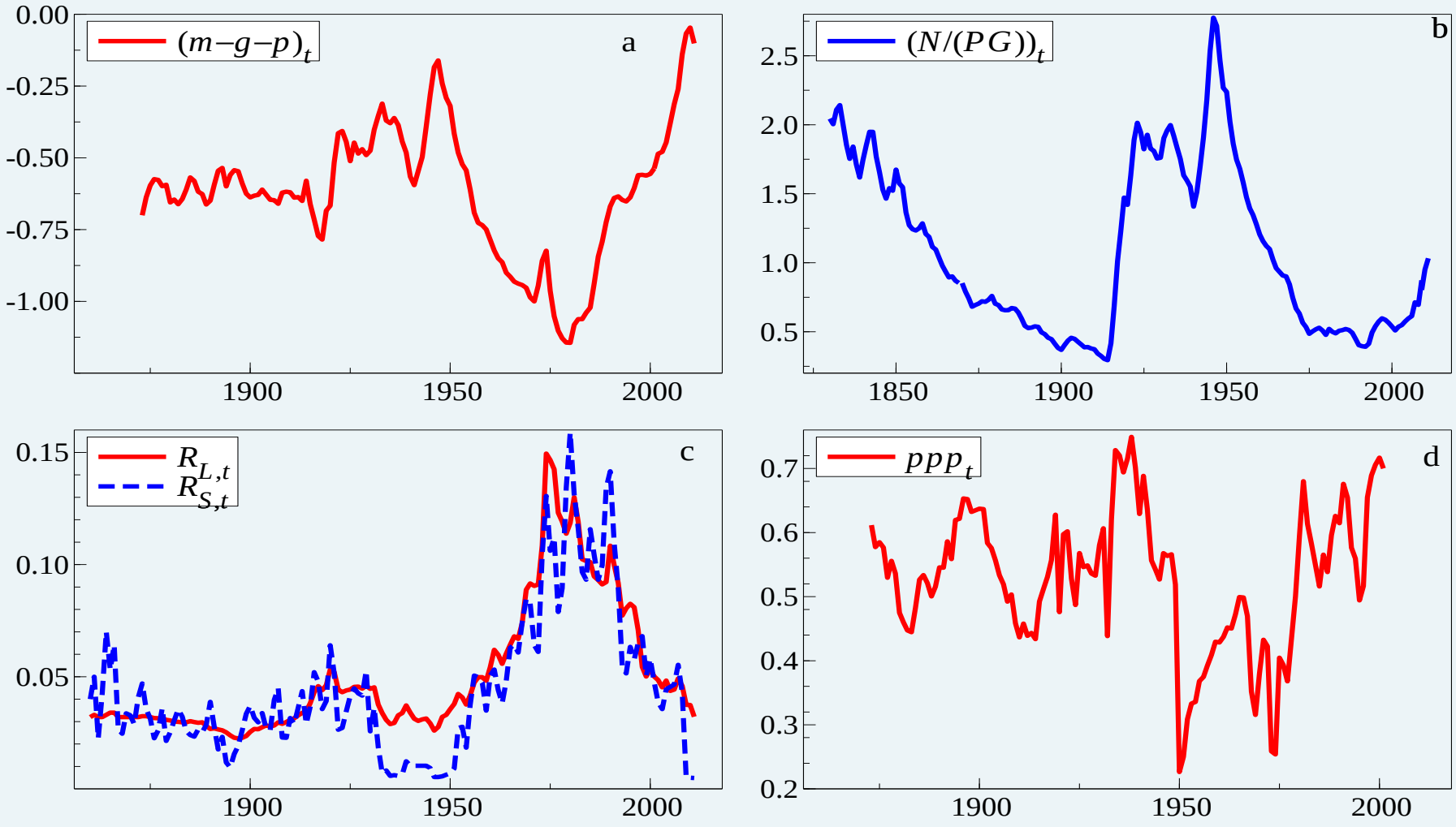


Figure: Time series of $(m-g-p)_t$, $(N/(PG))_t$, $R_{L,t}$ and $R_{S,t}$, and ppp_t.

Summary of lecture 1 key points

(A) Huge changes in wages and prices over last 150 years.
(B) Real wages increased greatly: close to the rise in productivity per worker.
(C) Matches simplest neo-classical model of equating marginal revenue per worker with marginal costs.
(D) Theory leads to systematic departures from data,
but data omit important changes in hours and skills etc.
(E) Many dramatic historical events, developments and periods.
(F) The growth rates of all the variables have also changed.
(G) **Macroeconomic data manifest both evolution and abrupt changes:** trends and sudden shifts are common.
(H) Formulated models of trends, both deterministic and stochastic.
(I) Broad money and national debt have increased dramatically, but kept similar ratios to GDP.

Next need to analyze, then model, sudden breaks.

Route map

(1) Macroeconomic data: evolution with abrupt change

(2) **Taming trends and breaks**

(3) Dependence between variables and over time

(4) Economic theory and statistical analysis: 2 key ingredients

(5) Modelling UK unemployment

(6) Modelling UK wages

(7) Modelling UK money demand

(8) Modelling UK prices

(9) Conclusion

Lecture 2 guide post

1. The levels of wages and prices trended greatly over the last 150 years, but their rates of change did not trend.
2. Wage and price inflation experienced many sudden large changes in their average values, called location shifts.
3. The growth of real wages doubled after the Second World War, probably due to the growth of labour and capital.
4. Most of the location shifts in wage and price inflation match, so real wages rarely jumped, called co-breaking, except for 1940.
5. Changes in (the log of) wages divided by prices do not trend, so 'tame' the huge rises in the nominal levels of wages and prices.
6. Those huge rises are also tamed by scaling real wages by productivity as that cancels their trends, called cointegration.
7. Indicator variables, zero except for unity over a short period to indicate its presence, can capture location shifts.
8. Rapid shifts may also be due to non-linear reactions.
9. Trends and breaks are 'removed' or 'modelled' by differencing or cointegration, and indicators or co-breaking respectively.

Victorian London in 1860

Again imagine you are living in London in 1860 on a worker's wage.

The UK operates under the 'Gold Standard', which fixes the price of gold in terms of £.

Gold discoveries in California and Australia shortly before briefly sustained general price rises, but were insufficient to support the rapid expansion of Western economies.

When increases in the output of goods and services continually exceed new gold supplies, the price level has to fall:
just a rise in price of gold.

Nominal wages and prices over 1860–1900 shown in Figure 16:
rose till early 1870s, fell back till the late 1880s (panel a).

Wage and price inflation (b) fluctuated considerably, but often negative.

Real wages lagged productivity growth (c).

Real wage changes (d) varied between $\pm 3\%$pa.

Wages and prices over 1860–1900

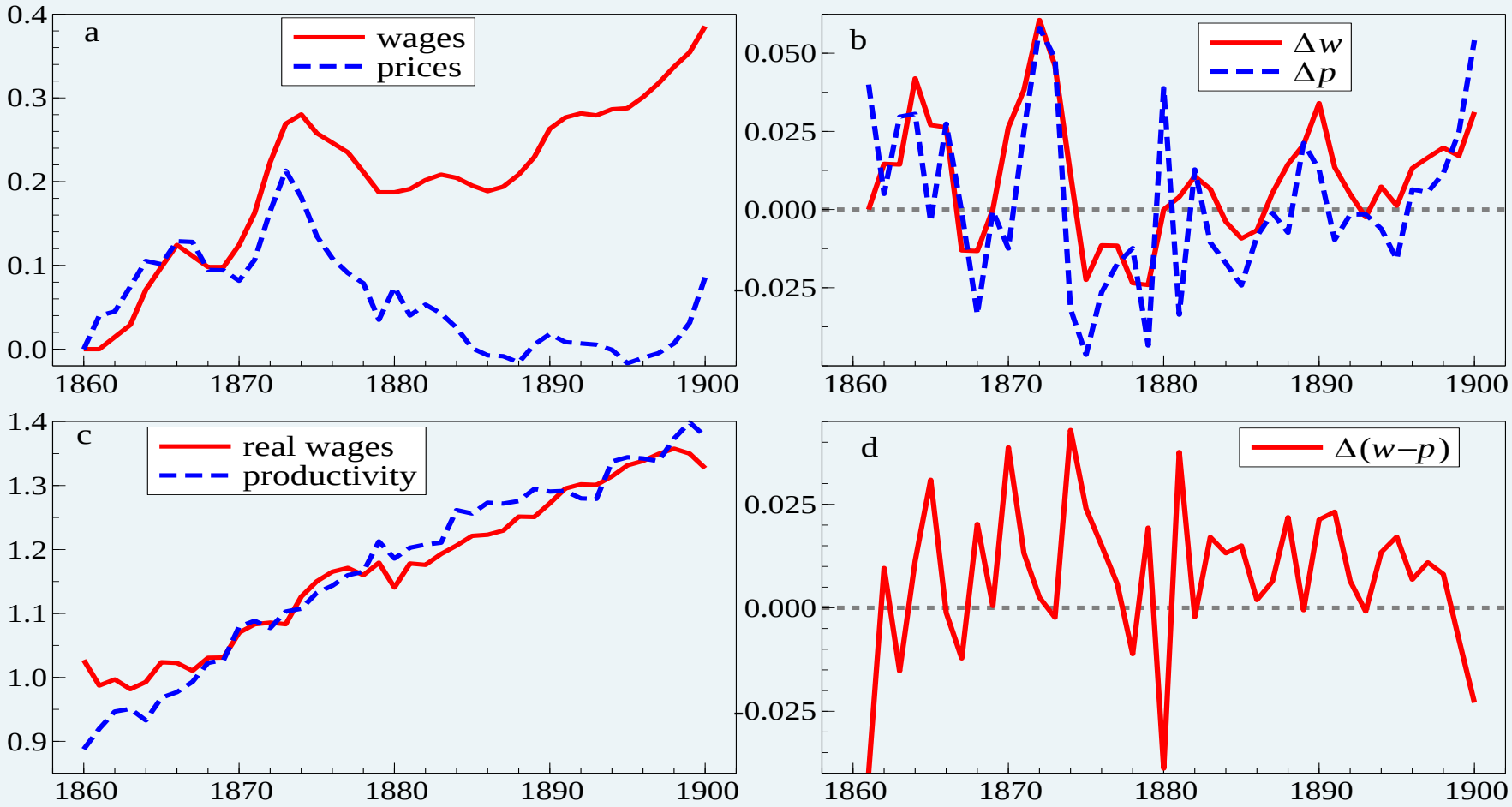


Figure: (a) Annual wages and prices; (b) wage and price changes; (c) real wages and productivity; (d) changes in real wages, all over 1860–1900.

The struggle to survive

Over a working life of 40 years, purchasing power of your wage would have risen less than 40%.
Started at an inadequate level, and ended at one.

Many older individuals forced into Workhouses, with much new construction in the 1860s.

Even by 1913, life was little better:
Pember Reeves (1913), spells out the hardships of London life.

The Great Depression did not help, but from Second World War, matters improved rapidly after the abandonment of the Gold Standard for Bretton Woods currency agreements,
replaced by floating exchange rates in 1970s.

That necessitated a mechanism to determine the price level.

Measuring wage and price inflation

Over the last decade, 'inflation targetting' has become the main instrument of economic policy, where the Bank of England moves interest rates to try and stabilize price inflation around 2.0% pa.

The Bank targets the consumer price index (CPI).
We are analyzing the gross domestic product (GDP) deflator, the price index that converts nominal GDP to real.
Our P_t is **annual**, whereas CPI is measured **monthly**.

Thus, $\Delta p_t = p_t - p_{t-1}$ is annual inflation, and when $\text{cpi}_t = \log(\text{CPI}_t)$, so is $\text{cpi}_t - \text{cpi}_{t-12} = \Delta_{12}\text{cpi}_t$.

Price indices have arbitrary units determined by a 'base year' where they are set to unity, or sometimes 100.
In logs, that affects the level, but not the changes.

We now apply our analysis to wage and price inflation, denoted Δw_t and Δp_t, shown in Figure 44.

Wage and price inflation

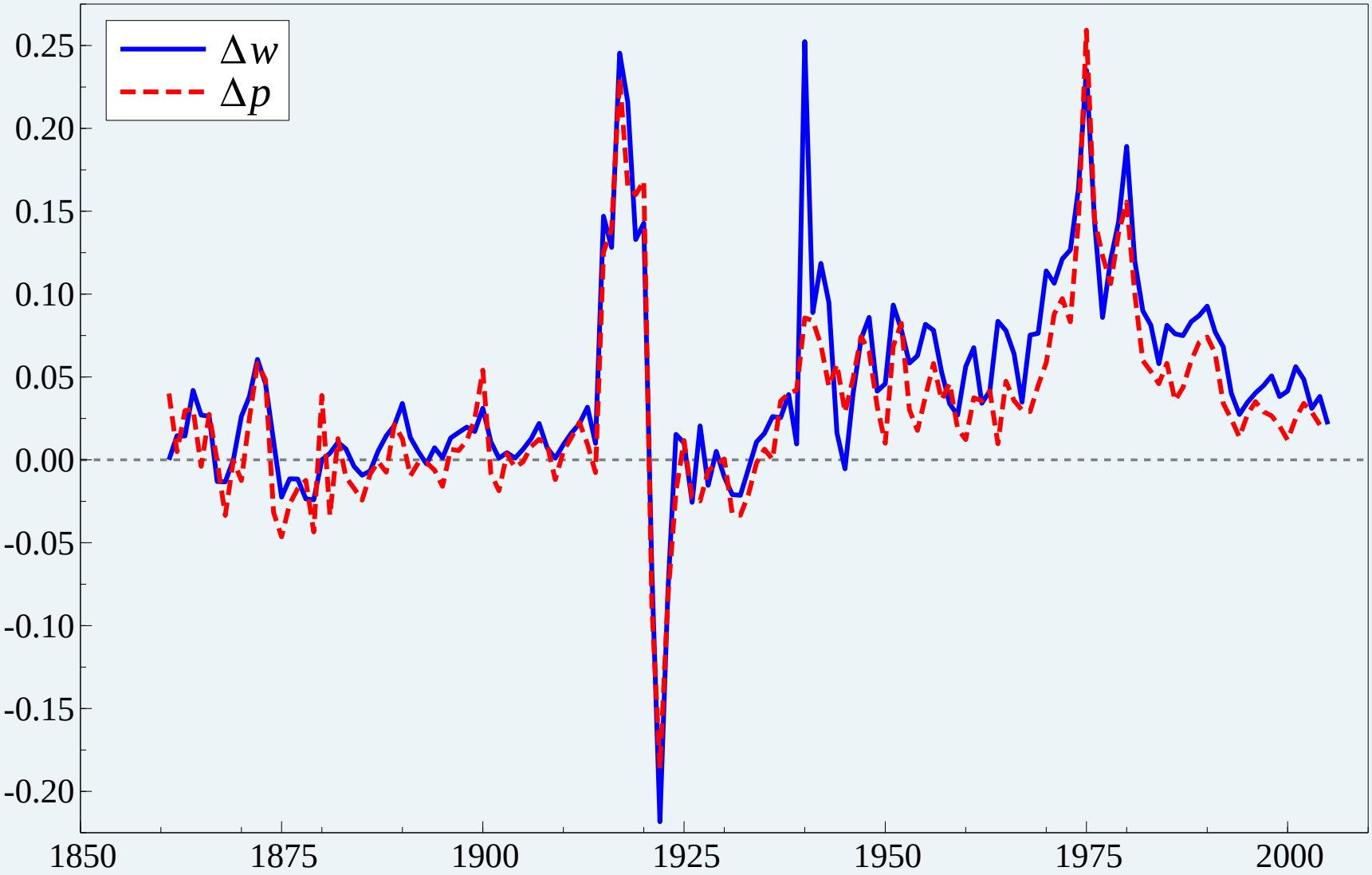


Figure: Annual wage, Δw_t, and price, Δp_t, inflation over 1860–2011.

Many new features:

1] neither Δw_t nor Δp_t trends overall, consistent with (11);

2] three episodes of very high inflation (above 20% pa):

a] during World War I (WWI) for both Δw_t and Δp_t;

b] during World War II (WWII) for wages; and

c] during the 1970s for both Δw_t and Δp_t;

3] also periods of sharp falls in wages and prices, especially 1920–22, when both fell about 20% p.a.;

4] **usually both variables move in similar ways**, but;

5] generally, $\Delta w_t \geqslant \Delta p_t$, especially after WWII;

6] both series relatively 'erratic', with many 'jumps'.

From this, can you guess what the change in real wages, $\Delta(w_t - p_t)$, 'looks like'? Figure 46 shows.

Real wage growth

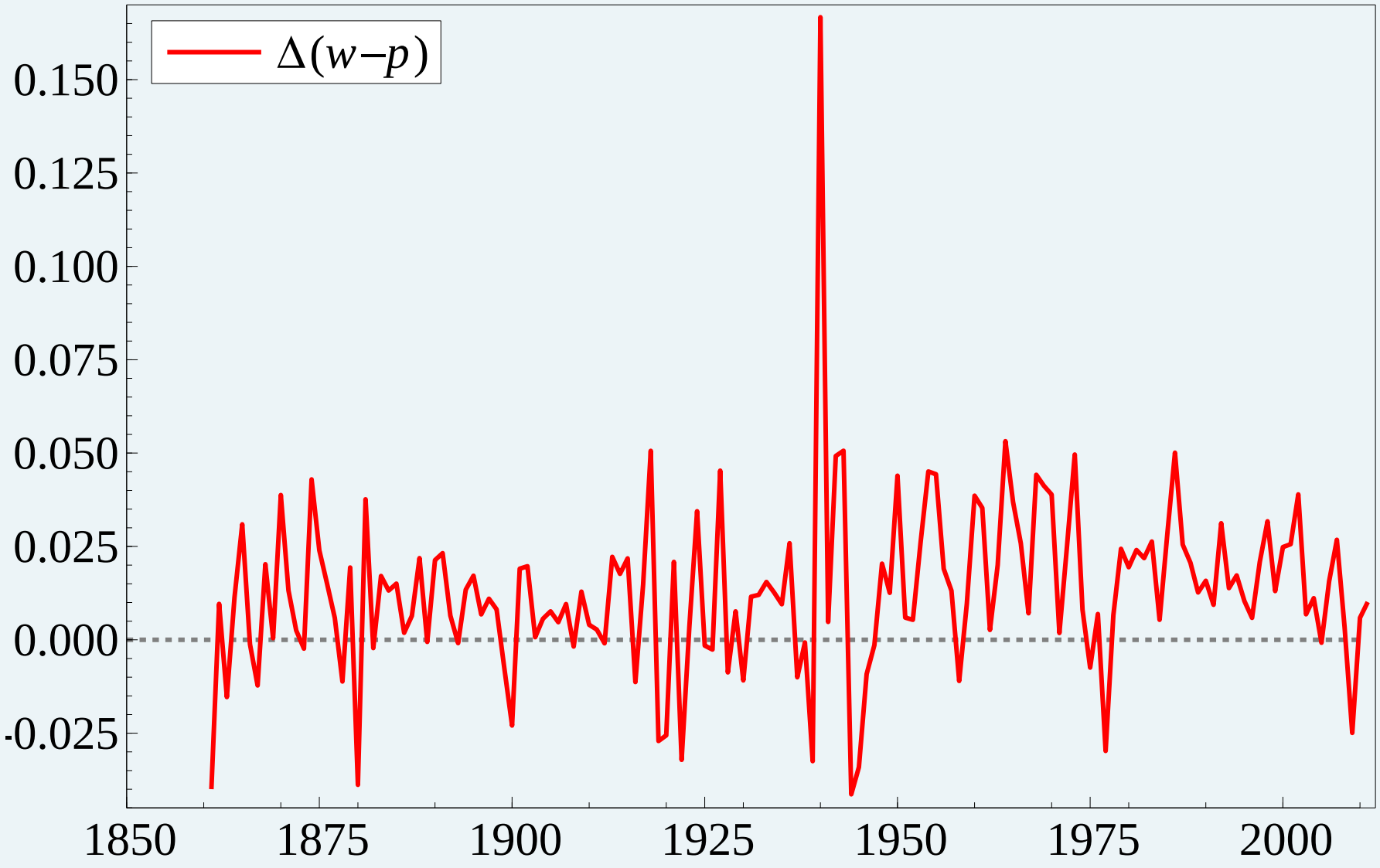


Figure: Real wage growth, $\Delta(w_t - p_t)$, over 1860–2011.

Increasing real incomes

Huge ‘spike’ in 1940, but can now see 19th Century behaviour shows a lot of up–down movements.

Latter half of 20th Century had higher mean, with more persistent increases in last few decades.

But might be hard to discern the small mean shift against the ‘noisy fluctuations’ in $\Delta(w - p)$, **even though growth doubled** (1.8% p.a., versus 0.9% p.a. pre-1945) as shown in Figure 48.

Growth rate of GDP per worker over 25-year intervals in Figure 49 shows prosperity is remarkably recent.
Real incomes double in under 33 years at 2.2% p.a.,
so grow more than 8-fold in a century,
and **64-fold** in 200 years,
but **only double every century at** 0.7% p.a. (rule of 72).

Add major improvements in hygiene, medical technology and longevity, leads to being much better off now: but why?

Real wage mean change

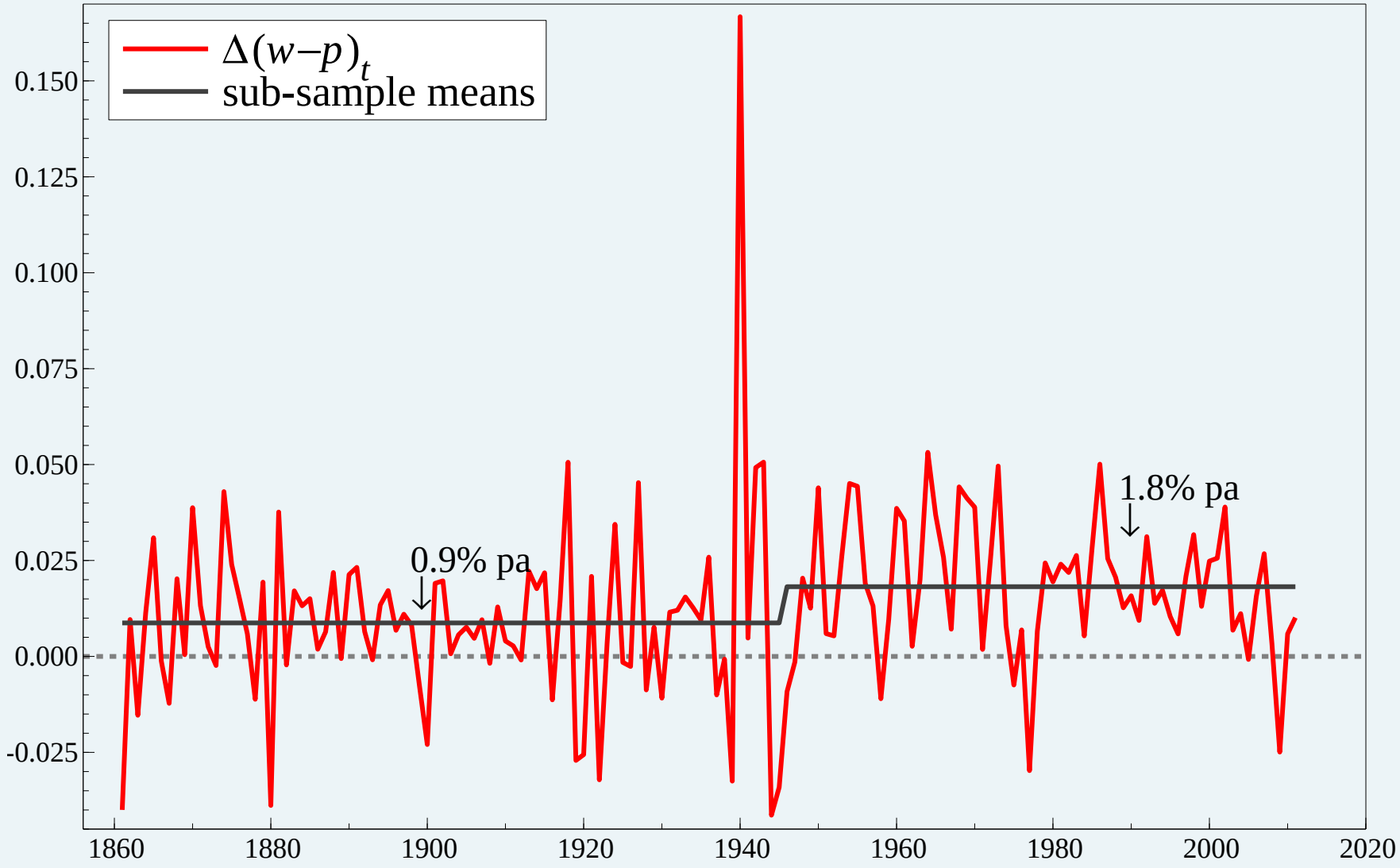


Figure: Real wage mean change over 1860–1945 and 1945–2011.

GDP per worker growth by 25-year intervals

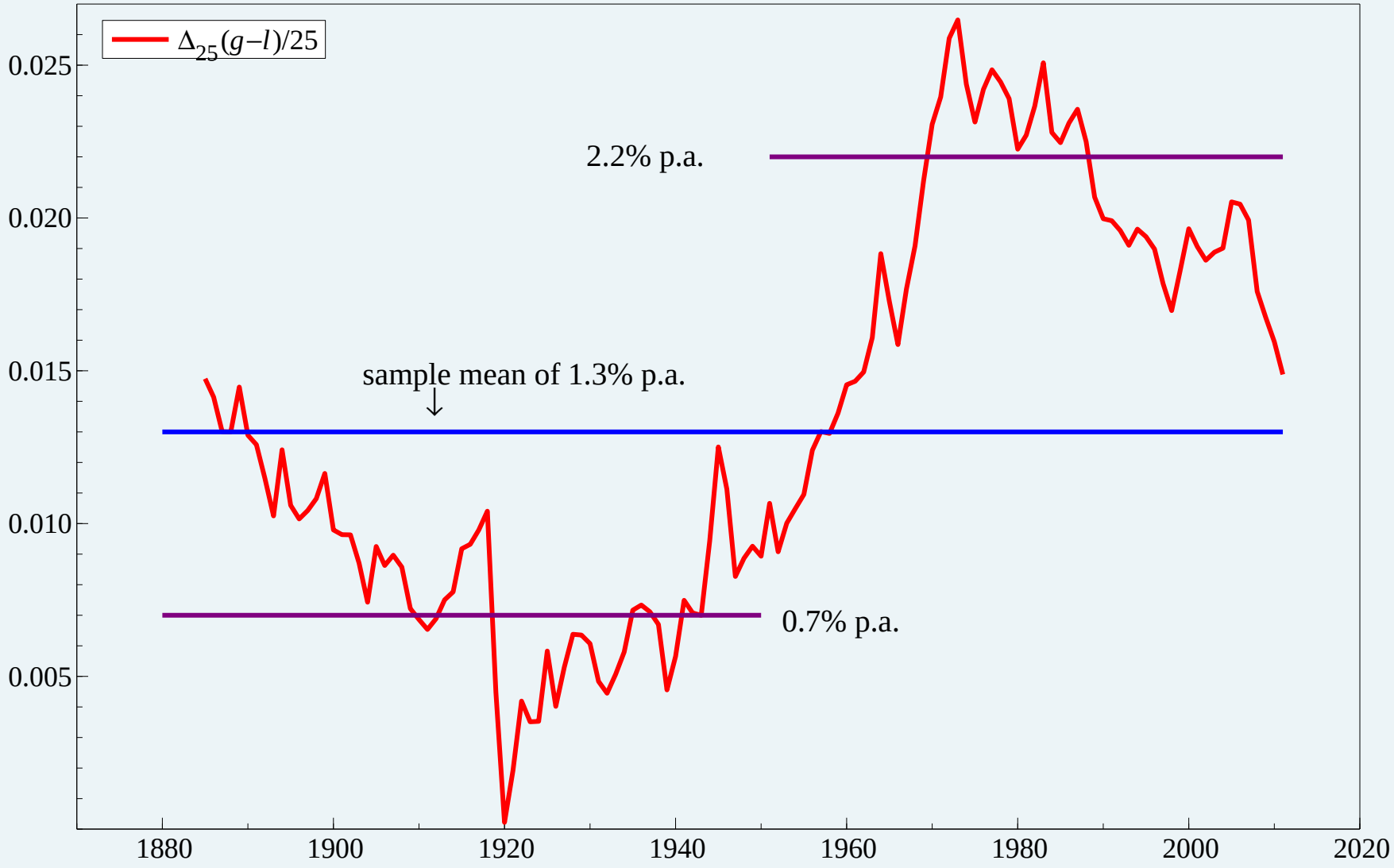


Figure: GDP per worker growth by 25-year intervals.

Growth of labour and capital

Very different behaviour of Δl_t and Δk_t in Figure 51, shown on same scale.

Employment has grown on average by 0.6% p.a., whereas capital has grown at 2% p.a., **about 3.5 times as quickly**.

A] Vastly more capital per worker now, so workers much more productive;

B] and capital itself is much more productive;

C] as well as workers embody much more education and skills (human capital);

D] although they work shorter hours per year–

E] still a huge rise in output per person employed;

F] yet employment more volatile than capital while growing more slowly: it is easier to sack workers than equipment or buildings.

Growth rates of employment and capital stock

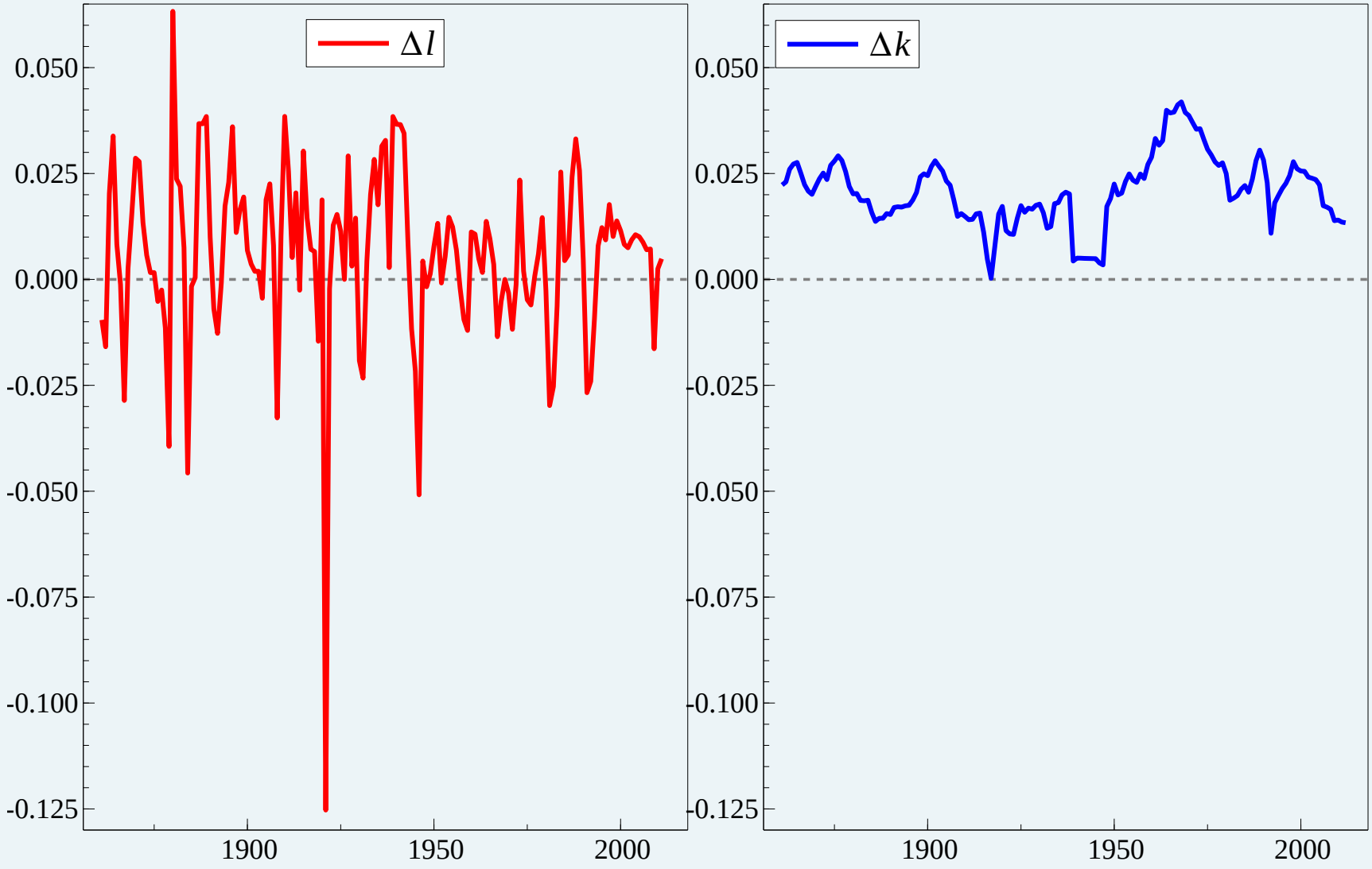
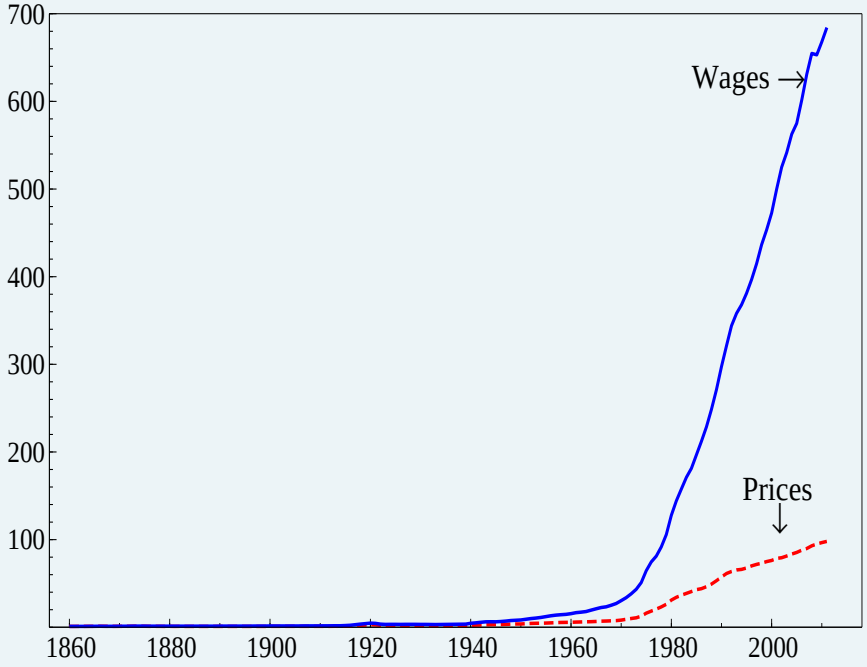


Figure: Growth rates of employment and capital stock over 1860–2011.

Do you remember Figure 6 in the left panel?

J-shaped over $[1, 700]$, with most early detail lost.



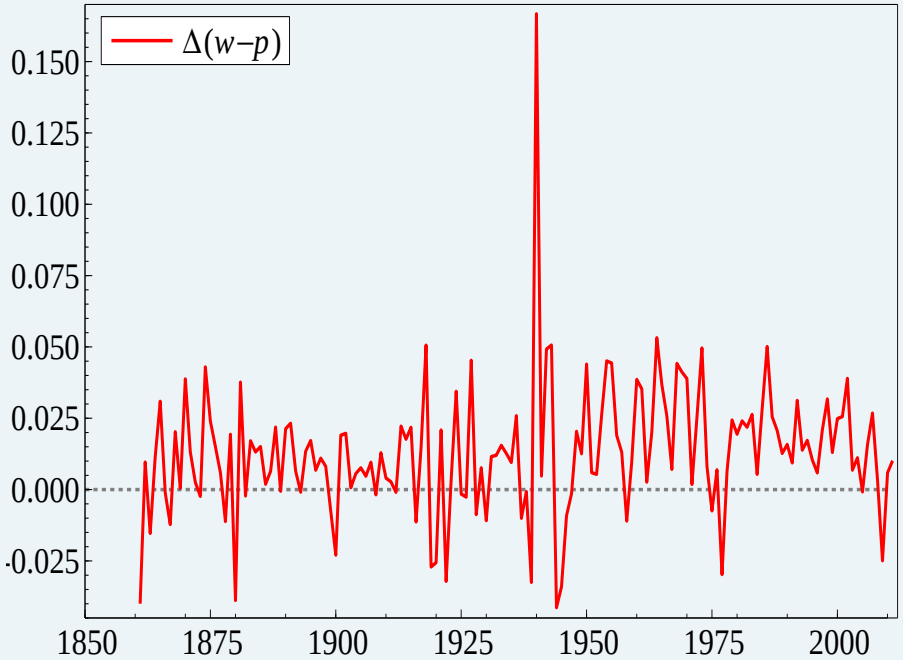


Figure 46 (right) shows considerable movement, similar over 150 years, yet $\Delta(w - p) \in [-0.03, 0.16]$.

That is our first step in 'modelling' wages and prices.

But there is another way to tame time-series trends:

Figure 53 records the (log) 'wage share', $w - p - g + l$.

‘Wage share’ *w-p-g+l*

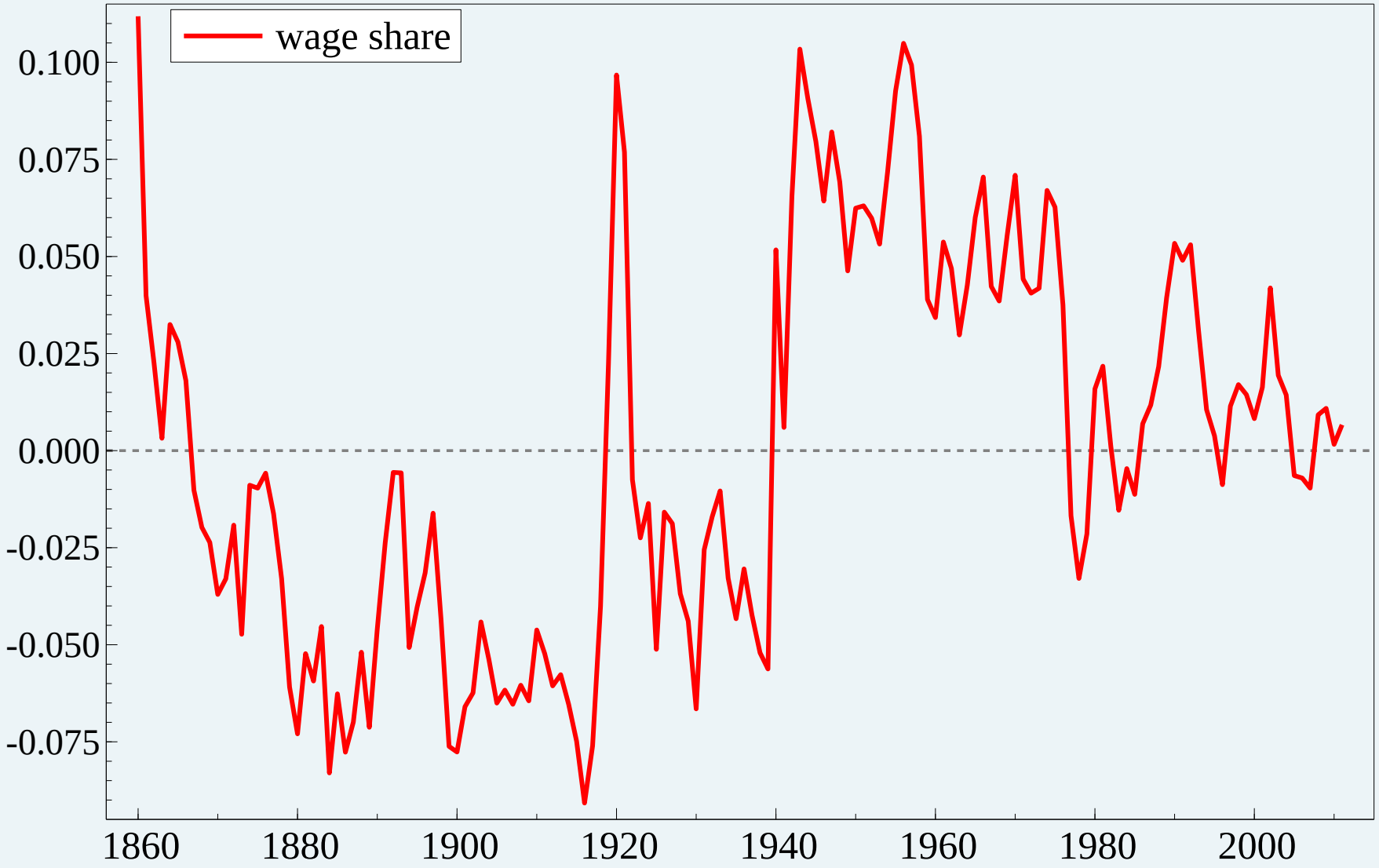


Figure: ‘Wage share’ $(w - p - g + l)_t$ over 1860–2011.

Both $w - p$ and $g - l$ are cumulative processes with apparent stochastic trends, discussed in Lecture 1, as Figure 25 showed.

Time series, x_t, with stochastic trends cumulate past errors:

$$x_t = \alpha + \beta t + \sum_{s=1}^{t} \epsilon_s \tag{12}$$

Integrals, $\int$, used to be written as elongated S mimicking a sum, so time series like (12) are called **integrated** of first order, denoted I(1). Linear combinations of I(1) processes usually also exhibit stochastic trends, but those may cancel between series.

For example, g and l are I(1), and so is $g - l$.

The 'wage share', $w - p - g + l$, is the gap between these, yet does not trend: **the combination cancelled the trends**.

Cointegration is the name when stochastic trends cancel.

Here $w - p$ and $g - l$ are I(1),
but $w - p - g + l$ is not integrated, denoted by I(0).

A single stochastic trend x_t is given by:

$$x_t = x_{t-1} + \beta + \epsilon_t = x_0 + \beta t + \sum_{s=1}^{t} \epsilon_s \tag{13}$$

where $\epsilon_t \sim \mathsf{IN}\left[0, \sigma_\epsilon^2\right]$ as in (10). Now consider the two variables, y_t, z_t, determined by equations dependent on x_t:

$$\begin{aligned} y_t &= \mu_0 + \lambda x_t + e_t \\ z_t &= \mu_1 + x_t + \nu_t \end{aligned} \tag{14}$$

where $\lambda \neq 0$, e_t and ν_t are random errors. Then y_t and z_t 'inherit' a **common** stochastic trend from x_t so are I(1). However:

$$y_t - \lambda z_t = \mu_0 - \lambda \mu_1 + e_t - \lambda \nu_t \tag{15}$$

so the stochastic trend x_t has been cancelled by the (unique) linear combination $y_t - \lambda z_t$ which only depends on I(0) errors.
Hence y_t and z_t are said to be **cointegrated**.
While (13)–(15) are a simple example, they illustrate the general idea of cointegration.

Alternatively, when two variables are directly linked by:

$$y_t = \beta_0 + \beta_1 z_t + \epsilon_t \tag{16}$$

where ϵ_t is a random error, and z_t is $\mathsf{I}(1)$, generated by:

$$z_t = z_{t-1} + \gamma + \nu_t \tag{17}$$

where ν_t is a random error, then differencing (16):

$$\Delta y_t = \beta_1 \Delta z_t + \Delta \epsilon_t \tag{18}$$

where (17) implies $\Delta z_t = \gamma + \nu_t$, so:

$$\Delta y_t = \beta_1 \gamma + \Delta \epsilon_t + \beta_1 \nu_t \tag{19}$$

and hence:

$$y_t = y_{t-1} + \delta + u_t \tag{20}$$

where $\delta = \beta_1 \gamma$ and $u_t = \Delta \epsilon_t + \beta_1 \nu_t$.
Thus, y_t is $\mathsf{I}(1)$, but cointegrated with z_t from (16) as $y_t - \beta_1 z_t$ is $\mathsf{I}(0)$.

Many groups of economic time series seem to cointegrate. Figure 58 shows the £ exchange rate index, e (left panel) the UK price level, p, and world prices, p_w, in the middle, and purchasing power parity, ppp$= e - p + p_w$ (right panel).

Despite opposite trends, ppp does not trend.

Cointegration defines an 'equilibrium trajectory', where departures induce an equilibrium correction, which moves the economy back towards that path–otherwise the economy would drift–but may converge back slowly.

Figure 59 shows the UK's long-term and short-term interest rates, R_L and R_S, and the spread, $R_L - R_S$, which 'cancels' much of their movement.

We will return to cointegration analysis later.

The next step is to investigate breaks in wage and price inflation over the last 150 years.

Purchasing power parity

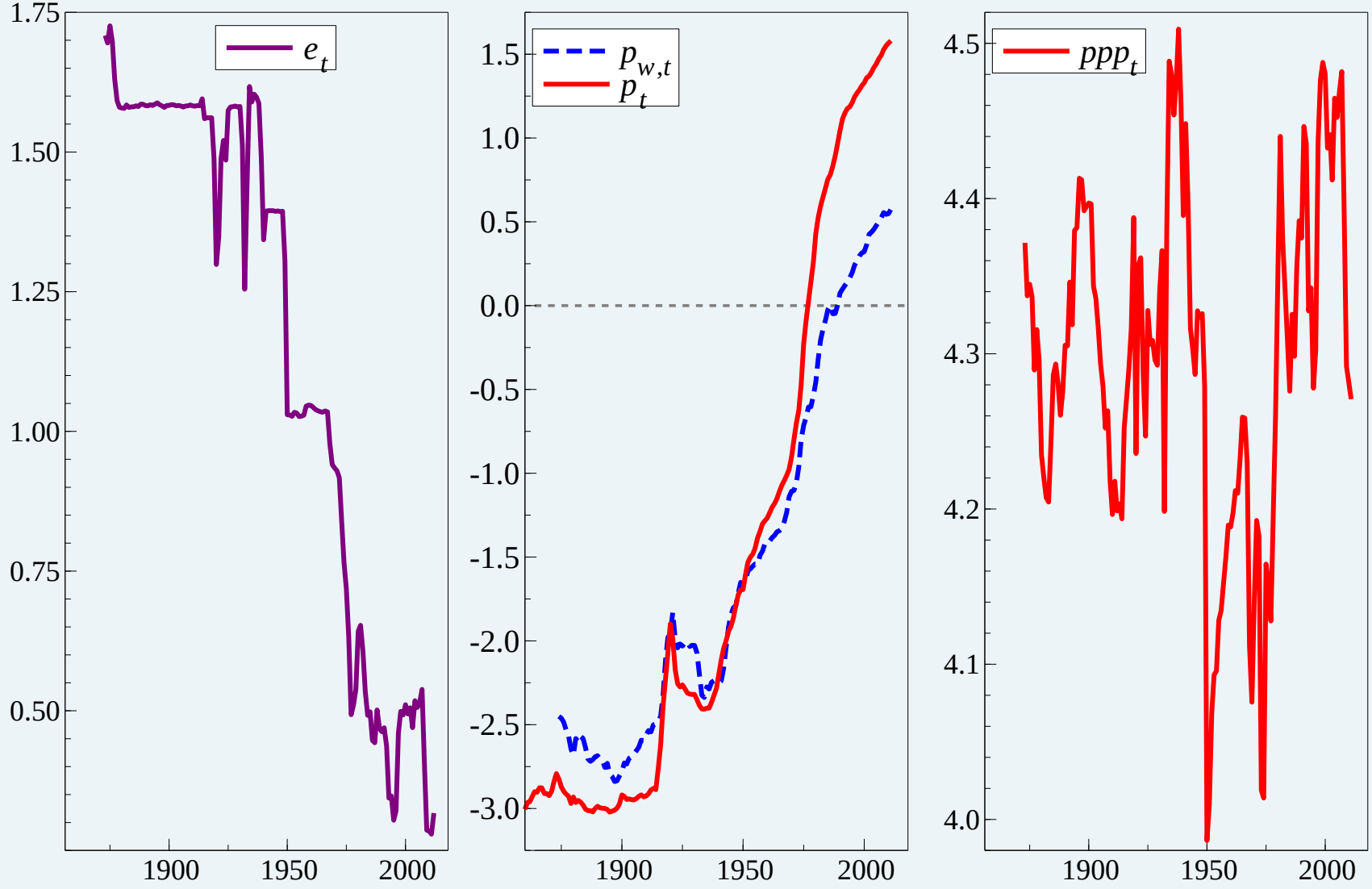


Figure: Exchange rates, UK and world prices, and purchasing power parity over 1860–2011.

Interest rate spread

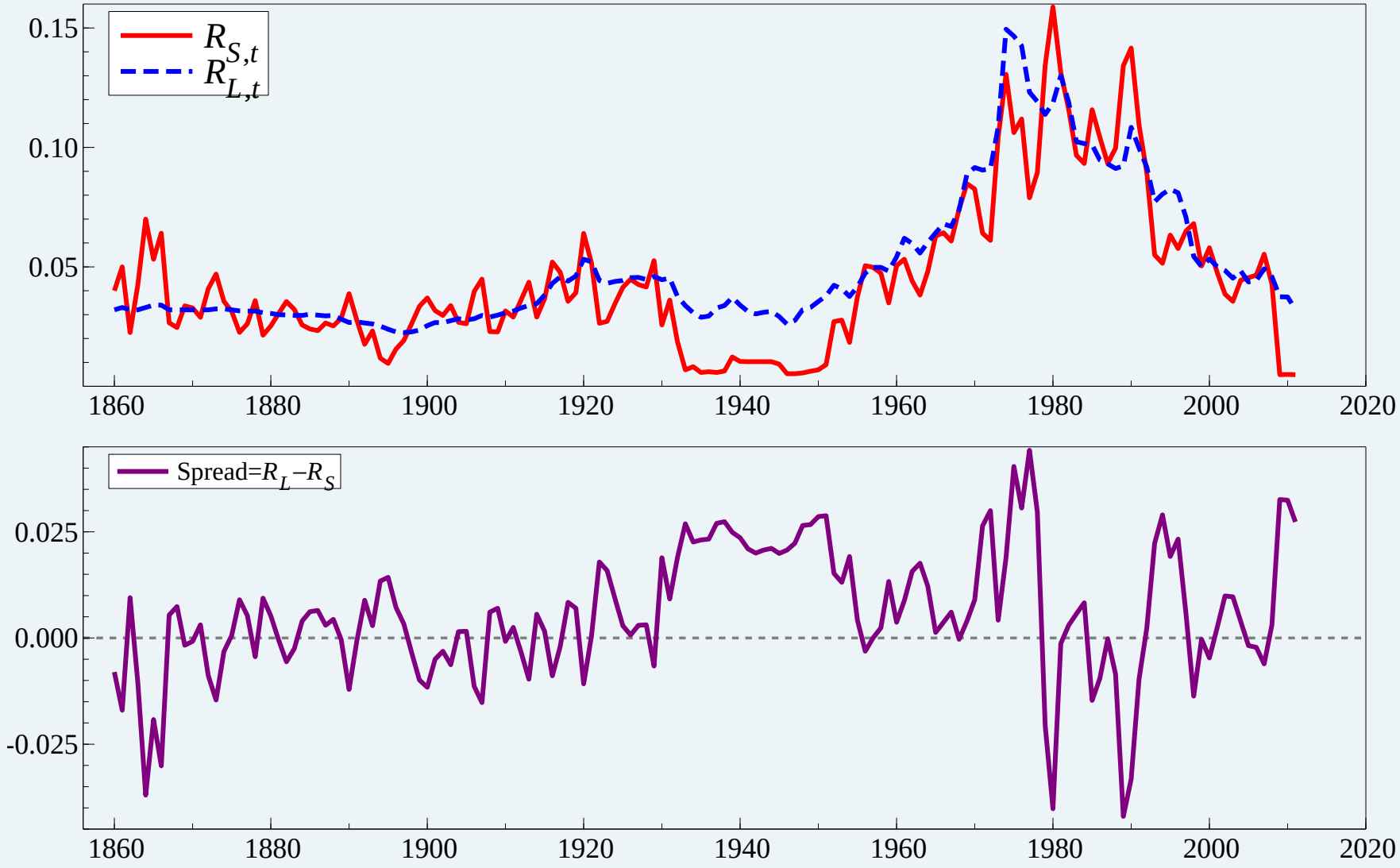


Figure: Interest rates and their spread over 1860–2011.

Evidence about breaks in nominal and real wage growth

Table: Shifts affecting Δw and $\Delta(w-p)$ in % p.a.

mean (SD)	1861–2011	pre-1945	post-1945
Δw	4.3% (5.9%)	2.2% (6.2%)	7.1% (4.2%)
$\Delta(w-p)$	1.3% (2.3%)	0.9% (2.6%)	1.8% (1.7%)

SD in parentheses is standard deviation, in same units as variables.

Means of Δw and $\Delta(w-p)$ much higher post-1945, yet more stable with smaller SDs post WWII.

Also, SD$[\Delta(w-p)]$ always much smaller than SD$[\Delta w]$.

Whole-period mean & SD mis-represent what happened

Major social structural changes in 1945–46:

- nationalization & introduction of NHS;
- unemployment insurance & pensions;
- Beveridge (1942, 1945) reports–mandate for low unemployment using Keynesian policies.

In fact, possibly **nine shifts** in the level of wage inflation.

UK inflation location shifts

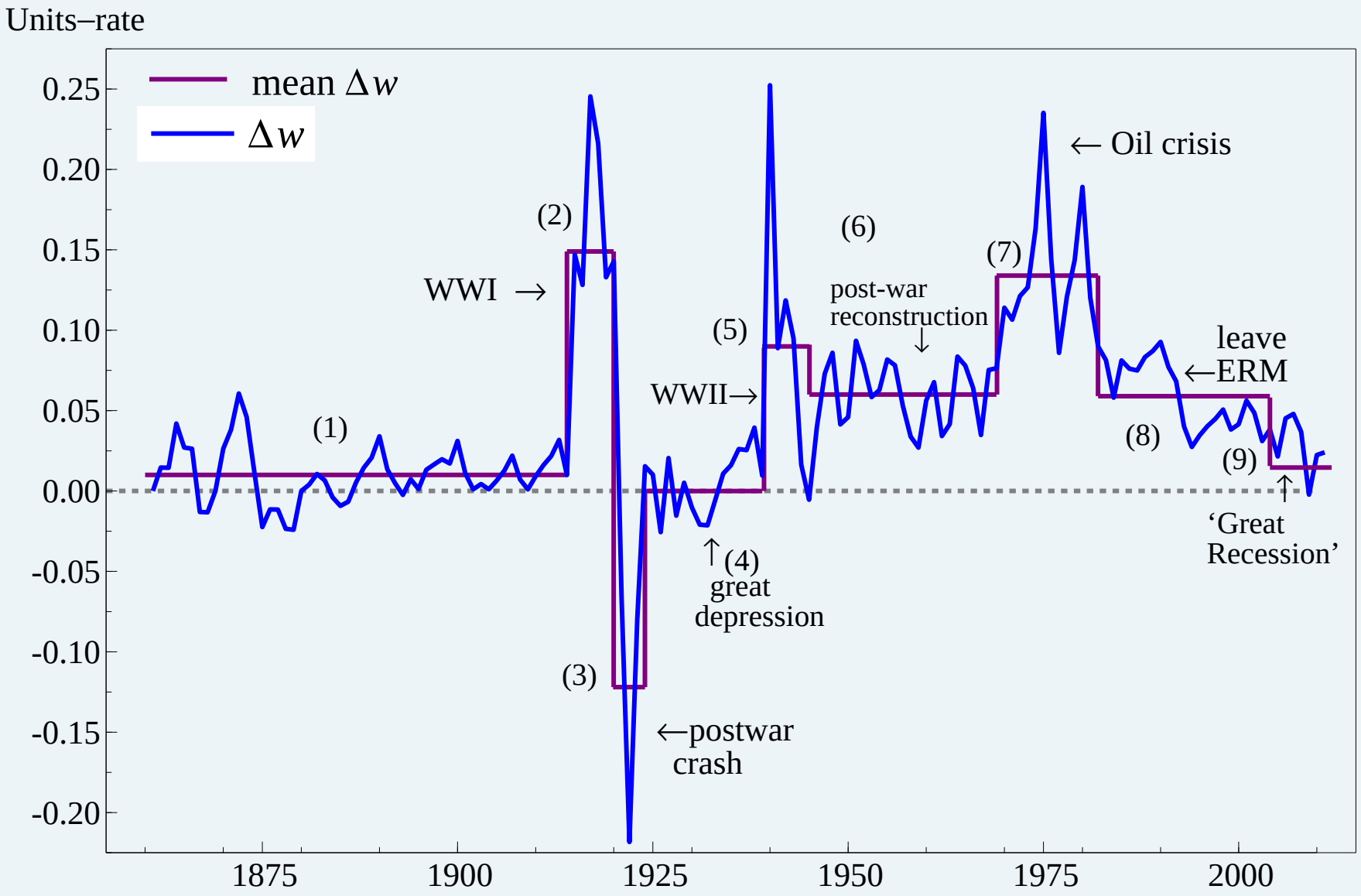


Figure: UK inflation location shifts.

Seem to be nine distinct epochs:

1] business-cycle era 1865 — 1914, 6 cycles of about 8 years;

2] huge rise in WWI, then;

3] large fall over 1921–1923;

4] near zero till start of WWII, then;

5] a huge 'spike' in 1940;

6] falls at end of war, then low and steady till mid 1970s;

7] sharp rise during the oil crises;

8] falls back after leaving ERM.

9] low levels from the 'Great Recession'.

Shifts in means shown in Figure 61 called **location shifts**.

Very different problem from trends: there have been many **major shifts in means (and variances) over time.**

Usually **unanticipated**, so lead to large forecast errors: well illustrated by recent financial crisis.

Table: Mean shifts over nine sub-periods in % p.a.

Sub-sample	Δw	Δp	$\Delta(w-p)$
1861 – 1913	1.00	0.20	0.80
1914 – 1920	14.6	14.0	0.60
1921 – 1923	−12.2	−11.9	−0.30
1924 – 1938	0.50	−0.50	0.90
1939 – 1945	8.20	5.90	2.30
1946 – 1968	6.00	3.90	2.10
1969 – 1981	13.4	11.9	1.60
1982 – 2004	5.90	3.90	2.00
2005 – 2011	2.80	2.28	0.52

These sub-sample divisions are chosen after the fact:
need general procedures for dealing with breaks.

Consider a single mean shift from μ_0 to μ_1 at time $T_1 < T$:

$$y_t = \begin{cases} \mu_0 + \epsilon_t & t \leqslant T_1 \\ \mu_1 + \epsilon_t & t > T_1. \end{cases} \tag{21}$$

where ϵ_t is a random error, with mean zero.

The overall mean of y_t is:

$$E[y_t] = \frac{1}{T}(\mu_0 \times T_1 + \mu_1 \times (T - T_1)) = \mu_0 + \frac{(T - T_1)}{T}(\mu_1 - \mu_0).$$

To model such a shift, use an **indicator variable**, $1_{\{t > T_1\}}$, which is zero till T_1 then unity after, so:

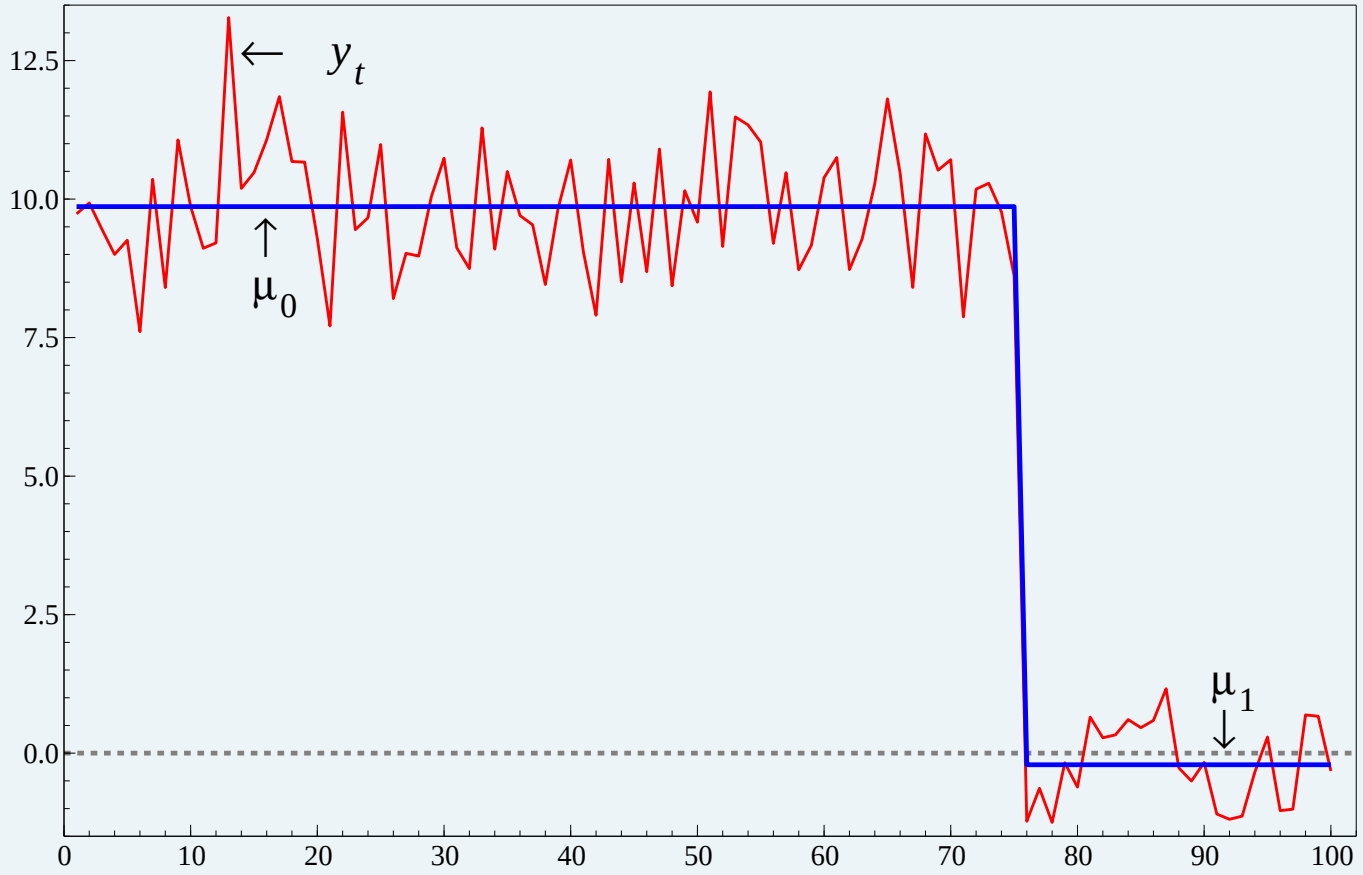
$$\begin{aligned} 1_{\{t>T_1\}} &= 0 \quad t \leqslant T_1 \\ 1_{\{t>T_1\}} &= 1 \quad t > T_1 \end{aligned}$$

and hence:

$$\begin{aligned} y_t &= \mu_0(1 - 1_{\{t>T_1\}}) + \mu_1 1_{\{t>T_1\}} + \epsilon_t \\ &= \mu_0 + (\mu_1 - \mu_0)\, 1_{\{t>T_1\}} + \epsilon_t \end{aligned} \tag{22}$$

Then (22) has an intercept of μ_0, with a **location shift** of $\mu_1 - \mu_0$ at time T_1. The next Figure illustrates.

Location shift example



Shift is $\mu_1 - \mu_0 = 10$ **standard errors** at $T_1 = 0.75T = 75$.

Need an indicator $1_{\{t \geqslant 75\}}$ for the second mean, $\mu_1 = 0$.

Figure 61 had nine mean shifts–so must model **multiple shifts**.

To model many shifts, use multiple **indicator variables**, $1_{\{1 \leqslant t < T_0\}}$, $1_{\{T_1 \leqslant t < T_2\}}$, $1_{\{T_3 \leqslant t < T_4\}}$ each of which is unity over the period shown, so:

$$y_t = \mu_0 + (\mu_1 - \mu_0)\, 1_{\{T_1 \leqslant t < T_2\}} + (\mu_2 - \mu_0)\, 1_{\{T_3 \leqslant t < T_4\}} + \cdots + \epsilon_t \tag{23}$$

Now (23) has an intercept of μ_0, with mean shifts of $\mu_1 - \mu_0$ at time T_1, $\mu_2 - \mu_0$ at time T_3, etc.

8 mean shifts in Figure 61 are 13.6%, -13.2%, -0.5%, 7.2%, 5.0%, 12.4%, 4.9% and 1.8% from an initial-sample mean of $\mu_0 = 1.0\%$.

Could use 9 indicators and no overall mean; but not both.

To 'explain' such behaviour requires some other variables that rise and fall in those patterns at the same times....

Called **co-breaking**, which is when breaks occur in several series, and a combination of them has fewer breaks.

Graphical evidence of co-breaking

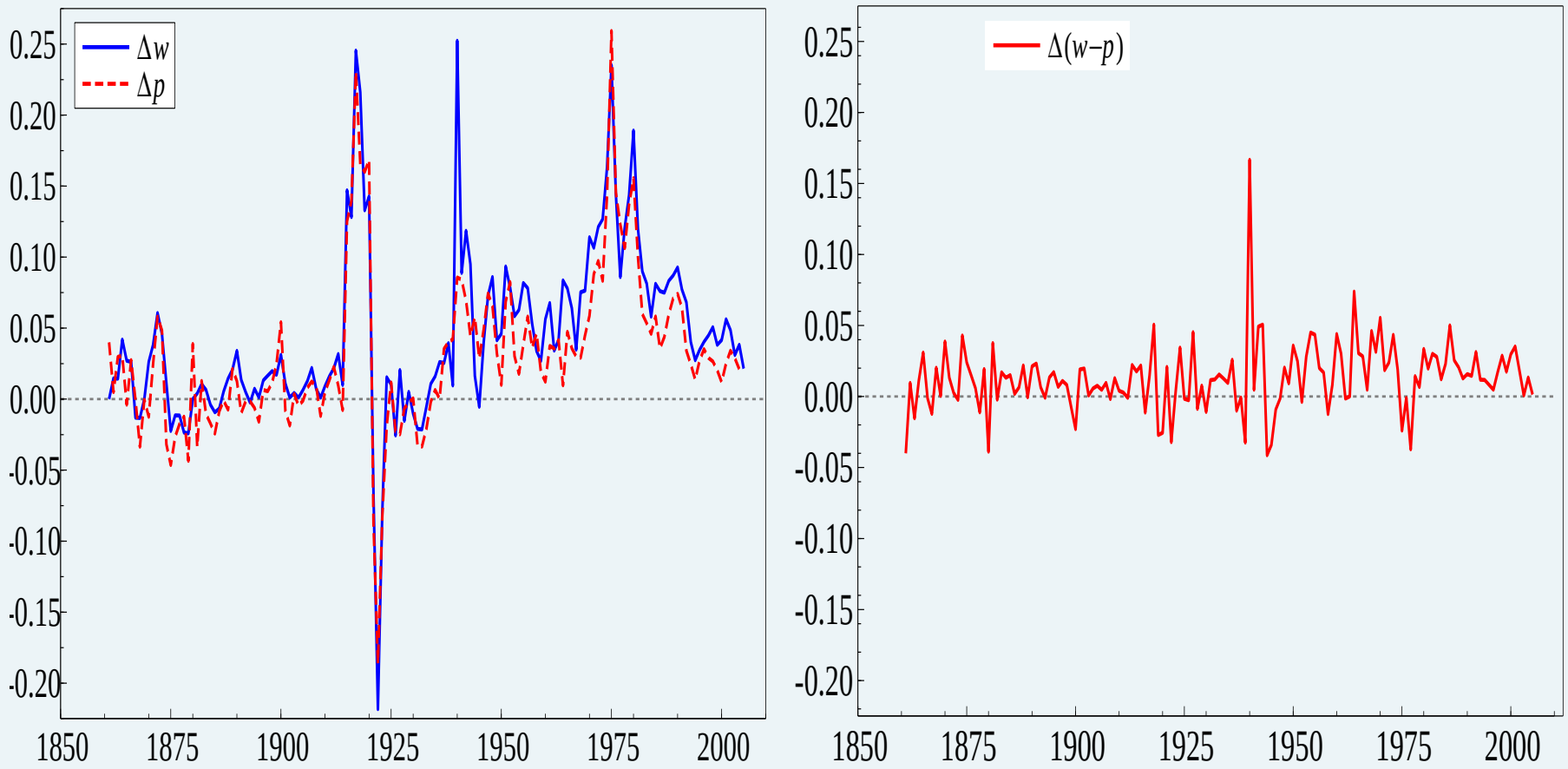


Figure 44 for wage and price inflation rates (left panel) shows many shifts compared to real wage changes (right).

Need to analyze co-breaking between Δw and Δp with many shifts, to produce $\Delta(w - p)$ with few.

For a single break, consider two equations:

$$\begin{aligned} y_t &= \mu_0 + (\mu_1 - \mu_0)\, 1_{\{t \geqslant T_1\}} + e_t \\ z_t &= \mu_0 + (\mu_1 - \mu_0)\, 1_{\{t \geqslant T_1\}} + \nu_t \end{aligned} \tag{24}$$

Then:

$$y_t - z_t = e_t - \nu_t \tag{25}$$

and the shift has cancelled for that unique linear combination.

While (24) is again a simple example, it illustrates co-breaking.

Thus, we have now seen that in economics, **trends and breaks are ubiquitous**, so have discussed how to handle:

a] **trends: by differencing and cointegration**;

b] **breaks: by indicators for shifts and co-breaking**.

First 'removes' the problem; second 'models' it.

Instead of a linear relation like $y_t = \beta_0 + \beta_1 x_t + \epsilon_t$,
y_t may be a non-linear function of x_t denoted $y_t = f(x_t) + \epsilon_t$.
Vast number of possible non-linear functions. Simplest is a polynomial:

$$y_t = \theta_0 + \theta_1 x_t + \theta_2 x_t^2 + \theta_3 x_t^3 + \nu_t \tag{26}$$

(further powers of x_t could be included).
Although non-linear in the variable x_t, (26) is linear in its parameters θ_i so can be estimated by regression
(to see that it is a linear model, re-define $x_t^i = z_{i,t}$ for $i = 1, 2, 3$).
(26) can also be used to test whether $f(x_t)$ is non-linear by testing the significance of x_t^2 and x_t^3.
Figure 70 shows graphs of a quadratic (first column), cubic (second column) and the exponential term $x_t e^{-|x_t|}$ (third column), representing a variety of non-linear reactions.

Three non-linear functions

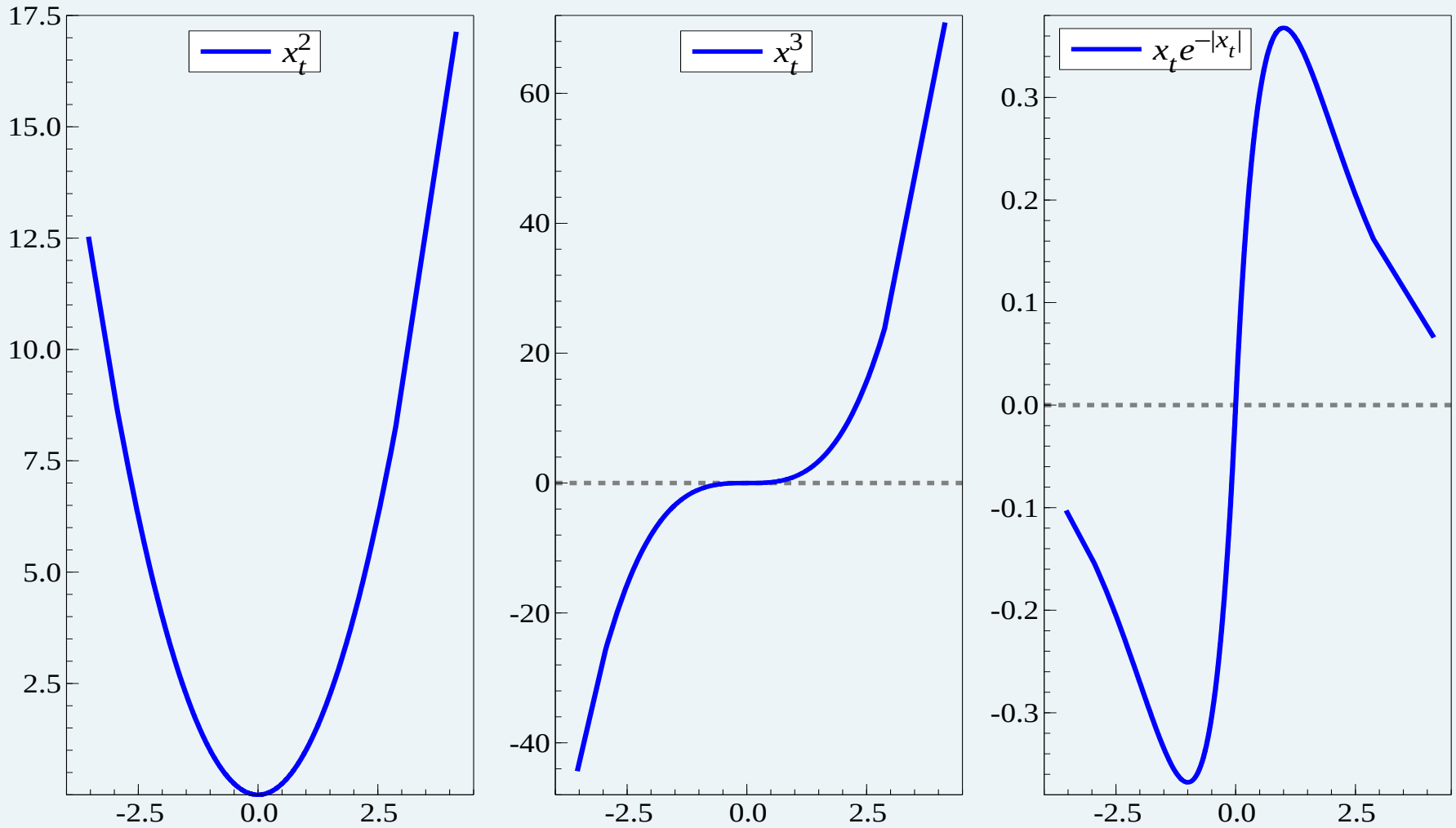


Figure: Three non-linear functions of x_t: quadratic, cubic, and an exponential

Modelling non-linear reactions by smooth transitions

A widely used non-linear model in macroeconomics is a *logistic smooth transition*, which allows for more than one 'regime' between which the economy moves more or less rapidly.

Let $s_t = (x_t - c)/\sigma_x$, so s_t is in units of the standard deviation, σ_x, and c is the threshold which triggers a shift, depending on how far x_t is from c. Also, let $\gamma > 0$ determine the rapidity of the adjustment.

The function $1/(1 + \exp(-\gamma s_t))$ then ensures a bounded outcome, as it becomes unity for large positive s_t, and zero for large negative.

With two regimes, express y_t as the following function of x_t:

$$y_t = \mu + \beta x_t + \mu^* \left(1 + \exp\left(-\gamma s_t\right)\right)^{-1} + \epsilon_t \tag{27}$$

In (27), μ and β are the original mean and slope, and $\mu + \mu^*$ is the mean after the shift to the second 'regime'.

Such a model is non-linear both in the variables and in the parameters.

Setting $\mu^* = 0$ makes γ disappear from the model, which raises what is called an identification problem, making it can be hard to select a model when parameters enter non-linearly.

Location shifts and non-linear reactions

Although many aggregate economic time series appear to exhibit location shifts, sudden movements in levels could also be the outcome of a non-linear reaction like (27) where γ is large.
Conversely, location shifts can approximate such non-linearities.

The wrong choice between a location shift and a non-linear reaction can have an adverse impact on models and forecasts, incorrectly extrapolating a non-existent shift, or a spurious non-linearity.

Fortunately, when the DGP is in either the class of shifts or non-linearities, the model selection methods described in Lecture 4 will usually make a good choice.
When observed jumps are due to the same non-linear reaction at different times, that provides a more parsimonious representation than many separate indicators, so is usually chosen;
and when location shifts are not connected, indicators usually replace an incorrect non-linear function.

We return to non-linear models in Lecture 6.

Summary of lecture 2 key points

(A) Wage and price **inflation** did not trend over 150 years.
(B) Both show many large sudden location shifts.
(C) Many of those location shifts match, so real wages rarely jump, with exception of a huge spike in 1940.
(D) Real-wage growth is trend free, but doubled after WWII.
(E) Rate of productivity growth changed: capital was more productive, capital per worker grew, with workers more educated and skilled.
(F) Real-wage growth 'tames' the huge rises in nominal levels of wages and prices.
(G) Can also 'tame' by **cointegration** cancelling trends.
(H) Next, 'removed' breaks by indicators.
(I) Allowed formulation of a model of **co-breaking**, cancelling breaks.
(J) Non-linear reactions are also possible, and may need modelled.
Next consider **dependence** between variables, and over time.

Route map

(1) Macroeconomic data: evolution with abrupt change

(2) Taming trends and breaks

(3) **Dependence between variables and over time**

(4) Economic theory and statistical analysis: 2 key ingredients

(5) Modelling UK unemployment

(6) Modelling UK wages

(7) Modelling UK money demand

(8) Modelling UK prices

(9) Conclusion

Lecture 3 guide post

1. Most macroeconomic variables are highly correlated, but their correlations are not constant over time.
2. Correlations between current levels and their previous values (called autocorrelations) are often near unity even 20 years apart.
3. Such findings are consistent with macroeconomic variables having stochastic trends, but being perturbed by shifts.
4. Matching that, autocorrelations between the differences of variables usually 'die out' rapidly as their distance apart increases.
5. Correlations between first differences are often low: real-wage growth removes stochastic trends of wages and prices, most of their breaks, and the autocorrelations of wage and price inflation.
6. The relationship between unemployment and wage inflation, known as the Phillips curve, shifts over time.
7. Non-constant relations are common in empirical macro economics, so important determinants may have been omitted.
8. To understand the behaviour of macroeconomic variables requires models that include all substantively relevant influences.

The levels of aggregate economic variables are highly intercorrelated.

Table: Correlations between macroeconomic time series

1860–2011	p	w	g	l	k
p	1.000	**0.998**	**0.946**	0.819	**0.952**
w	–	1.000	**0.964**	0.848	**0.967**
g	–	–	1.000	**0.950**	**0.997**
l	–	–	–	1.000	**0.939**
k	–	–	–	–	1.000

All ten correlations large and positive, and eight exceed 0.9, shown in bold.

Figure 77 records their time series as a group (with UK population, *pop*), matched by means and ranges.

Figure 78 shows a range of scatter plots.

Many aggregate UK economic time series

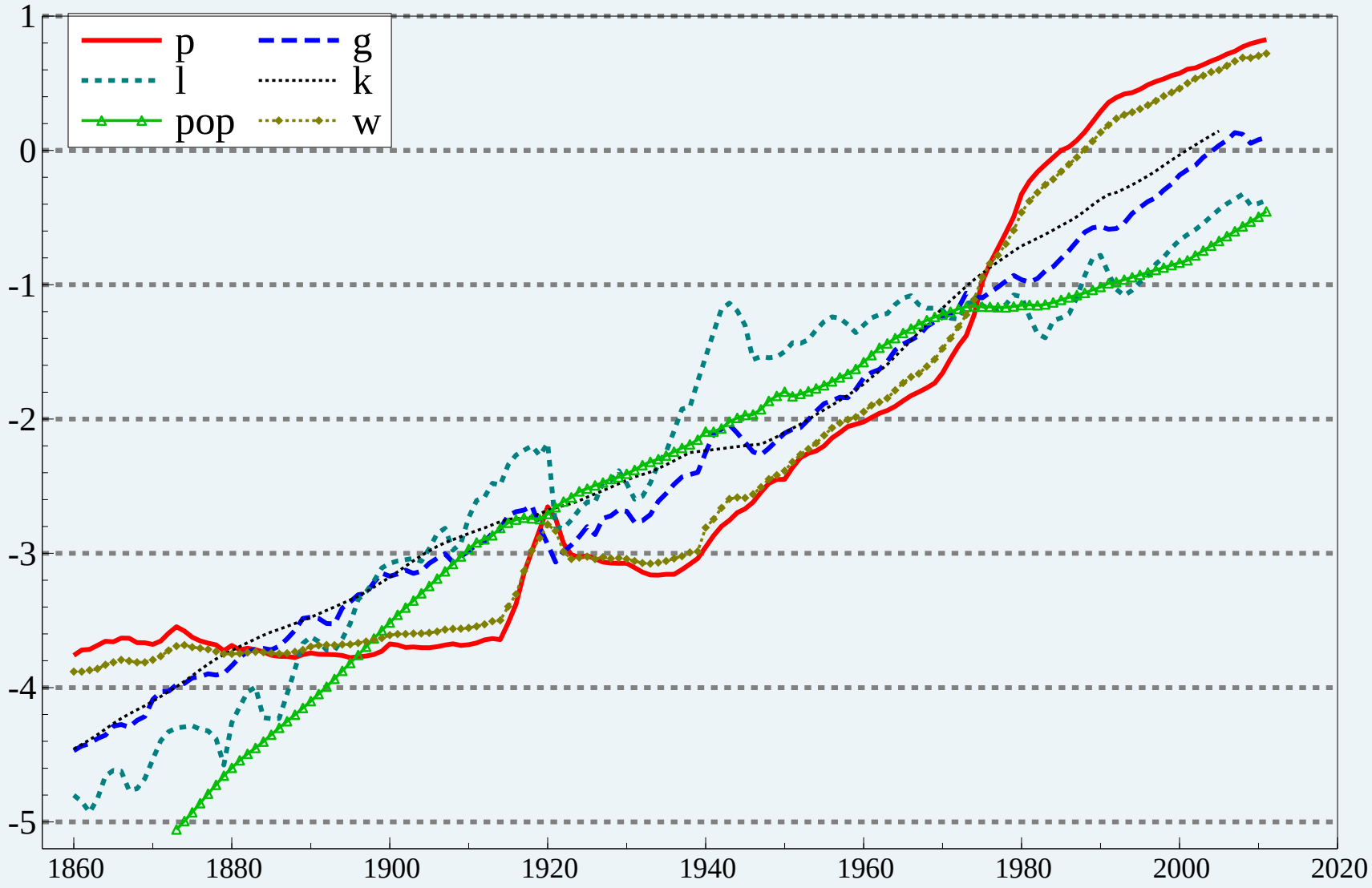


Figure: Many aggregate UK economic time series.

Scatter plots of UK economic variables

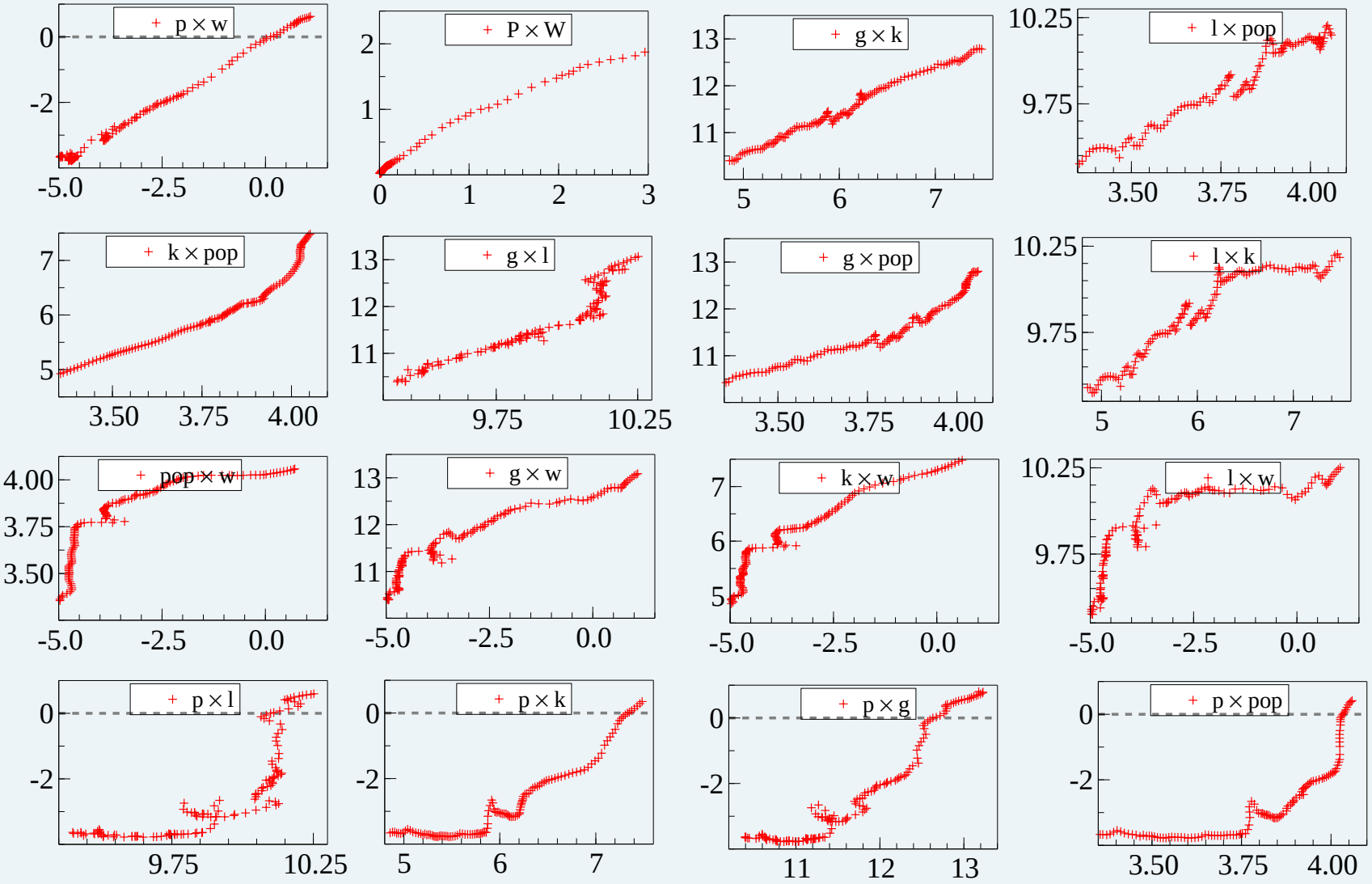


Figure: Scatter plots of UK economic variables.

Six key features of the scatter plots in Figure 78 are common across many macroeconomic time series.

1] quite unlike cross-section scatter plots;

2] points are plotted–but look like lines;

3] all share feature of going from lower left to upper right;

4] most far from straight lines, even rows 1 & 2;

5] some rise sharply early, then 'flatten off' (row 3);

6] others 'flat', then suddenly rise (row 4).

Conclude: correlations high but vary over time.

Many variables 'proxy' each other (spurious relations), and relations are not constant: but a third difficulty lurks.

Correlations between successive values of each variable are very high: moving from $\text{corr}(x_t, y_t)$ to $\text{corr}(x_t, x_{t-1})$ called an autocorrelation.

Figure 80 records plots of x_t against x_{t-1}.

Plots of x_t against its lagged value x_{t-1}

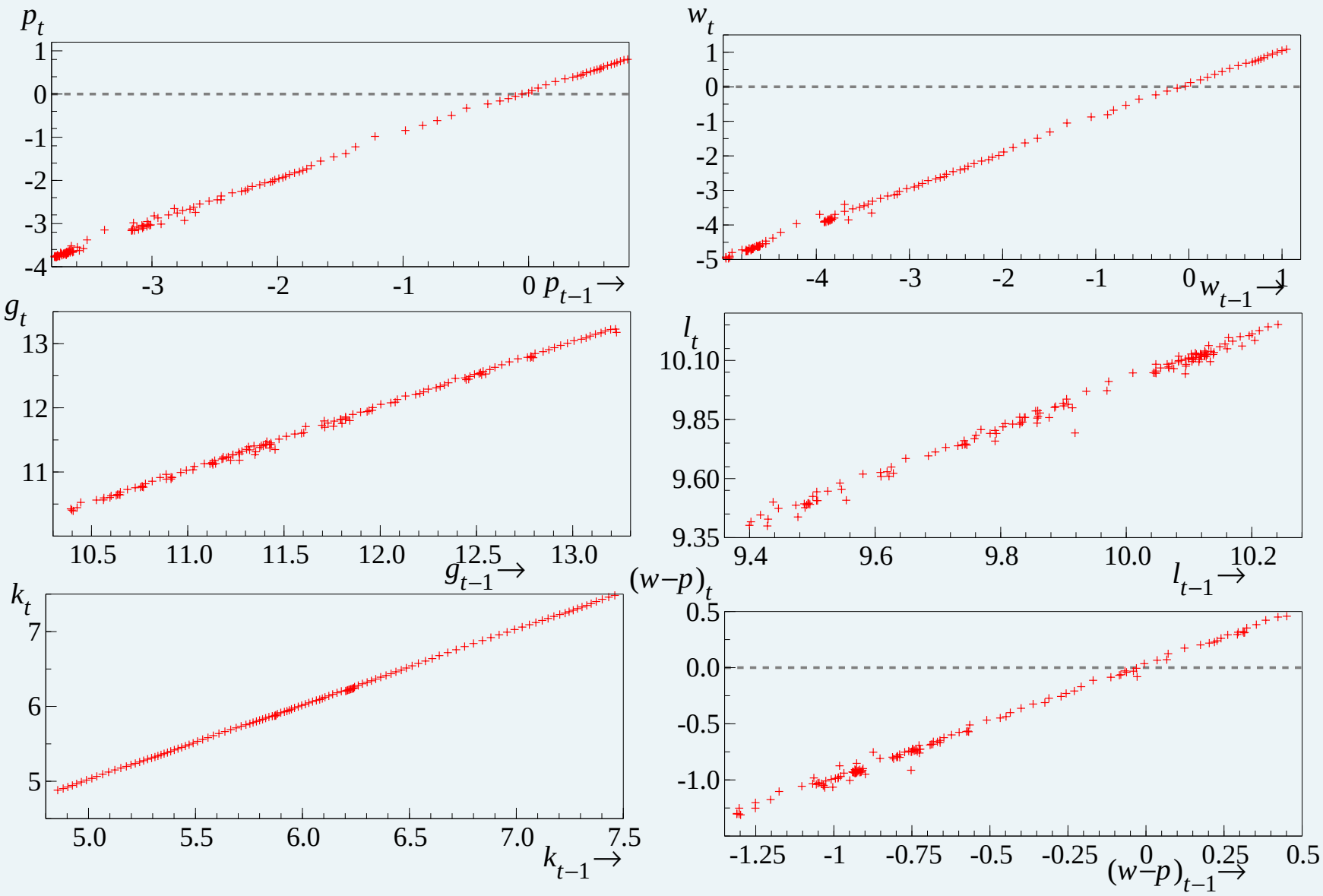


Figure: Plots of x_t against its lagged value x_{t-1}.

Almost perfect straight lines for all 6 variables, despite their rather different time series graphs seen earlier.

Once $\text{corr}(x_t, x_{t-1})$ is thought about, realize can get a whole string $\text{corr}(x_t, x_{t-2})$, $\text{corr}(x_t, x_{t-3})$, ..., $\text{corr}(x_t, x_{t-s})$ say.

Table 4 shows these 'autocorrelations' for $s = 5$ lags.

Table: Autocorrelations for macroeconomic time series

$\text{corr}(x_t, x_{t-s})$	x_{t-1}	x_{t-2}	x_{t-3}	x_{t-4}	x_{t-5}
p_t	0.999	0.998	0.996	0.993	0.991
w_t	0.999	0.998	0.997	0.995	0.993
g_t	0.999	0.996	0.996	0.994	0.993
l_t	0.996	0.990	0.983	0.977	0.974
k_t	0.9999	0.9997	0.9994	0.9989	0.9983

All large and positive: **smallest is 0.974**, consistent with strong trends of the form $x_t = x_{t-1} + \beta + \epsilon_t$ as in (9).

Easiest to graph autocorrelations for many periods:
a plot of $\text{corr}(x_t, x_{t-s})$ against s.

Called a **correlogram**.

Figure 83 shows these.

All are close to unity for 20 years–almost complete 'persistence':
$\text{corr}(x_t, x_{t-20}) \simeq 1$ for six of the series.

Dashed lines show approximate 95% confidence bands for the null hypothesis that each autocorrelation is zero:
all very far outside the null band throughout.

None 'die out' as length s increases.

Why does that happen?

Correlograms of levels of variables

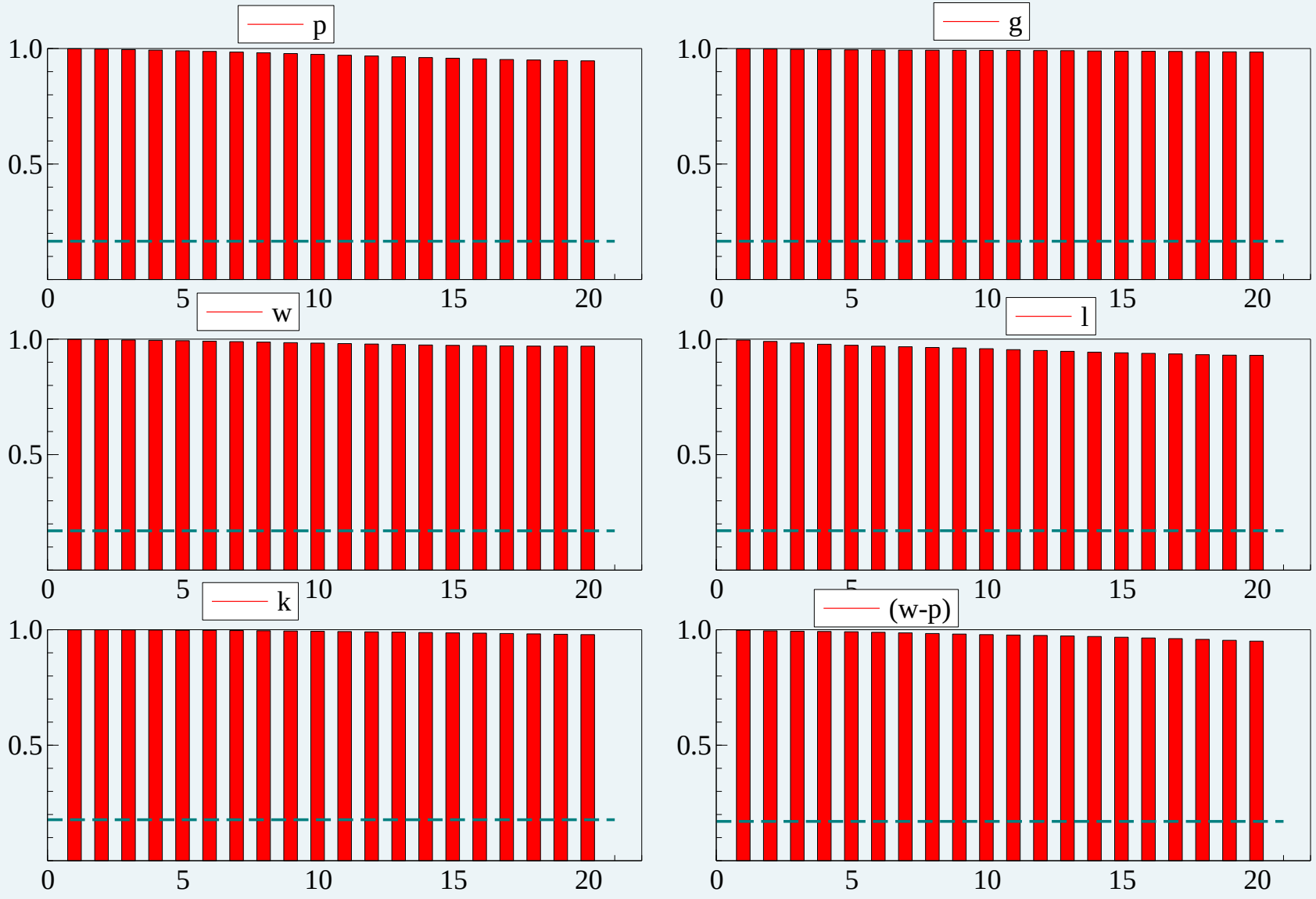


Figure: Correlograms of levels of variables.

When $x_t = x_{t-1} + \beta + \epsilon_t$, we saw that:

$$x_t = x_0 + \beta t + \sum_{s=1}^{t} \epsilon_s \tag{28}$$

Hence, x_t depends on t epsilons. As the ϵ_s are all independent, the variance of the sum is the sum of the variances, so $\text{Var}[x_t] = t\text{Var}[\epsilon_t]$. Even with no trend so $\beta = 0$:

$$x_{t-20} = x_0 + \sum_{s=1}^{t-20} \epsilon_s \tag{29}$$

which has 20 epsilons fewer. So x_t and x_{t-20} share $(t-20)$ epsilons, leading to $\text{Cov}[x_t, x_{t-20}] = (t-20)\text{Var}[\epsilon_t]$.
Finally, $\text{Var}[x_{t-20}] = (t-20)\text{Var}[\epsilon_t]$.
All 3 components of correlation $\text{corr}[x_t, x_{t-20}]$ share $\text{Var}[\epsilon_t]$ so:

$$\text{corr}[x_t, x_{t-20}] = (t-20)/\sqrt{t(t-20)} = \sqrt{1-20/t} \simeq 1 \tag{30}$$

when t is large. Even higher autocorrelations result if $\beta \neq 0$.

Now graph correlograms of changes: corr$(\Delta x_t, \Delta x_{t-s})$.
Removes common **cumulative** epsilons and trend if $\beta \neq 0$.

Figure 86 shows these correlograms of differences.

Rather different from each other, and very different from autocorrelations of levels.

Wage and price **inflation** autocorrelations start high, and are positive for about 10 years—some evidence of 'persistence'—but Δg_t, Δl_t near zero after one period.

Correlogram for Δk_t (net investment) more like I(1).

Here all correlations 'die out' as length s increases.

Considerable 'cancellation' for $w - p$ relative to p and w:

so $\Delta(w - p)$ **has removed:**
the common stochastic trends of p and w;
most of their breaks; and their high autocorrelations.

Correlograms of differences of variables

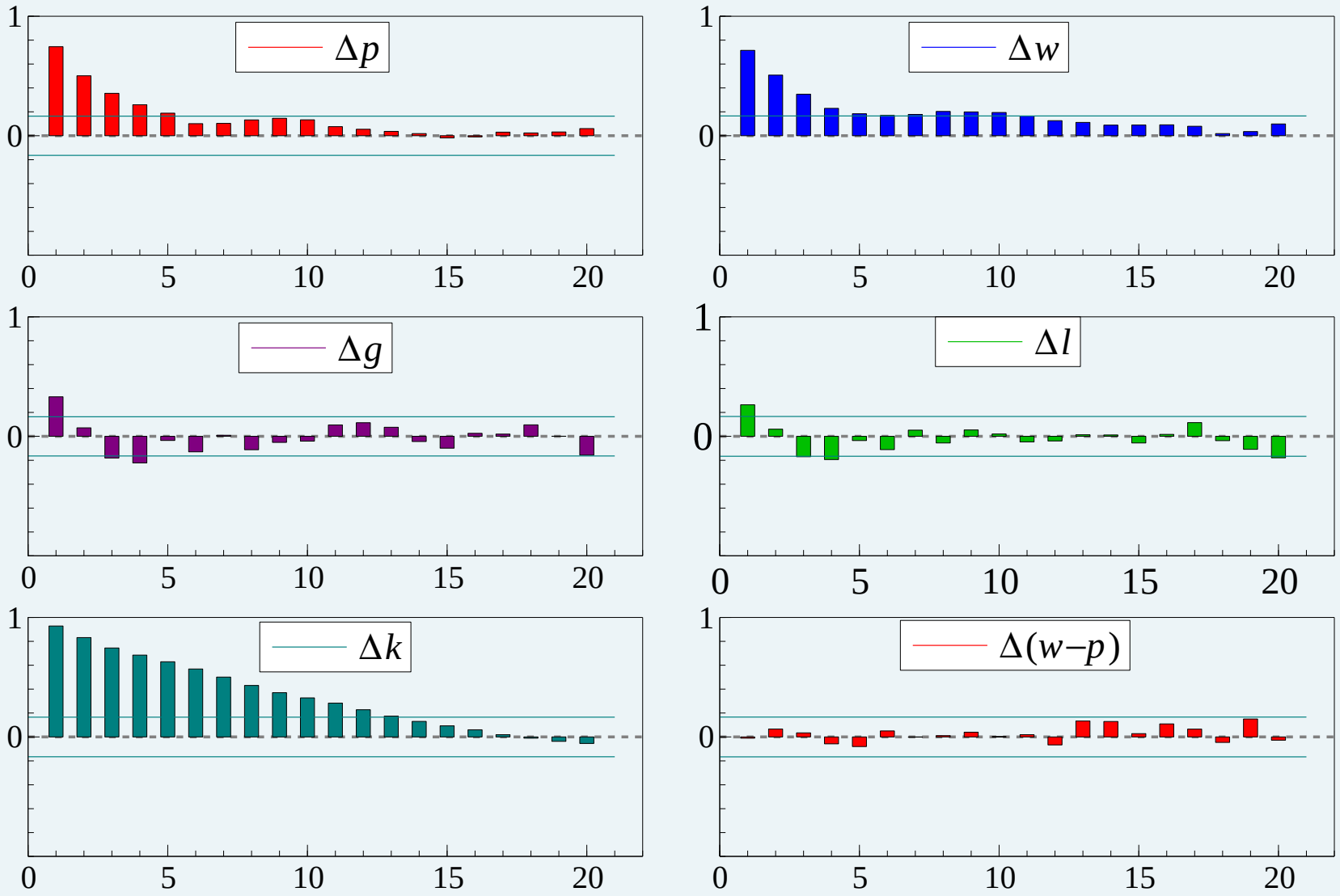


Figure: Correlograms of differences of variables.

Correlations between differenced variables

Correlations between variables are also greatly altered by differencing: see Figure 88 for $\text{corr}(\Delta x_{j,t}, \Delta x_{k,t})$, $j \neq k$.

Completely different from correlations between levels, and rather different from each other.

First three are quite high and positive, for Δw & Δp; Δg & Δl; and $\Delta(w - p)$ & Δw.

Next three also quite close but negative, between ΔU_r & Δl; Δg & ΔU_r; and Δw & ΔU_r, where U_r is the unemployment rate, the next variable we will consider.

Final three are all relatively uncorrelated, between $\Delta(w - p)$ & ΔU_r; Δl & Δk; and Δk & ΔU_r.

We will now apply what we have learned to the evidence on UK unemployment.

Many correlations between differences

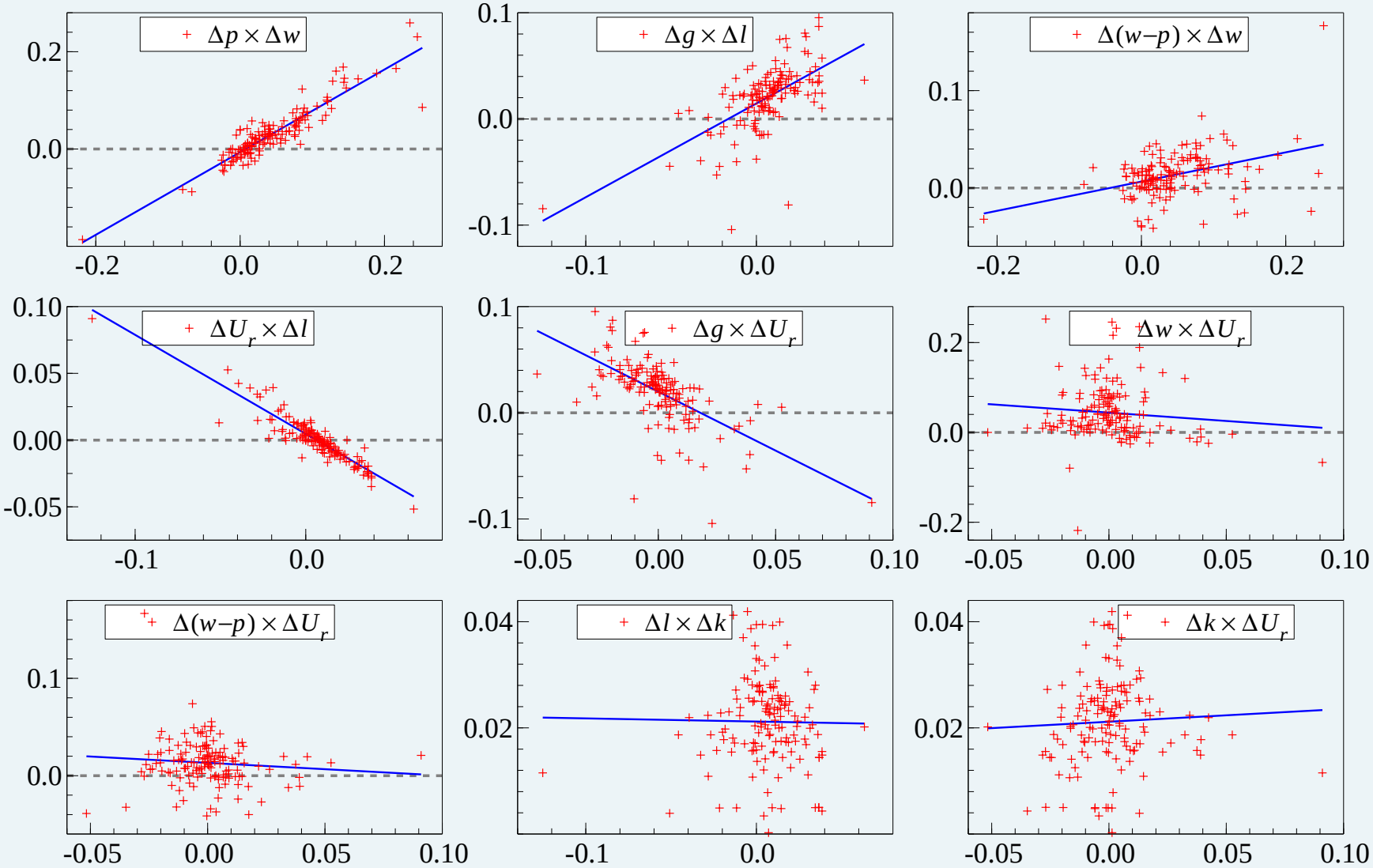


Figure: Many correlations between differences.

UK unemployment

Unemployment, U_n, is the difference between
the **supply of labour willing to work**, L^s,
and the **demand for that labour**, L^d, so $U_n = L^s - L^d$.

The **unemployment rate**, U_r, is U_n/L^s.

It is often assumed that employment, L, equals L^d:
firms do not employ unnecessary labour,
but there is good evidence of cyclical 'labour hoarding'.

L^d is a derived demand to produce goods and services:
and varies as the profitability and volume of sales change.

Total labour supply depends on population of 'working age', its quality and hours of work–all of which have changed greatly over the last 150 years–and on 'willingness to work' at current wages.
L^s is measured by surveys of 'willingness to work':
and U_n is also directly measured by registrations.
All the data are estimates, prone to revision–but let's see what happened: Figures 90–93 show the history.

UK unemployment rate 1860–1918

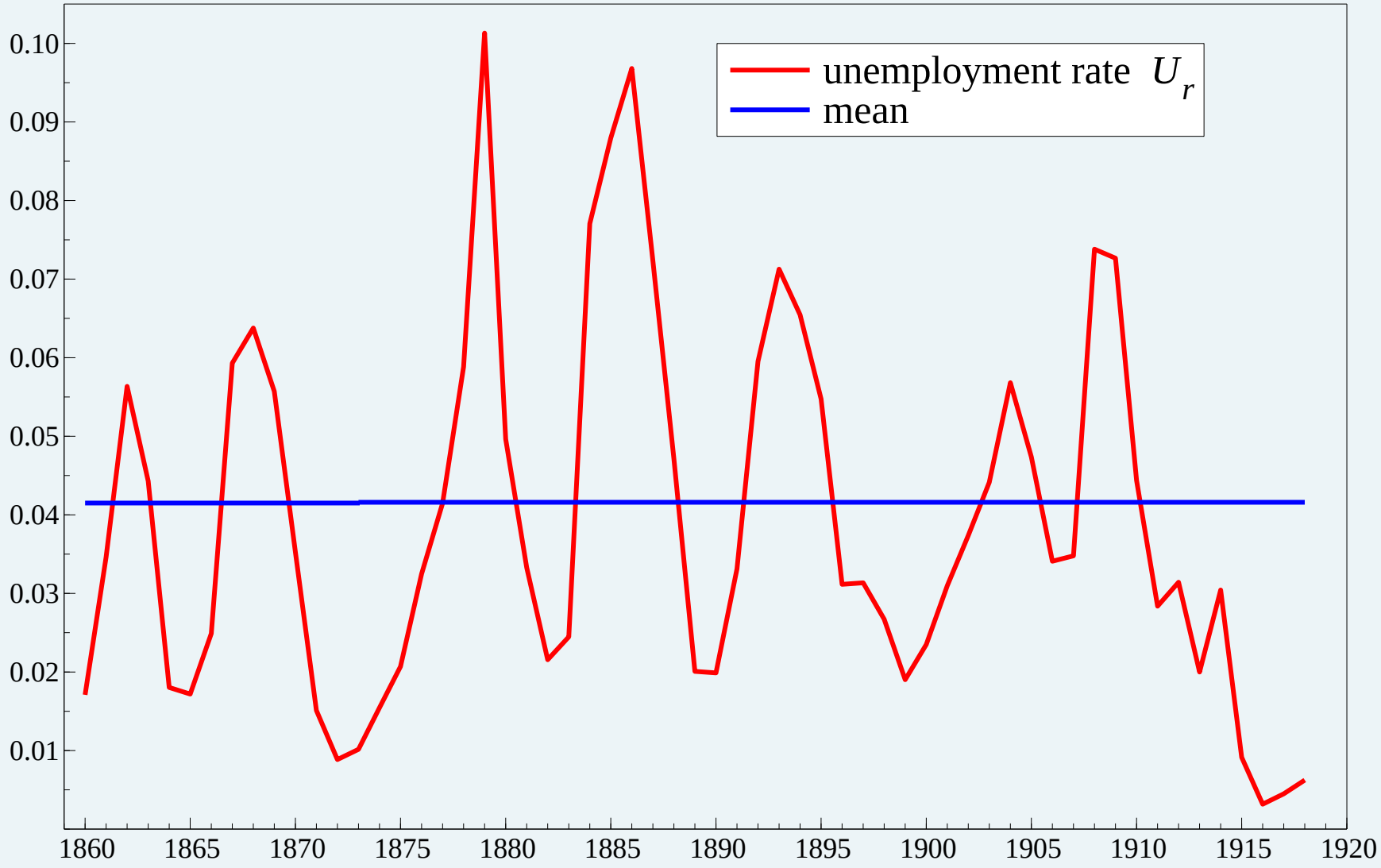


Figure: UK unemployment rate 1860–1918.

UK unemployment rate 1860–1940

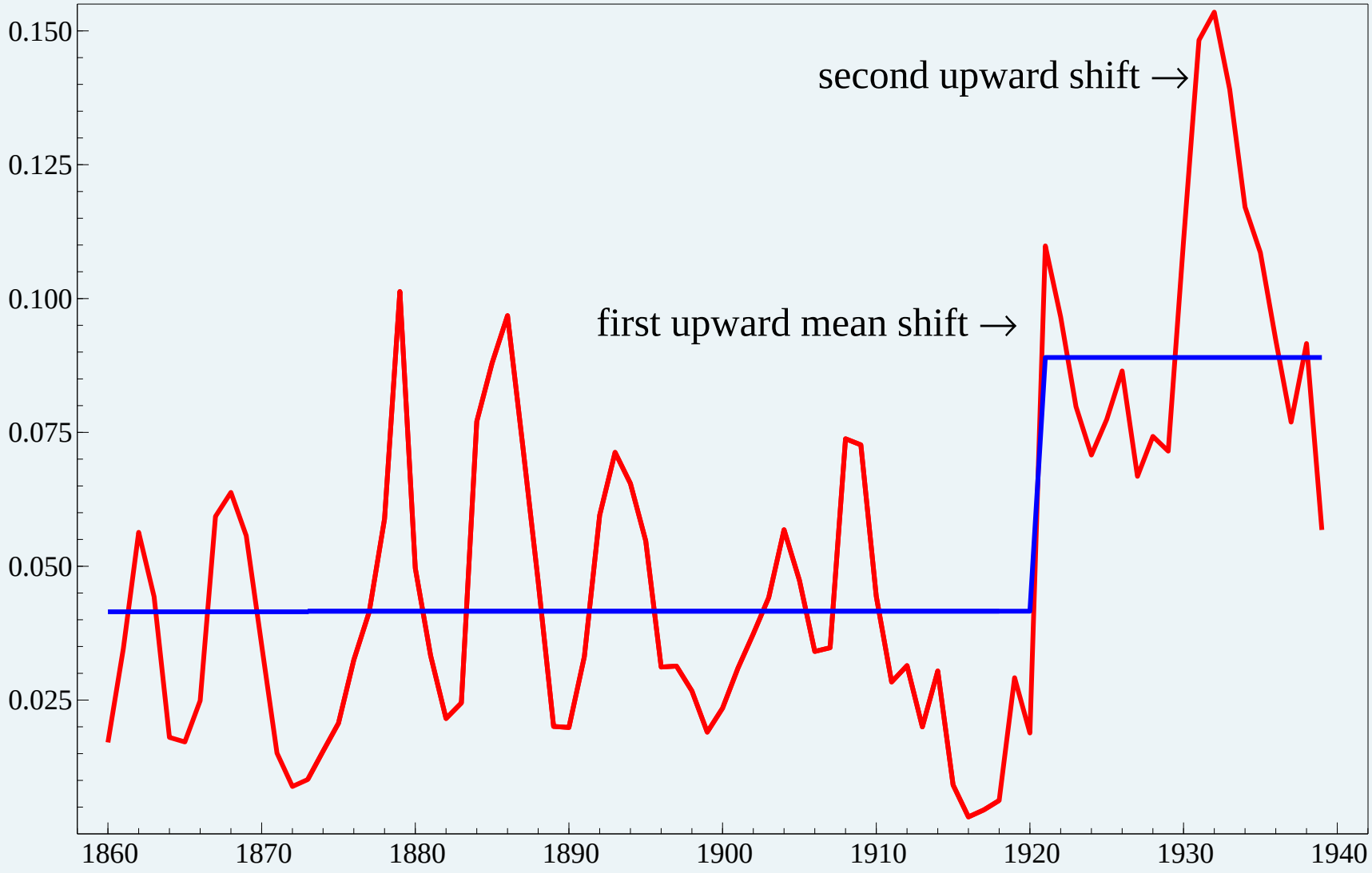


Figure: UK unemployment rate 1860–1940: major upward shifts.

UK unemployment rate 1860–1980

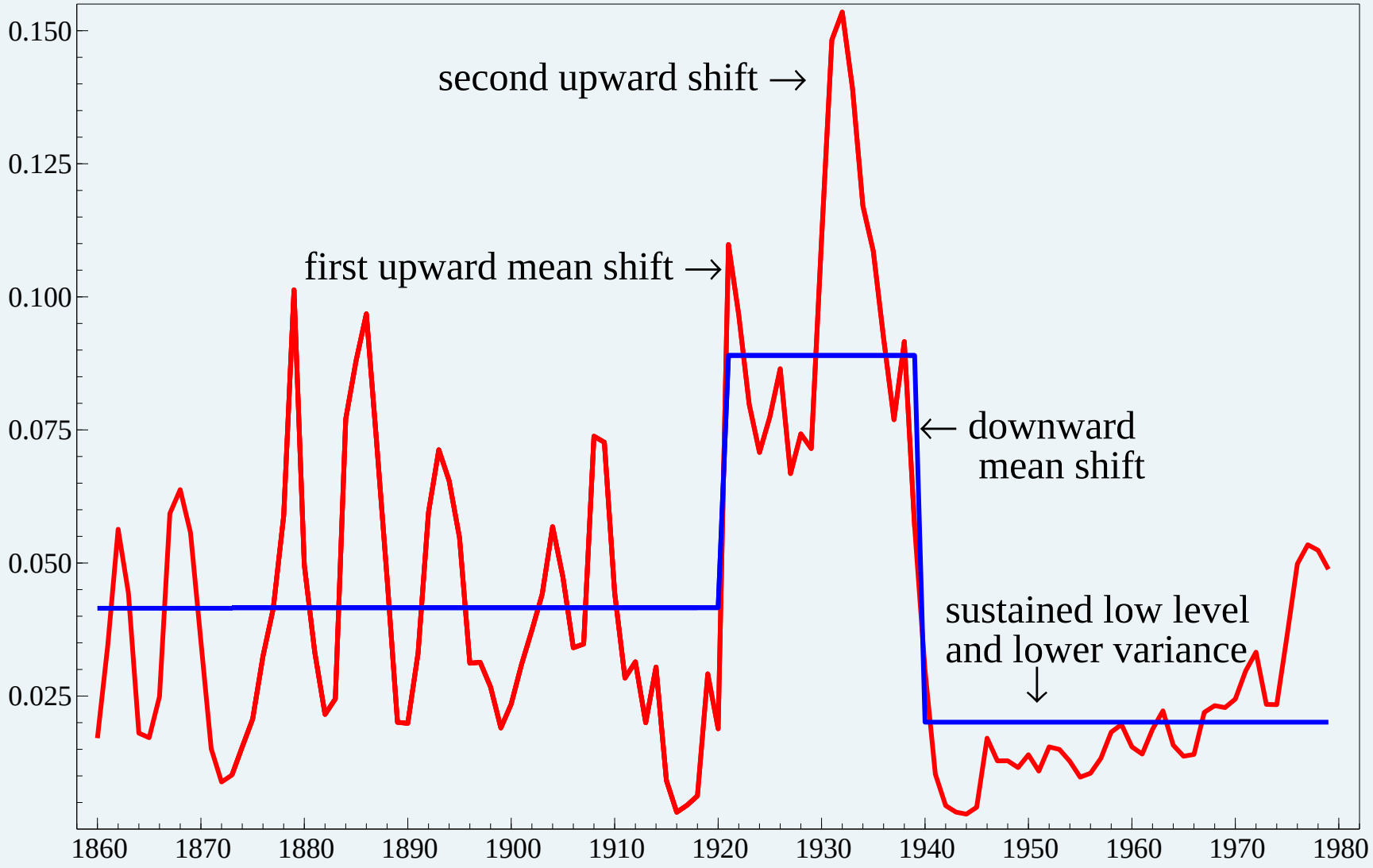


Figure: UK unemployment rate 1860–1980: major downward shift.

UK unemployment rate 1860–2011

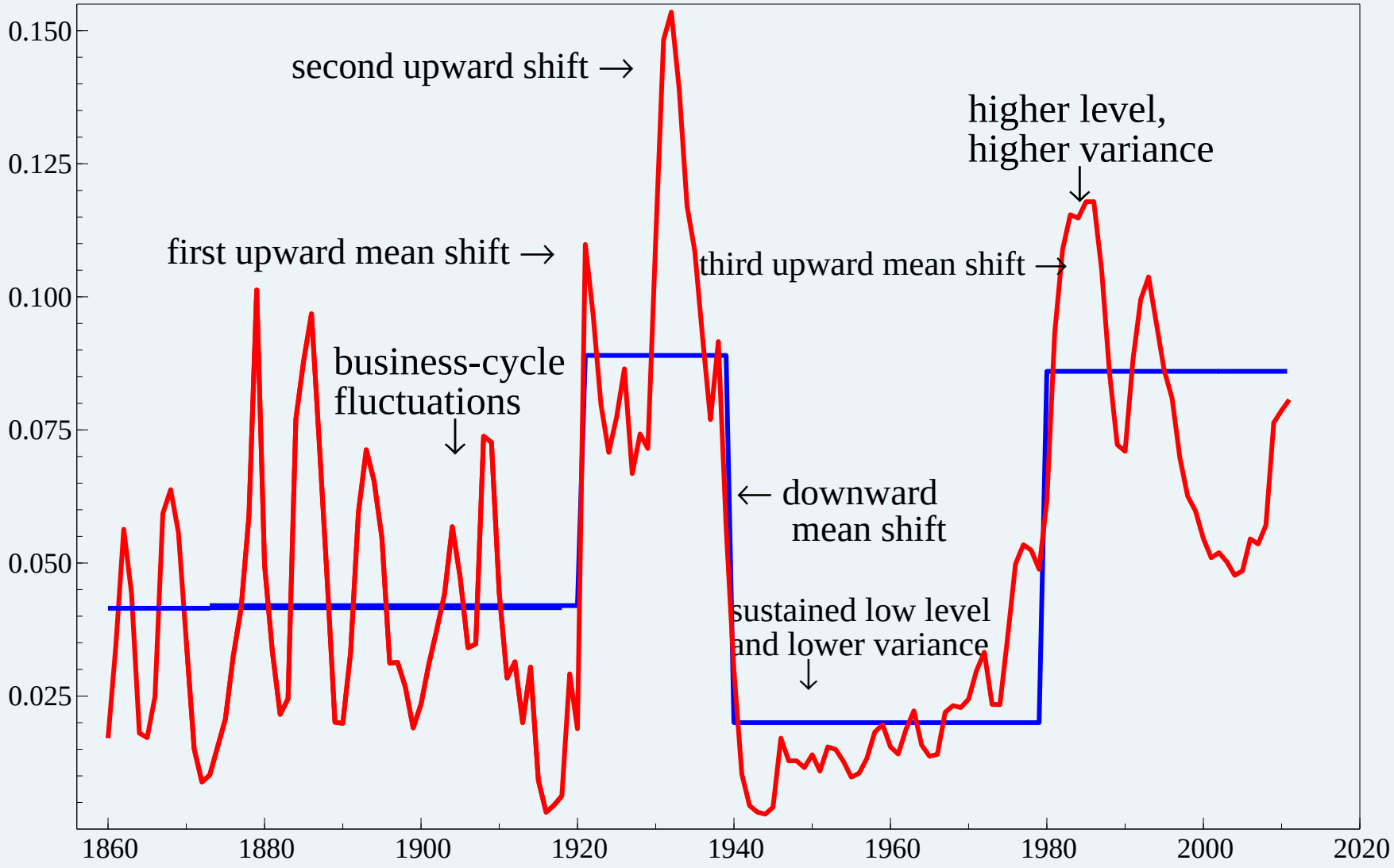


Figure: UK unemployment rate 1860–2011: another major upward shift.

Causes and consequences of unemployment

Major shifts in the behaviour of unemployment have occurred, large up and down changes in its mean, usually very rapidly.

Variance of unemployment has also changed from business-cycle era, high during and after WWI and in the inter-war period, to low post WWII, then high again from the late 1970s.

As Figure 95 reveals, most are associated with major events: wars, financial crises and depressions, inappropriate policy regimes.

Two key issues:
what determines unemployment;
and what does it determine?

Will start with the second:
does unemployment determine inflation?

Events affecting UK unemployment 1860–2011

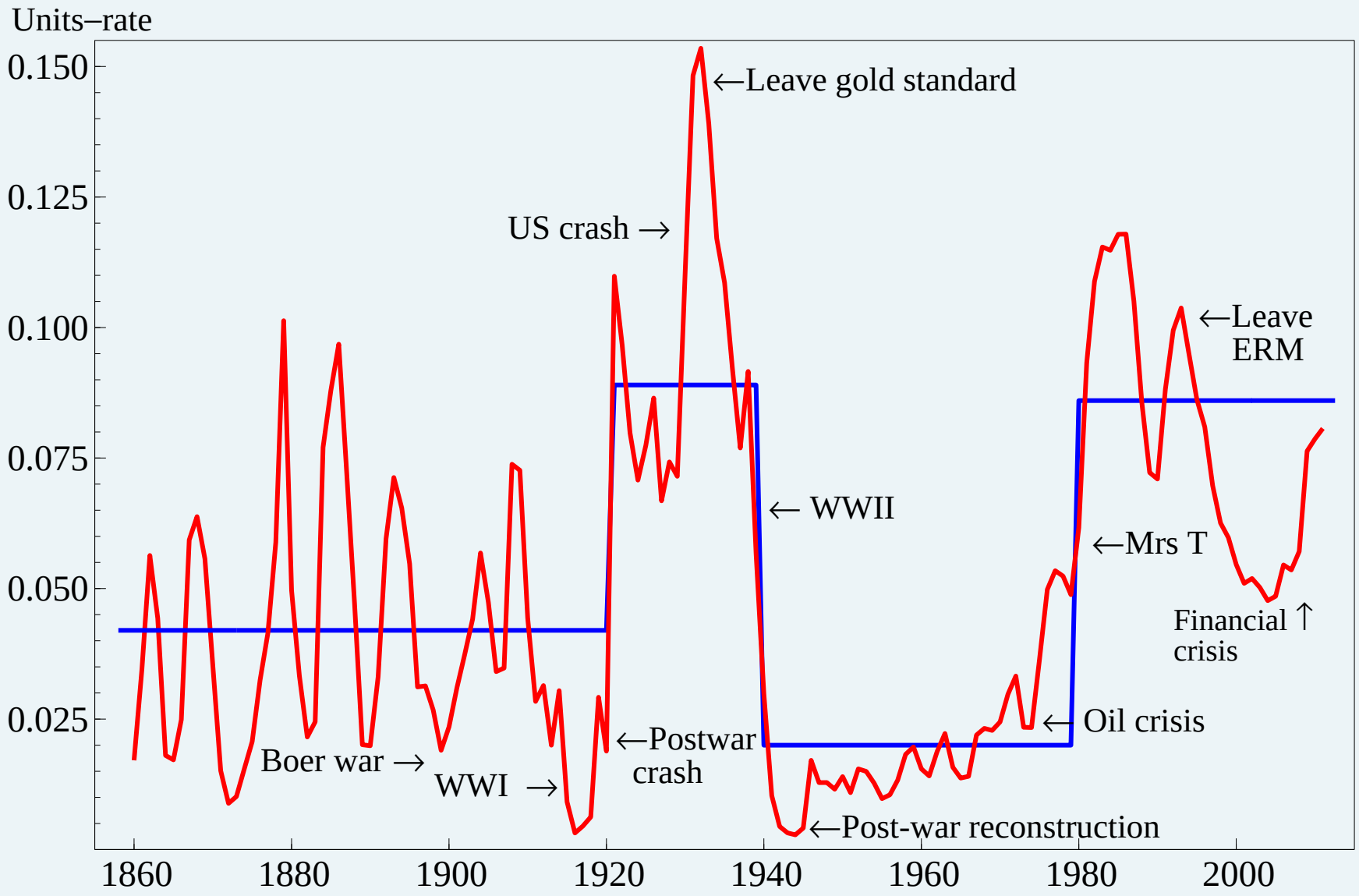


Figure: Events affecting UK unemployment 1860–2011.

Famous curve derived by Bill Phillips (1958) in which unemployment seems to determine inflation: $\Delta w_t = f(U_{r,t}) + e_t$.
Figure 96 illustrates over 1860–1913: lower unemployment, or higher demand for labour, associated with higher wage inflation.

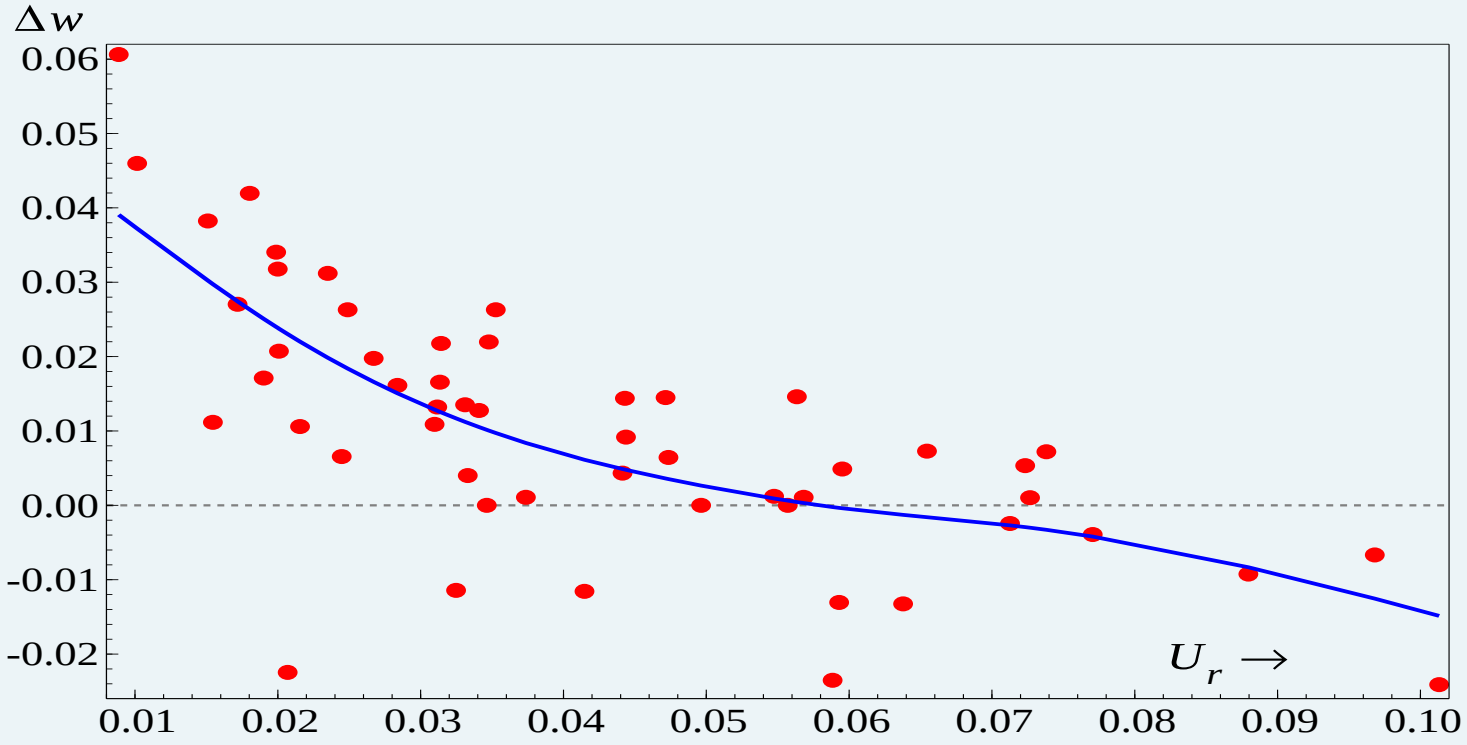


Figure: Plot of nominal wage changes Δw against unemployment U_r.

Shifts in inflation-unemployment relation

Phillips curve seems fine in 19th-early 20th century.

Before we consider the later evidence, remember we saw 8 shifts in Δw and 4 in U_r:
some shifts are **not** going to match.

Sometimes called 'breakdown' of the Phillips curve.

Figure 98 shows the curve for four sub-periods:

two reveal an implausible 'rising' curve.

Could reflect that other factors affect the link–
or no genuine link.

Also saw earlier that $\Delta(w - p)$ had only 2 shifts:

inflation also influences nominal wage changes.

Figure 99 shows real-wage curves for the four sub-periods:

unclear what is happening because of huge 1940 change,
but less evidence of shifts—or of a link.

Shifts in inflation-unemployment 'trade-off'

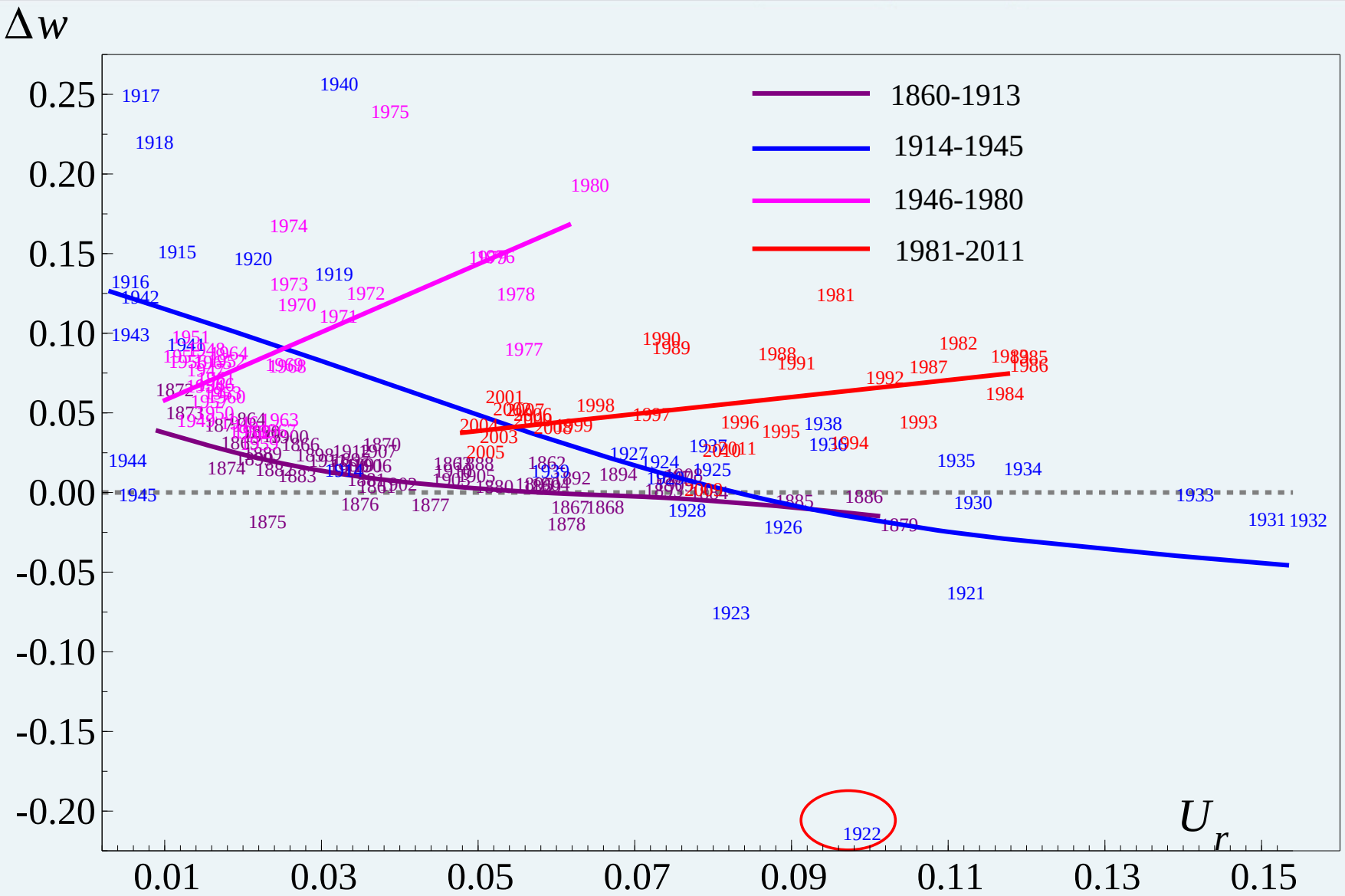


Figure: Shifts in inflation-unemployment 'trade-off'.

Shifts in real wage-unemployment 'trade-off'

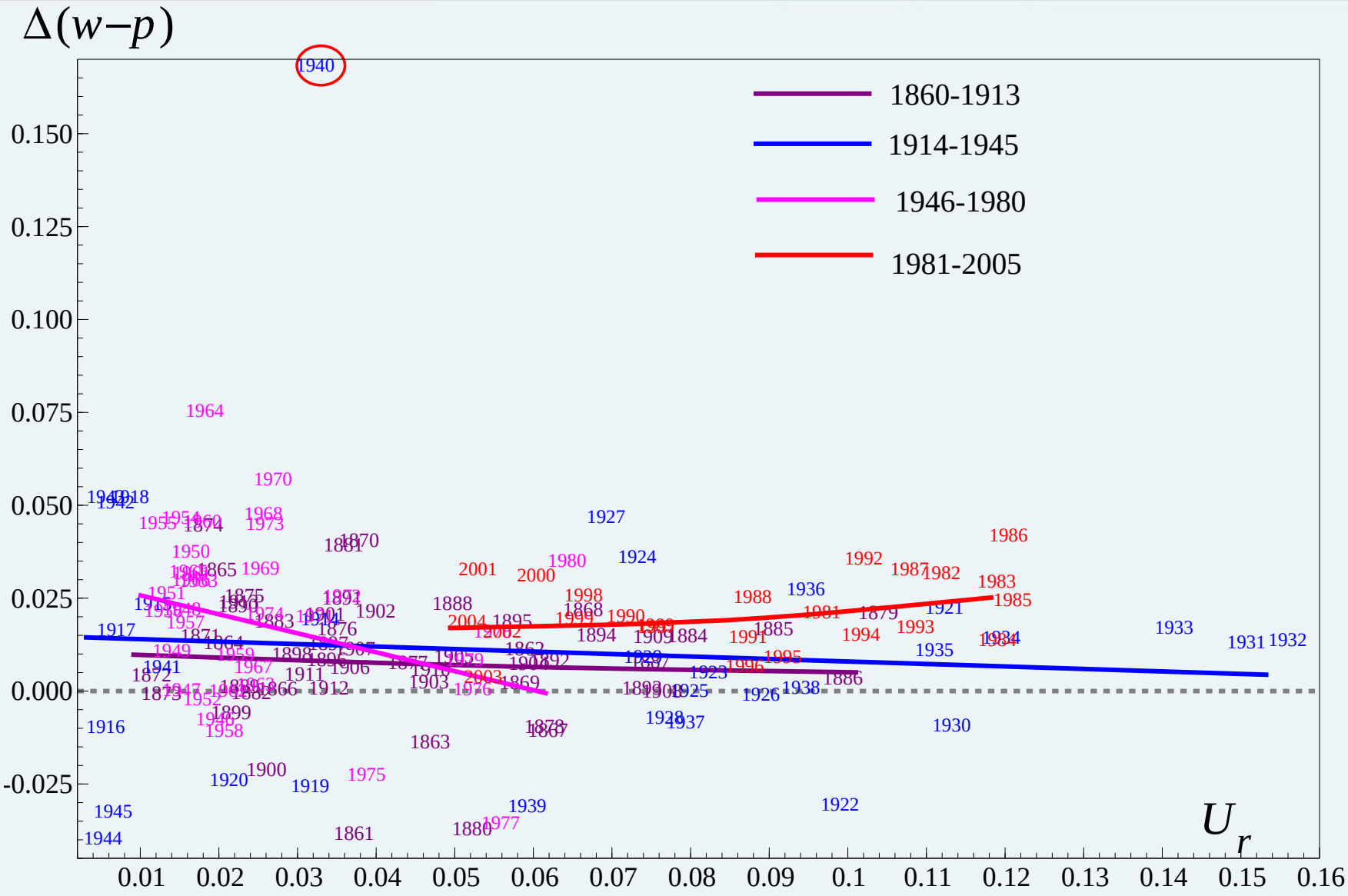


Figure: Shifts in real wage-unemployment 'trade-off'.

That last graph ignores our finding that
$w - p$ cointegrates with $g - l$.

Figure 101 records the link between $\Delta(w - p)$ and $\Delta(g - l)$.

The curves in Figure 99 also ignore the **dynamic links**, namely the close relations between any variable x_t and its lagged value x_{t-1}.

Figure 102 joins each point to the next for 1860–1913 to reveal 'Phillips loops' by decades.

The movements between points are systematic, following a 'time line', and loop around the red curve.

Need to allow for all the relevant influences, breaks, dynamics, and trends to really understand how economics variable are determined....

So next explore how that can be done without too much stress.

Changes in real wages versus changes in productivity

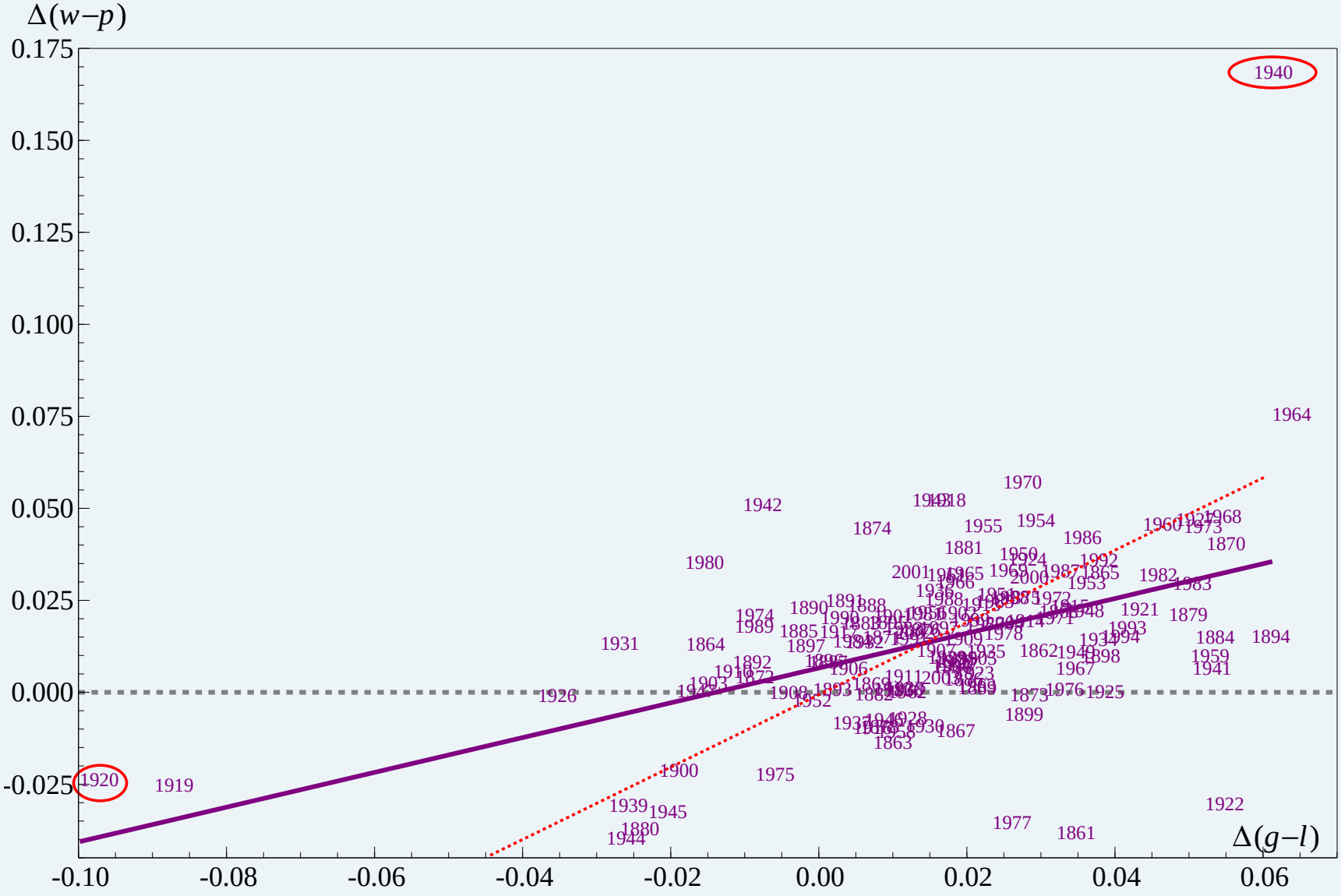


Figure: Changes in real wages versus changes in productivity.

Dynamics in inflation-unemployment trade-off

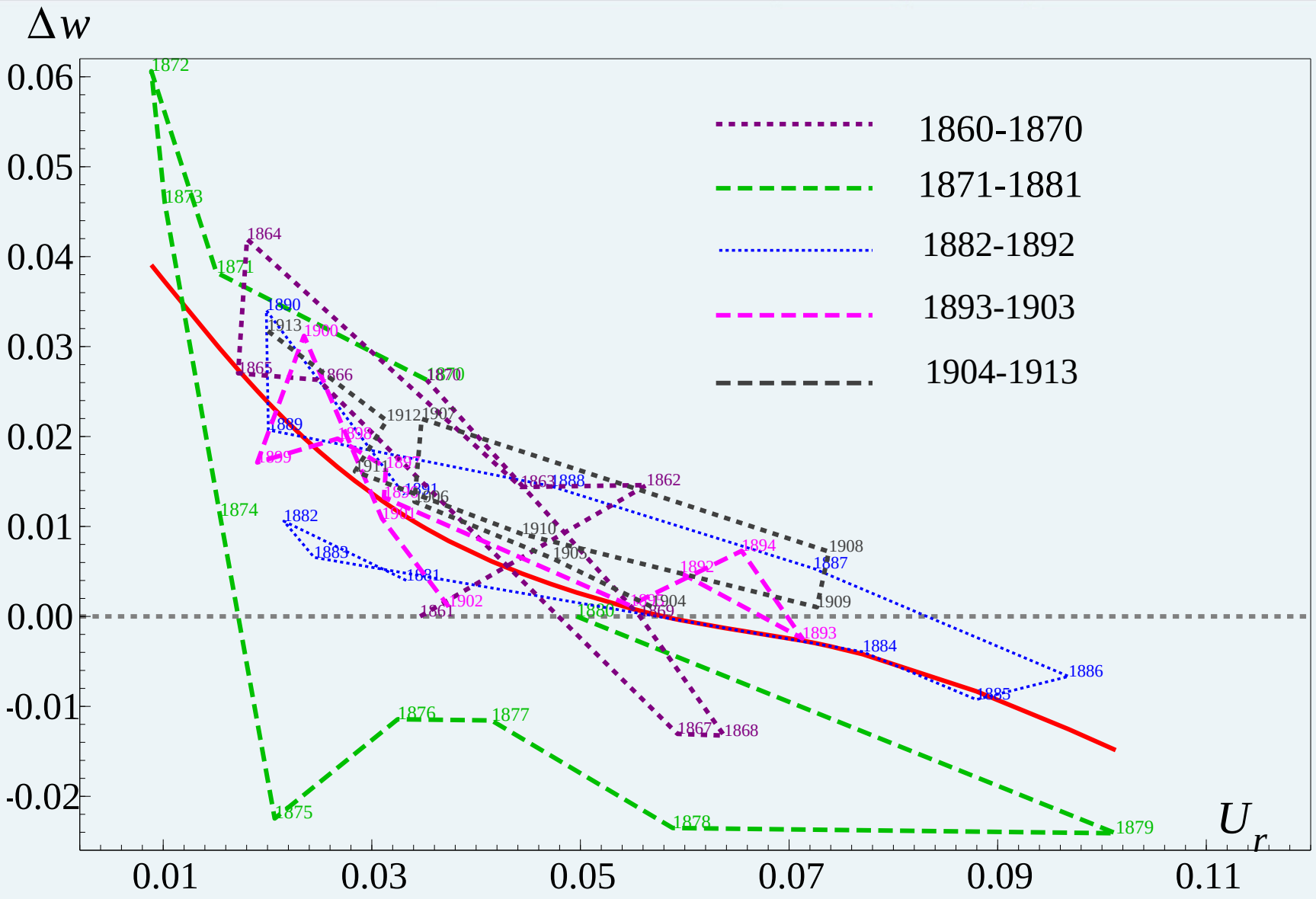


Figure: Dynamics in inflation-unemployment trade-off.

Summary of lecture 3 key points

(A) High correlations between many macroeconomic variables, but changing over time.
(B) Very high correlations between variables and their lags: corr(x_t, x_{t-s}) near unity even up to $s = 20$.
(C) But corr$(\Delta x_t, \Delta x_{t-s})$ often 'dies out' rapidly.
(D) Both (B) & (C) consistent with stochastic trends.
(E) Real-wage growth 'tames' the stochastic trends of p and w, most of their breaks, and even the autocorrelations of wage and price inflation.
(F) Correlations between differenced variables often low.
(G) Simple **unemployment-wage inflation relation** shifts.
(H) Need to model all relevant variables, their dynamic reactions, non-linearities, breaks, and trends.

Must now develop methods for handling these influences jointly.

Route map

(1) Macroeconomic data: evolution with abrupt change

(2) Taming trends and breaks

(3) Dependence between variables and over time

(4) **Economic theory and statistical analysis: 2 key ingredients**

(5) Modelling UK unemployment

(6) Modelling UK wages

(7) Modelling UK money demand

(8) Modelling UK prices

(9) Conclusion

Lecture 4 guide post

1. Theoretical analyses of economic behaviour offer valuable insights, but are incomplete explanations of macroeconomic data.
2. Statistical models are also theories, but of the DGP.
3. Such models, like regressions equations, have unknown parameters that must be estimated from the available data.
4. The 'best' estimators, and their resulting distributions, are obtained assuming that the statistical model is the DGP.
5. To ensure a good approximation, all substantively relevant variables, dynamics, breaks, and non-linearities must be included.
6. If a model does not represent the main features of the DGP, estimates can be poor and inferences will usually be invalid.
7. Distributions of estimators and tests can be hard to derive analytically, but can be illustrated using simulation methods applied to computer-generated data.
8. Dynamic models are essential for macroeconomic time series.
9. Simple unemployment equations demonstrate that not all models are useful, but graphs and mis-specification tests can reveal flaws.

Economic theory provides abstract models of the behaviour of economic agents. Often non-stochastic, assuming a 'steady state', with many ceteris paribus clauses.

Consider an economic analysis which suggests:

$$y = f(z) \tag{31}$$

where y depends on n 'explanatory' variables z.
Form of $f(\cdot)$ in (31) depends on:
the specific utility or loss functions of agents,
the constraints they face, & the information they possess.

Analyses usually assume: a form for $f(\cdot)$, that $f(\cdot)$ is constant, that only z matters, & that z is 'exogenous' (determined outside the model).

Must aggregate across heterogeneous individuals
whose endowments shift over time, often abruptly.

Helpful starting point—but many aspects must be 'data determined': need to specify a **statistical model** for (31).

A **statistical model** is also a theory–
of how the data are generated.
The simplest example for $y = f(z)$ in (31) is a **linear** model, with **independent normal** errors, a single **exogenous** regressor, z_t, and **constant parameters**, β_0 and β_1 for some unit of time t:

$$y_t = \beta_0 + \beta_1 z_t + \epsilon_t \text{ where } \epsilon_t \sim \mathsf{IN}\left[0, \sigma_\epsilon^2\right], \tag{32}$$

where successive ϵ_t are Independently drawn from a Normal distribution, with mean $\mathsf{E}[\epsilon_t] = 0$, variance $\mathsf{Var}[\epsilon_t] = \sigma_\epsilon^2$, and are **independent of** z_t, so that $\mathsf{E}[\epsilon_t | z_t] = 0$.

Why consider a formulation like (32)?
(i) growth rates of aggregate data are often near normal:
see Figure 108;
(ii) regression works best for normal distributions;
(iii) with normality as a baseline, 'outliers' can be defined;
(iv) generalizations to many regressors are easy;
(v) linearity is a useful first approximation, and can be generalized;
(vi) dynamics can be added easily.

Densities of first and second differences of some UK macroeconomic variables

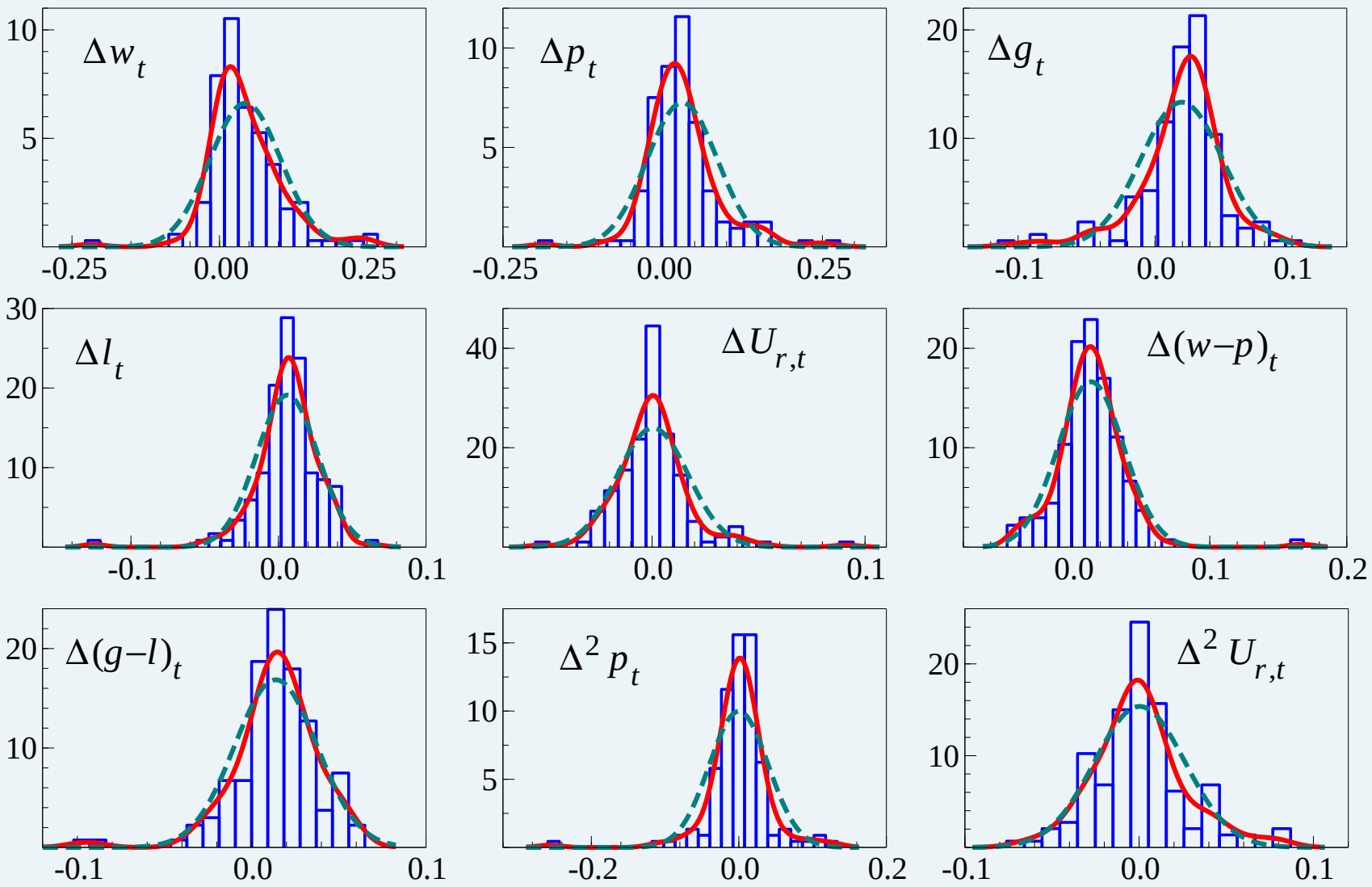


Figure: Densities of first and second differences of UK macroeconomic variables.

From (32), taking expectations of both sides conditional on z_t, and using $\mathsf{E}[\epsilon_t|z_t] = 0$ as $\mathsf{E}[\epsilon_t] = 0$ in general, then $\mathsf{E}[y_t|z_t] = \beta_0 + \beta_1 z_t$. Thus:

$$y_t|z_t \sim \mathsf{IN}\left[\beta_0 + \beta_1 z_t, \sigma_\epsilon^2\right]. \tag{33}$$

In (33), observations are generated by a random 'shock' ϵ_t, perturbing the agents' 'plan' $\beta_0 + \beta_1 z_t$, to produce the independent outcomes $\{y_t\}$.

Even in this simple setting, there are three **unknowns**: 2 regression parameters, β_0 & β_1, and error variance, σ_ϵ^2.

Need systematic procedures for calculating values of unknown parameters, when the data are generated by the statistical model, and criteria to determine which choices are 'best'.

Least squares, maximum likelihood and instrumental variables are most common statistical methods: background to that material is provided in Hendry and Nielsen (2007, Chapters 1–5).

Here we will discuss least squares.

In simplest case of known $\beta_0 = 0$ in (32), least-squares estimator (OLS) for sample of size T multiplies:

$$y_t = \beta_1 z_t + \epsilon_t \tag{34}$$

by z_t to get:

$$z_t y_t = \beta_1 z_t^2 + z_t \epsilon_t \tag{35}$$

and averaging over the sample observations:

$$\frac{1}{T}\sum_{t=1}^{T} z_t y_t = \beta_1 \left(\frac{1}{T}\sum_{t=1}^{T} z_t^2\right) + \frac{1}{T}\sum_{t=1}^{T} z_t \epsilon_t \tag{36}$$

As $E[z_t \epsilon_t] = 0$, set the sample average $\frac{1}{T}\sum_{t=1}^{T} z_t \epsilon_t$ to zero, so:

$$\widehat{\beta}_1 = \left(\frac{1}{T}\sum_{t=1}^{T} z_t y_t\right) / \left(\frac{1}{T}\sum_{t=1}^{T} z_t^2\right) \tag{37}$$

which can be calculated from data on $(y_1 \ldots y_T, z_1 \ldots z_T)$.

The residuals are the deviations of the fitted values $\widehat{y}_t = \widehat{\beta}_1 z_t$ from the data y_t:

$$\widehat{\epsilon}_t = y_t - \widehat{\beta}_1 z_t \tag{38}$$

and it can be proved that $\sum_{t=1}^{T} \widehat{\epsilon}_t^2$ is the smallest sum of squared residuals that can be achieved by the choice of $\widehat{\beta}_1$ in (37).
We can estimate σ_ϵ^2 by:

$$\widehat{\sigma}_\epsilon^2 = \frac{1}{(T-1)} \sum_{t=1}^{T} \widehat{\epsilon}_t^2 \tag{39}$$

The divisor of $(T-1)$ in (39) is called the degrees of freedom and is one less than T because we have estimated one parameter, $\widehat{\beta}_1$, which constrains the residuals.

To derive the properties of $\widehat{\beta}_1$ in (37) as an estimator for the unknown β_1, first substitute for y_t from (32), using $\beta_0 = 0$, which delivers:

$$\widehat{\beta}_1 = \beta_1 + \frac{\frac{1}{T}\sum_{t=1}^{T} z_t \epsilon_t}{\frac{1}{T}\sum_{t=1}^{T} z_t^2} = \left(\frac{1}{\frac{1}{T}\sum_{t=1}^{T} z_t^2}\right) \frac{1}{T}\sum_{t=1}^{T} z_t \epsilon_t \tag{40}$$

In (40), the $\{z_t\}$ are fixed by conditioning, so only the $\{\epsilon_t\}$ are stochastic, and as $\widehat{\beta}_1 - \beta_1$ is a **linear function** of the $\{\epsilon_t\}$:

$$\mathsf{E}\left[\widehat{\beta}_1\right] = \beta_1$$

so $\widehat{\beta}_1$ is an unbiased estimator of β_1 in this simple setting. Also:

$$\mathsf{Var}\left[\widehat{\beta}_1\right] = \frac{\sigma_\epsilon^2}{\sum_{t=1}^{T} z_t^2} \tag{41}$$

As the $\{\epsilon_t\}$ are normally distributed, the distribution of $\widehat{\beta}_1$ is:

$$\widehat{\beta}_1 \sim \mathsf{N}\left[\beta_1, \frac{\sigma_\epsilon^2}{\sum_{t=1}^{T} z_t^2}\right] \tag{42}$$

Easiest to understand such formulae from 'artificial data': use random numbers for $\epsilon_t \sim \mathsf{IN}[0, \sigma_\epsilon^2]$, for $t = 1, 2, \ldots, T$, setting $\sigma_\epsilon^2 = 1$.
To generate $\{z_t\}$, I used $z_t = 0.005t + 0.8z_{t-1} + \nu_t$, $\nu_t \sim \mathsf{IN}[0, 1]$.
I set $\beta_0 = 1$ & $\beta_1 = 2$ to generate $y_t = 1 + 2z_t + \epsilon_t$ as in (32).
Although I know the DGP, the computer estimation program does not!
Thus, generate $T = 100$ observations on ϵ_t, ν_t and z_t then $y_t = 1 + 2z_t + \epsilon_t$. Estimating β_0 & β_1 by least squares delivers:

$$\widehat{y}_t = \underset{(0.12)}{1.26} + \underset{(0.05)}{1.96}\, z_t$$

$$R^2 = 0.942 \quad \widehat{\sigma}_\epsilon = 1.062 \tag{43}$$

In (43), R^2 is the square of the correlation between y_t and $\widehat{y}_t$ called the squared multiple correlation, $\widehat{\sigma}_\epsilon$ is the residual standard deviation, and coefficient standard errors, denoted $\mathsf{SE}[\widehat{\beta}]$ below (square roots of the estimates of $\mathsf{Var}[\widehat{\beta}]$ in (41)) are shown in parentheses.
The estimates of β_0, β_1 & σ_ϵ are quite close to the true values; and the fit is 'good' as measured by R^2.

It is helpful to graph various aspects of the outcome.
Figure 115 illustrates four features in its 4 panels.
Panel a records the actual outcomes y_t
and the fitted values: $\widehat{y}_t = \widehat{\beta}_0 + \widehat{\beta}_1 z_t = 1.26 + 1.96 z_t$.
As can be seen, the 'tracking' is close: $\widehat{y}_t$ is similar to y_t.
Panel b shows the residuals: $\widehat{\epsilon}_t = y_t - \widehat{\beta}_0 - \widehat{\beta}_1 z_t$ as in (38), but scaled by $\widehat{\sigma}_\epsilon$, so $\widehat{\epsilon}_t / \widehat{\sigma}_\epsilon$ are plotted.
Consistent with panel a, the residuals look 'random' with most lying between ± 2 (95% interval if normal).
Panel c shows their histogram and a smoothed density estimate, with a normal density for comparison.
Panel d reports the correlogram of the residuals: like panel b, there is little evidence of residual autocorrelation.
All the graphs are consistent with the assumptions made: hardly a surprise as the model is correct.

Fitted and actual values and residuals

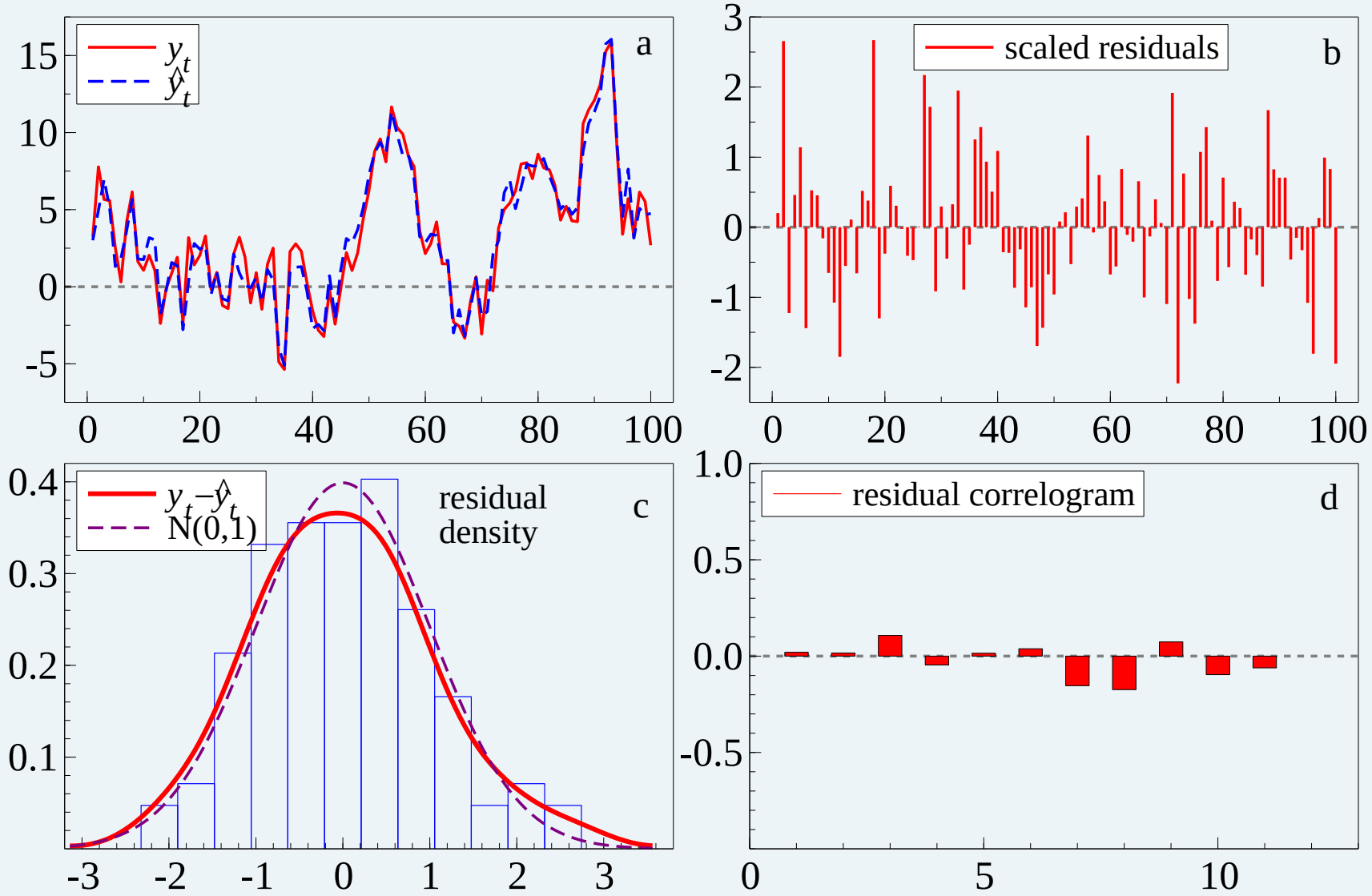


Figure: Fitted and actual values (a), residuals (b), residual density (c), and residual correlogram (d).

From (42):

$$\frac{\sqrt{\sum_{t=1}^{T} z_t^2}\left(\widehat{\beta}_1 - \beta_1\right)}{\sigma_\epsilon} \sim \mathsf{N}\left[0, 1\right] \tag{44}$$

Setting $\beta_1 = 0$ in (44) when that is true yields:

$$\frac{\widehat{\beta}_1 \sqrt{\sum_{t=1}^{T} z_t^2}}{\sigma_\epsilon} \sim \mathsf{N}\left[0, 1\right] \tag{45}$$

but that would be false when $\beta_1 \neq 0$.

Thus, although β_1 and σ_ϵ are never known, (45) points towards a possible way to test hypotheses about β_1: replace β_1 in (44) by assumed values, and check if the outcome is consistent with $\mathsf{N}[0, 1]$.

Using an estimate $\widehat{\sigma}_\epsilon$ of σ_ϵ in the denominator on the left-hand side of (44), however, changes the distribution from Normal to a Student's t as follows (Student was the pseudonym for W.S. Gosset).

We can test the **null hypothesis** (denoted H_0) $H_0{:}\beta_1 = 0$ by:

$$t_{\beta_1=0} = \frac{\widehat{\beta}_1}{SE[\widehat{\beta}_1]} \tag{46}$$

where $t_{\beta_1=0}$ is distributed as Student's t under H_0.

The **alternative hypothesis** is $H_1{:}\beta_1 \neq 0$ and when H_1 is true, (46) has a non-zero mean and the distribution is said to be 'non-central'.

The test procedure is to reject H_0 when the value of (46) is far from 0.

The probabilities of departures from the mean of zero under the null hypothesis for t statistics are tabulated in many sources, but most econometric software packages calculate them automatically.

When the sample size T is large, Student's t is close to a standard Normal under the null, but has a wider spread when T is small. Unlike the Normal, the t-distribution depends on the degrees of freedom, $T - n$ where n is the number of parameters estimated.

Two key values under H_0 for moderate T (larger than 50, say) are that:
(a) $|t_{\beta_1=0}| \geqslant 2$ occurs approximately 5% of the time, and
(b) $|t_{\beta_1=0}| \geqslant 2.68$ occurs about 1% of the time.

As such a large value is likely to occur less than once in a hundred times under H_0, it is more reasonable to assume the null hypothesis is false, so reject it in favour of the alternative that $\beta_1 \neq 0$.

A model of $y|z$ is defined by its **distribution** $f_y(y|z, \beta)$, characterized by the vector of n **parameters** $\beta = (\beta_1 \ldots \beta_n)$: in (33), $f_y(y|z, \beta)$ was $y_t|z_t \sim \mathsf{IN}\left[\beta_0 + \beta_1 z_t, \sigma^2_\epsilon\right]$.

Need to estimate the unknown β from a sample $(y, z) = (y_1 \ldots y_T, z_1 \ldots z_T)$ by estimators $\widehat{\beta} = g(y|z)$.

Estimators and tests have *sampling distributions*: $f_\beta(\widehat{\beta}|\beta)$.

Such distributions usually assume the model $f_y(y|z, \beta)$ is the process which generated $(y_1 \ldots y_T)$: called the data-generation process (DGP), $D_y(y_1 \ldots y_T|z, \theta)$.

Derive estimators and their distributions *as if* $f_y(y|z, \beta)$ was the DGP.

Want estimators to correctly reflect 'true' parameter values: e.g., $\mathsf{E}[\widehat{\beta}] = \beta$, where E denotes the expected or average value, so the estimator is unbiased, preferably with a small variance.

When the data are artificial, we can generate thousands of samples of $\{y_t, z_t, t = 1, 2, \ldots, T\}$, estimate the parameters by OLS in every sample, as in (36), and graph the resulting distributions.

Figure 121 illustrates the densities of:

$\widehat{\beta}_0$ in panel a: close to normal with a mean of 1 and a standard deviation of 0.14 (somewhat above $1/\sqrt{T} = 0.1$ as z_t does not have a mean of zero);

$\widehat{\beta}_1$ in panel b: close to normal with a mean of 2 and a standard deviation of 0.06;

$\widehat{\sigma}_\epsilon$ in panel c: quite close to normal with a mean of 1;

R^2 in panel d: skewed left with a mean of 0.92;

$t_{\beta_0=0}$ in panel e, and $t_{\beta_1=0}$ in panel f:

both are calculated by $t_{\beta_i=0} = \widehat{\beta}_i/SE[\widehat{\beta}_i]$,

and correctly reject their null hypothesis about 100% of the time.

Overall, our one sample in (43) was 'representative'.

Distributions of parameter estimates and tests

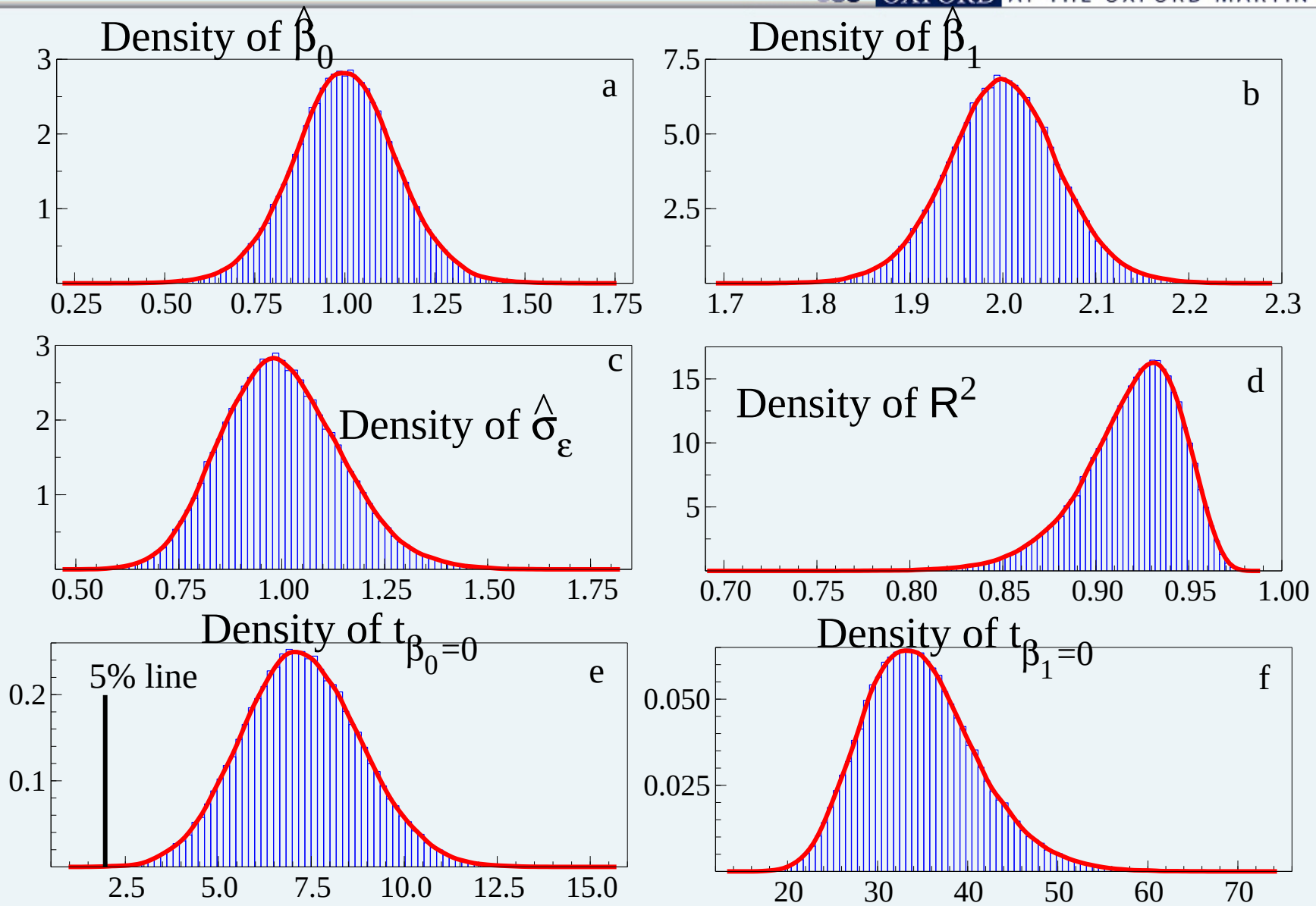


Figure: Distributions of parameter estimates (a)–(b), residual standard error (c), R^2 (d), and t-tests of the null hypothesis (e)–(f).

Such derivations require that the model is the DGP:
$f_y(y|z, \beta) = D_y(y|z, \theta)$ so $f_y(\cdot) = D_y(\cdot)$ and $\beta = \theta$.

Not going to get correct results if $f_y(y|z, \beta) \neq D_y(y|z, \theta)$.

Econometrics has developed many tests of whether the model is the DGP. Such tests often reject: unclear how to proceed if so.

Trends and breaks are common features of economic time series: above, developed some statistical models for these.

Saw **multivariate interdependence** in macro economics: w, p, g, l, k, U_r, e, R_L, R_S, p_w show **multiple regression** needed.

Time-series variables also **highly dependent over time**: need **dynamic** models, as reactions not always instantaneous.

Relationships need not be **linear**, so allow for **non-linearities**.

Macro-economic variables often **jointly determined** so regressors may be **endogenous**.

Models must include all these features to generate 'realistic' economic data.

These are important difficulties

Consequently:

(A) Econometric models must allow for all the complications of **stochastic trends, breaks, dynamics, non-linearities and interdependence jointly**.

(B) Do not know the macroeconomy DGP, so must **specify a class of models** general enough to include DGP as a special case.

(C) Can '**embed' economic ideas** in that general model, as part of explanation.

(D) Such a general class of models may include dozens, perhaps hundreds, of variables–**far too large for humans to handle**, **but computers can**.

(E) As we do not know the DGP, must **select the 'best choice'** from the initial general class.

(F) Finally, **check the resulting model describes available data**.

Will see how to proceed by modelling unemployment rate after discussng **exogeneity**.

Consider two variables with the joint Normal distribution:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} \sim \mathsf{IN}_2 \left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right] \tag{47}$$

As $\mathsf{corr}[y_t, z_t] = \sigma_{12}/\sqrt{\sigma_{11}\sigma_{22}}$, **correlation does not entail causation**. Indeed, the conditional expectation of z_t given y_t is $\mathsf{E}[y_t|z_t] = \beta_0 + \beta_1 z_t$ leading to the regression model:

$$z_t \mid y_t \sim \mathsf{N}\left[\mu_2 + \gamma_{21}\left(y_t - \mu_1\right), \omega_{22}\right]. \tag{48}$$

In (48), $\gamma_{21} = \sigma_{12}/\sigma_{11}$ with $\omega_{22} = \sigma_{22} - \sigma_{12}^2/\sigma_{11}$.
But the opposite relation from y_t on to z_t instead leads to:

$$y_t \mid z_t \sim \mathsf{N}\left[\mu_1 + \beta_{12}\left(z_t - \mu_2\right), \omega_{11}\right] \tag{49}$$

where $\beta_{12} = \sigma_{12}/\sigma_{22}$ with $\omega_{11} = \sigma_{11} - \sigma_{12}^2/\sigma_{22}$.
Both (48) & (49) equally 'good' regressions: which to use?

On the information given, cannot choose between directions of regression: y_t **and** z_t **are simultaneously determined**.

Rare to have so little structure:
no trends, no breaks, no other determinants of either variable.

If either variable y_t or z_t depended on an 'outside' influence, say w_t, can use that to choose which equation to model.

If w_t directly affected z_t, say, and only influenced y_t through z_t, then could model regression in (49).

Need w_t to be **exogenous**, as we now discuss.

Consider agents who plan to keep their 'permanent' consumption $\mathsf{E}[y_t] = \mu_1$ proportional to their 'permanent' income $\mathsf{E}[z_t] = \mu_2$ by $\mu_1 = \kappa\mu_2$. An investigator estimates the relation:

$$y_t = \beta_{11} + \beta_{12} z_t + \nu_t \quad \text{where} \quad \nu_t \sim \mathsf{IN}\left[0, \omega_{11}\right] \tag{50}$$

expecting $\widehat{\beta}_{11} \approx 0$, so interprets $\widehat{\beta}_{12}$ as $\widehat{\kappa}$. Although (50) entails:

$$\mathsf{E}[y_t] = \beta_{11} + \beta_{12}\mathsf{E}[z_t] \tag{51}$$

when (47) is the DGP, (50) also entails:

$$\mathsf{E}[y_t \mid z_t] = \left(\kappa - \frac{\sigma_{12}}{\sigma_{22}}\right)\mu_2 + \frac{\sigma_{12}}{\sigma_{22}} z_t \tag{52}$$

Hence, $\widehat{\beta}_{12}$ estimates the ratio of the covariance to the variance of z_t. For $\widehat{\beta}_{12}$ to estimate κ requires that $\kappa = \sigma_{12}/\sigma_{22}$: **the means $\mu_1 = \kappa\mu_2$ must be linked by the same parameter κ as the covariance**: $\sigma_{12} = \kappa\sigma_{22}$. If so, then $\beta_{11} = 0$ and $\beta_{12} = \kappa$.
When that condition holds, z_t is said to be **weakly exogenous** for the parameters of (50) (see Engle, Hendry and Richard, 1983).

Do not have complete and correct economic theories from which to derive 'correct' statistical models.

As do not know DGP, must postulate theory-based statistical model.

Two hypothetical models of UK unemployment rate $U_{r,t}$:

first is that a high wage share causes unemployment as labour 'too expensive';

second is that high unemployment leads to high unemployment from 'discouraged workers'.

Formulate first as the linear regression:

$$U_{r,t} = \beta_0 + \beta_1(w_t - p_t - g_t + l_t) + \epsilon_t \tag{53}$$

and the second becomes the autoregression:

$$U_{r,t} = \gamma_0 + \gamma_1 U_{r,t-1} + \nu_t \tag{54}$$

Both are 'straw' examples to illustrate how **not** to proceed.

Estimation of (53) yields using data from 1860 to 2004 yields:

$$\widehat{u}_{r,t} = \underset{(0.10)}{0.40} - \underset{(0.06)}{0.19} \; (w_t - p_t - g_t + l_t)$$

$$R^2 = 0.078 \;\; \widehat{\sigma}_\epsilon = 0.034 \tag{55}$$

Estimates 'seem significant'—in that the $t_{\beta_i=0}$ statistics appear to reject their null hypotheses—but will question that shortly.

If so, a high wage share **lowers** unemployment, which is the 'wrong' sign.

The fit is very poor: $R^2 = 0.078$ suggests most of movements in unemployment are not explained by (55).

Numerous problems shown in Figure 129.

Wage share model of UK unemployment

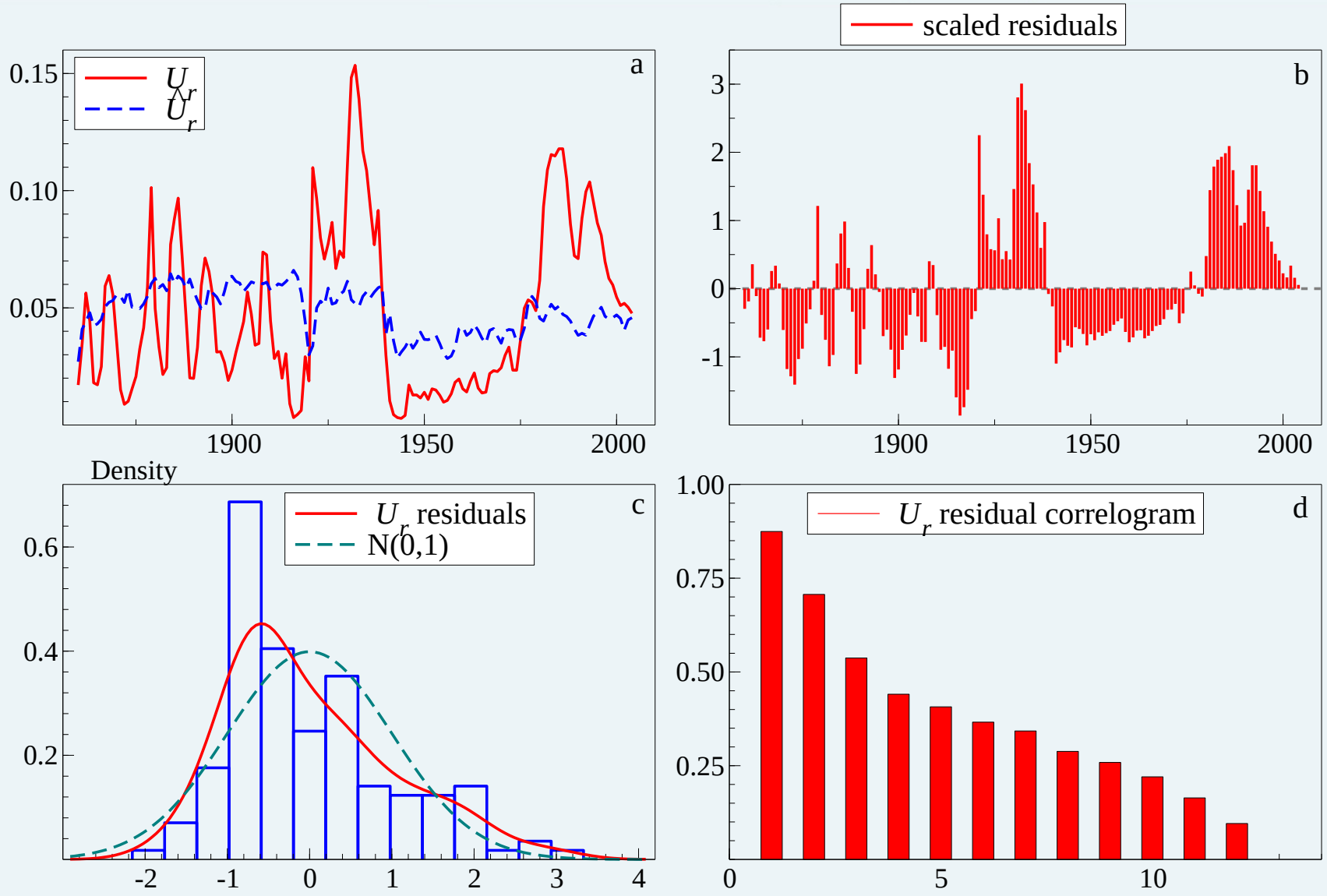


Figure: Graphical statistics for wage share model of UK unemployment.

Panel a shows the movements in the fitted line, $\widehat{u}_{r,t}$, namely $0.4 - 0.19(w_t - p_t - g_t + l_t)$, which does not have the correct 'time series profile' to explain unemployment.

The scaled residuals, $(u_{r,t} - \widehat{u}_{r,t})/\widehat{\sigma}_\epsilon$, in panel b move systematically and are far from 'random.

Panel d shows their correlogram:
highly positively autocorrelated as far back as 10 years.

Panel c plots the residual histogram, with an estimate of the density and a normal density for comparison.
There is 'ocular' evidence of some non-normality.

Note, conversely, that unemployment by itself cannot explain the wage share: $R^2 = 0.078$ must be the same.

Now consider the performance of the 'rival' model in (54).

Estimation of (54) yields :

$$\widetilde{u}_{r,t} = \underset{(0.002)}{0.006} + \underset{(0.04)}{0.88}\, u_{r,t-1}$$

$$R^2 = 0.78 \quad \widehat{\sigma}_\nu = 0.016 \tag{56}$$

The fit is much better, $R^2 = 0.78$: some movements in unemployment are explained by (56)–Figure 132 panel a.

The residuals in panel b are less systematic, but there is a large 'spike' or 'outlier' in 1920, **even though least squares tries to minimize squared residuals**, so there is nothing in the model to explain that jump in unemployment.

The residual correlogram in panel d is much 'flatter' than for (55), and the residual histogram in panel c is closer to the normal density, with a large outlier.

Autoregressive model of UK unemployment

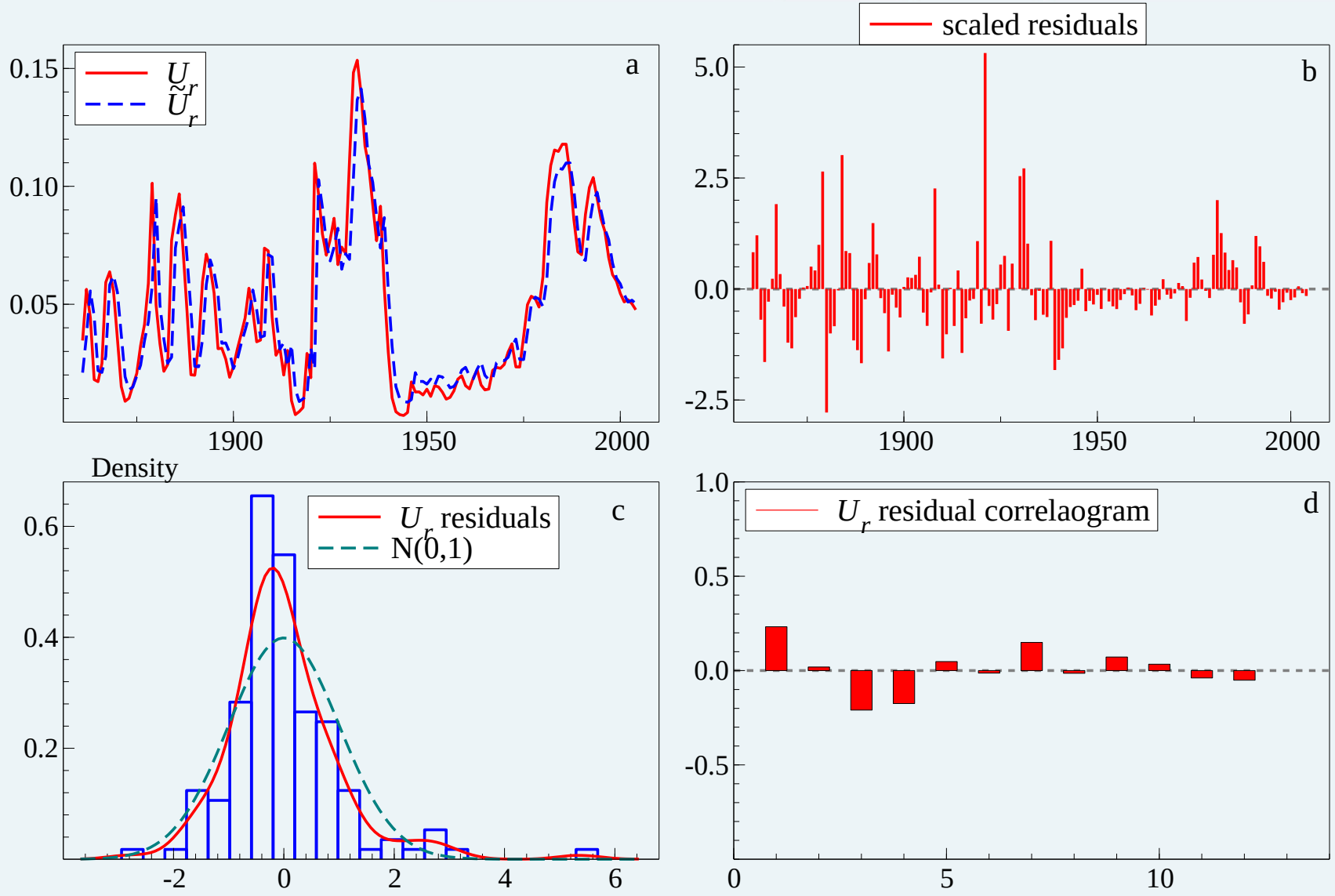


Figure: Graphical statistics for autoregressive model of UK unemployment.

What lessons can we learn?

First, simple theories and models may not be sufficient:
many forces drive the movements of macroeconomic data.

Second, easy to see failure of the simplest model:
learn that empirical findings can contradict assumptions.

Third, some models fit much better than others:
learn that not all models are born equal.

Fourth, postulating simple models then finding they fail is not going to be a productive methodology: it could go on endlessly.

Many data features not explained by economic theory.

Must allow for all the major determinants jointly:
stochastic trends, breaks, dynamics, non-linearities and interdependence.

Econometric modelling needed to 'match' theoretical framework to data, not just 'estimate fully-specified theory-model'.

How can we do that? **Let the computer take the strain**.

Six main null hypotheses to test about empirical models:

1. homoskedastic, innovation errors $\{\epsilon_t\}$;
2. weakly exogenous conditioning variables z_t;
3. constant, invariant parameters of interest, β;
4. theory-consistent, identified structures;
5. data-admissible on accurate observations; and
6. explain findings of rival models (encompassing).

Errors are homoskedastic when they have the same variance over the sample, and are innovations if not autocorrelated;
weak exogeneity was discussed above, and justifies conditioning;
parameters are invariant if β is constant when z_t changes;
data-admissible cannot generate impossible values, e.g., negative U_r.

Congruent models satisfy first three conditions.

Super exogeneity requires weak exogeneity and parameters invariant to shifts in z_t.

Evaluating empirical models

Econometric modelling involves discovering *sustainable relationships*, and rejecting models otherwise–construction and destruction.

Test if proposed models are congruent, so 'match':

1. the past: fit should only deviate from observations by homoskedastic, innovation residuals;
2. the present: contemporaneous variables should be exogenous;
3. the future: parameters should be constant and invariant to policy interventions;
4. theory information: model must have unique parameters related to theory;
5. measurement information: observations should be accurate, and the model data admissible;
6. results of other models: encompass their findings.

Non-congruent models fail evidence on 1.–3: see (55).

General-to-specific modelling

General-to-specific, *Gets*, a central approach to develop well-specified relationships.

Begin from general unrestricted model (GUM):
most general, estimable, model reasonable initially,
given sample size of data, previous empirical evidence,
subject-matter theory, institutional and measurement information.

GUM allows for everything that might matter logically:
many variables, dynamics, breaks, non-linearities, trends, etc.,
structured by a careful theoretical analysis & data transformations.

To model, first check GUM matches the evidence,
then simplify to acceptable parsimonious, interpretable, econometric model.

Test final model is valid by explaining other models' results.

We will use automatic techniques programmed to build models by a general-to-specific model selection strategy.

Every stage well defined, so a computer can do it:
Gets is just a step up from a computer calculating regression estimates.

Computers can handle far larger initial general models than any human.

Used to idea of a simple regression as in (34):
now need to imagine dozens of variables in the GUM:

$$y_t = \sum_{i=1}^{N} \beta_i z_{i,t} + \epsilon_t \tag{57}$$

where N might be large and some of $z_{i,t}$ are lags, shift indicators, functions.

Then computer eliminates all the $z_{j,t}$ with insignificant estimated coefficients, yet such that result remains well-specified.

We have seen static equations of the form:

$$y_t = \beta_0 + \beta_1 z_t + \epsilon_t \tag{58}$$

and autoregressive equations such as:

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \nu_t \tag{59}$$

so combine these in a more general dynamic model:

$$y_t = \beta_0 + \beta_1 z_t + \beta_2 y_{t-1} + \beta_3 z_{t-1} + \epsilon_t \tag{60}$$

In (60), y_t responds to changes in z_t, in its own lag, or previous value, y_{t-1}, and to the lag z_{t-1}, that relation being perturbed by a random error $\epsilon_t \sim \mathsf{IN}[0, \sigma^2_\epsilon]$ as in (32).

Thus (60) adds inter-dependence (z_t) to dynamics (y_{t-1}, z_{t-1}), so two of the four key ingredients now included.

When z_t has a stochastic trend, (60) reflects that as well.

To interpret (60), transform it by subtracting y_{t-1} from both sides to create the first difference on the left-hand side:

$$y_t - y_{t-1} = \beta_0 + \beta_1 z_t + (\beta_2 - 1)\, y_{t-1} + \beta_3 z_{t-1} + \epsilon_t \qquad (61)$$

Next, subtract $\beta_1 z_{t-1}$ from $\beta_1 z_t$, to create a difference, and add $\beta_1 z_{t-1}$ to $\beta_3 z_{t-1}$ (to keep the equation balanced) so that:

$$\Delta y_t = \beta_0 + \beta_1 \Delta z_t - (1 - \beta_2)\, y_{t-1} + (\beta_1 + \beta_3)\, z_{t-1} + \epsilon_t \qquad (62)$$

which reveals that β_1 is the impact of Δz_t on Δy_t.
Now collect the terms in y_{t-1} and z_{t-1} when $|\beta_2| < 1$ as:

$$\Delta y_t = \beta_0 + \beta_1 \Delta z_t - (1 - \beta_2)\,(y_{t-1} - \kappa_1 z_{t-1}) + \epsilon_t \qquad (63)$$

where $\kappa_1 = (\beta_1 + \beta_3)/(1 - \beta_2)$.

Finally, it is convenient to collect the intercept with the last term as well, writing (63) as:

$$\Delta y_t = \beta_1 \Delta z_t - (1 - \beta_2)(y_{t-1} - \kappa_0 - \kappa_1 z_{t-1}) + \epsilon_t \tag{64}$$

where $\kappa_0 = \beta_0/(1 - \beta_2)$.

When change ceases, so $\Delta y_t = \Delta z_t = 0$, or $y_t = y_{t-1} = y$ and $z_t = z_{t-1} = z$, with no shocks, so $\epsilon_t = 0$, then (64) becomes $y = \kappa_0 + \kappa_1 z$, which is the equilibrium.

The model in (64) is called an 'equilibrium-correction' mechanism (often abbreviated to **EqCM**) as the change in y_t 'corrects' to the previous deviation $(y_{t-1} - \kappa_0 - \kappa_1 z_{t-1})$ from equilibrium at a rate depending on $(1 - \beta_2)$.

Summary of lecture 4 key points

(A) Economic-theory models provide invaluable insights; but rarely allow for all variables and sudden changes.

(B) Statistical models, $f_y(y|z, \beta)$, are theories of the data-generation process (DGP), $D_y(y|z, \theta)$.

(C) Unknown parameters β are estimated from data, but estimation methods assume the statistical model is the DGP.

(D) For $f_y(\cdot)$ to coincide with $D_y(\cdot)$, must model all substantively relevant variables, dynamics, breaks, non-linearities and trends.

(E) Statistical properties of estimators and tests can be easily illustrated using computer-generated data.

(F) Start empirical analysis from a general unrestricted model (GUM).

(G) Simple unemployment equations showed **not all models equally useful**.

(H) Graphical statistics reveal a great deal about how well an estimated model describes the time-series evidence.

Next apply these ideas to model our empirical data.

Route map

(1) Macroeconomic data: evolution with abrupt change
(2) Taming trends and breaks
(3) Dependence between variables and over time
(4) Economic theory and statistical analysis: 2 key ingredients
(5) **Modelling UK unemployment**
(6) Modelling UK wages
(7) Modelling UK money demand
(8) Modelling UK prices
(9) Conclusion

Lecture 5 guide post

1. Economic-theory models rarely allow for the sudden shifts seen in data, but may still help inform a statistical model specification.
2. Theory can guide the formulation of a GUM when combined with previous evidence and institutional knowledge, incorporating dynamics, non-linearities, breaks and trends.
3. Linear dynamic equations can be transformed to EqCMs, eliminating stochastic trends by cointegration.
4. Multiple shifts can be tackled by impulse-indicator saturation (denoted IIS), with little impact when there are no shifts or outliers.
5. Such GUMs too large for humans, with more variables than observations, but not for automatic model selection software.
6. An EqCM model of unemployment describes the evidence quite well, and is constant over 2005–2011 (the 'Great Recession').
7. That model is not a 'causal' explanation, as a key regressor is contemporaneous with components that are undoubtedly affected by unemployment, but still has some economic policy implications.

Poverty and unemployment

Poverty was perhaps the greatest problem you would face living in 1860, with attendant problems of malnourishment and disease, but it would be greatly exacerbated by unemployment.

Essentially no other source of income for most than their earnings (or the Workhouse).

Garraty (1979) estimates that unemployment existed since the Middle Ages, then in the form of sturdy beggars.

As Figure 95 showed, unemployment rates fluctuated considerably before WWI, and could reach 10% of the labour force, but were dwarfed by the persistently high levels in the interwar period, a phenomenon that regrettably returned after the benign post-war reconstruction (and Keynesian) era, with the Oil Crises of the 1970s and Mrs Thatcher's economic policies.

It is obviously important to understand the behaviour of unemployment much better than the simple models in (55) and (56).

Postulating a better empirical model of unemployment

Could include wage share $(w - p - g + l)_t$ and its lagged value in the autoregressive model of $U_{r,t}$ as in (60).
Adds little: $R^2 = 0.79$ when it was 0.78.

Instead, will assume employment increases when hiring is profitable, and falls if not.
No good data on profit changes, but have a 'proxy'–namely a variable that is usually closely related.
1] Changes in revenues are linked to changes in GDP: Δg.

2] Close link between $(w - p)_t$ and $(g - l)_t$ seen above suggests labour costs and revenues are equilibrated.

3] Capital costs depend on real borrowing costs: $(R_L - \Delta p)_t$.

Combining, approximate the behaviour of changes in profits by the difference between the proxies for costs and for revenues:
$(R_L - \Delta p - \Delta g)_t$ called d_t—see Figure 146.

Profits proxy and unemployment

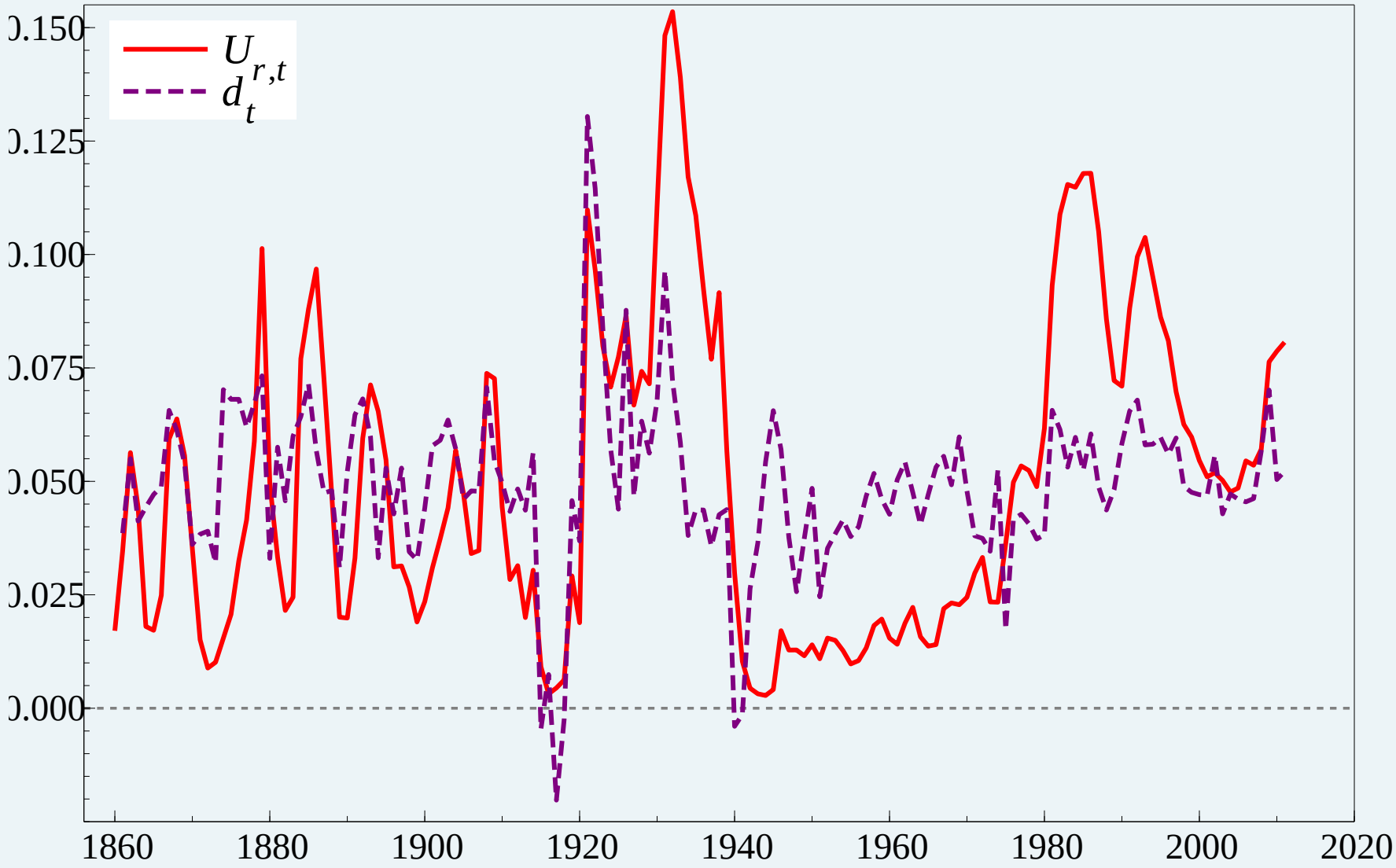


Figure: Profits proxy and unemployment.

The paths of the two time series have much in common: so let's model $U_{r,t}$ as in (60) using d_t for z_t:

$$\widehat{U}_{r,t} = \underset{(0.002)}{0.007} + \underset{(0.035)}{0.86}\, U_{r,t-1} + \underset{(0.024)}{0.24}\, d_t - \underset{(0.027)}{0.10}\, d_{t-1}$$

$$R^2 = 0.88 \quad \widehat{\sigma}_\epsilon = 0.013 \tag{65}$$

The fit is better than either previous model, and the impacts of both d_t and its lag are statistically significant:
divide each estimated coefficient by its standard error to get a statistic that has a t_{140} distribution under the null that each coefficient is zero, and compare to lying outside ± 2.6 to reject the null hypothesis at the 1% level.
Figure 148 records the actual $U_{r,t}$ and fitted $\widehat{U}_{r,t}$ values, residuals $\widehat{\epsilon}_t = U_{r,t} - \widehat{U}_{r,t}$, their density and correlogram.

Graphical statistics of the dynamic unemployment model

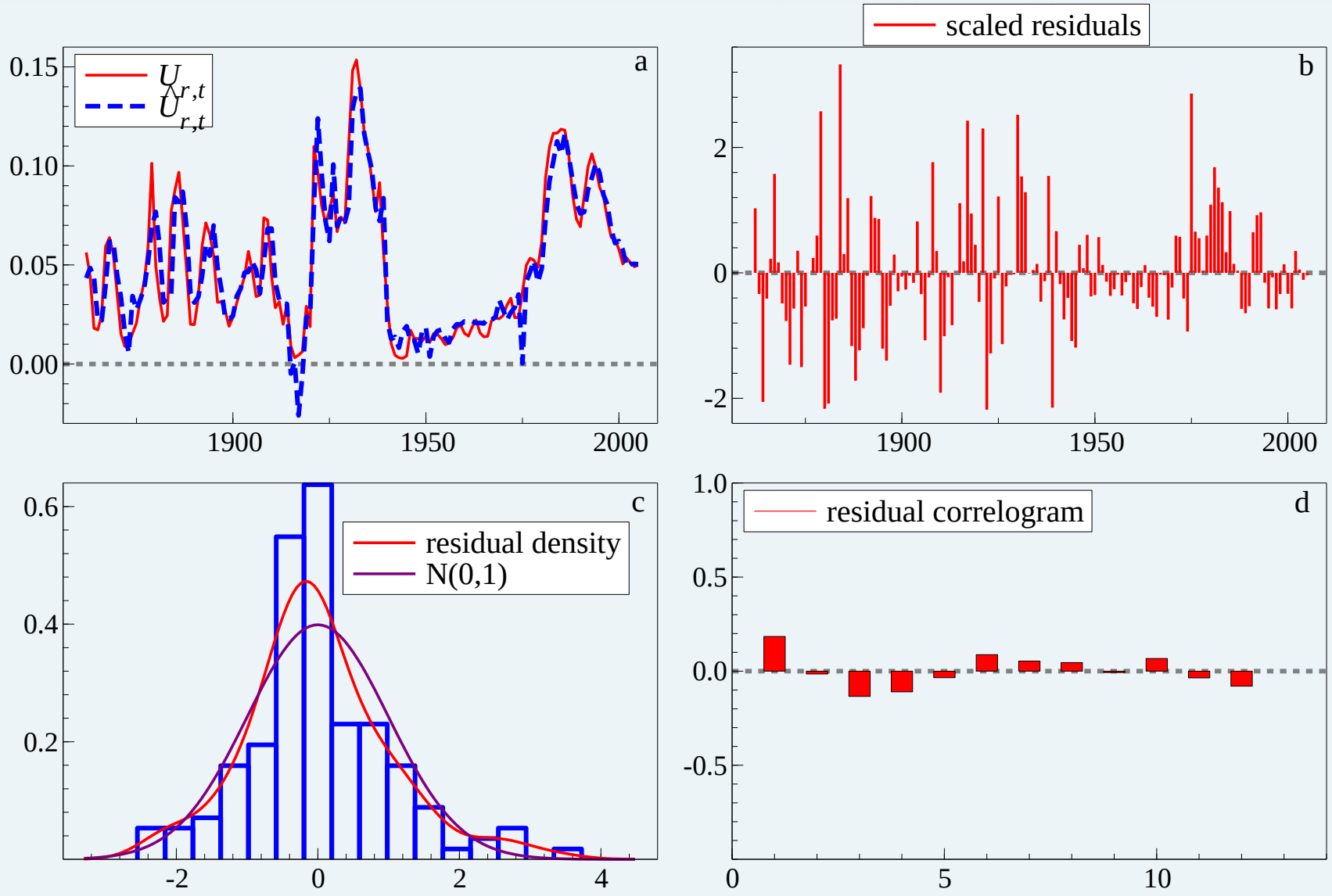


Figure: Graphical statistics of the dynamic unemployment model.

As $\widehat{\beta}_0 = 0.007$, $\widehat{\beta}_1 = 0.24$, $\widehat{\beta}_2 = 0.86$, and $\widehat{\beta}_3 = -0.10$, then $\widehat{\kappa}_0 = 0.007/0.14 = 0.05$ and $\widehat{\kappa}_1 = (0.24 - 0.10)/0.14 = 1.0$.

Thus, written as in (64), then (65) becomes:

$$\Delta\widehat{U}_{r,t} = 0.24\Delta d_t - 0.14\left(U_{r,t-1} - 0.05 - 1.0 d_{t-1}\right) \qquad (66)$$

(these coefficients are rounded), so the equilibrium is:

$$U_r = 0.05 + d$$

or 5% unemployment when $d = 0$, which is its mean.

Unemployment rises or falls by approximately 1% for every 1% increase or decrease in $d = (R_L - \Delta p - \Delta g)$.

The immediate effect of a change in d is an impact of $\pm 0.24\%$, so unemployment only moves part of the way to the eventual impact of 1% and that creates a disequilibrium.

Then, 14% of that deviation from equilibrium is removed each period.

Although our dynamic model in (66) is sensible and interpretable, it has an important restriction–we only allowed for 1 lag, so excluded lagged changes like $\Delta \mathrm{U}_{\mathrm{r,t-1}}$ and $\Delta \mathrm{d}_{\mathrm{t-1}}$ (or longer).

Those are easily added to equations like (66), and doing so delivers:

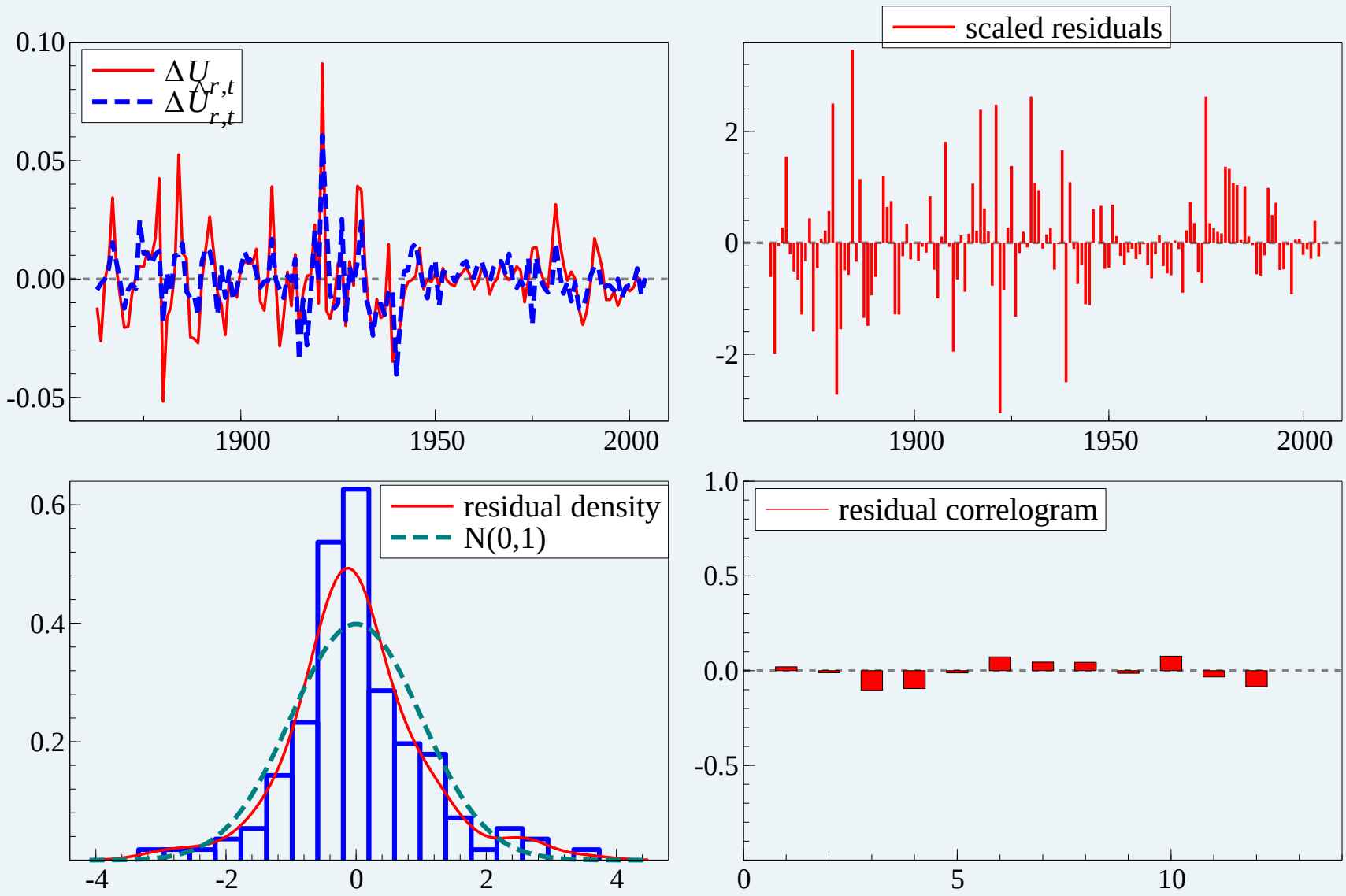
$$\Delta \widehat{\mathrm{U}}_{\mathrm{r,t}} = \underset{(0.07)}{0.16}\,\Delta \mathrm{U}_{\mathrm{r,t-1}} + \underset{(0.02)}{0.24}\,\Delta \mathrm{d}_{\mathrm{t}} - \underset{(0.03)}{0.12}\,(\mathrm{U}_{\mathrm{r,t-1}} - 0.05 - \mathrm{d}_{\mathrm{t-1}})$$

$$(\mathrm{R}^*)^2 = 0.48 \quad \widehat{\sigma}_\epsilon = 0.012 \tag{67}$$

$(\mathrm{R}^*)^2$ is when a constant is added–the much smaller value than, say (65), is because the dependent variable is $\Delta \mathrm{U}_{\mathrm{r,t}}$: $\widehat{\sigma}_\epsilon$ is smaller, so the model is an improvement.

Adding $\Delta \mathrm{U}_{\mathrm{r,t-1}}$ was significant, but $\Delta \mathrm{d}_{\mathrm{t-1}}$ was not.

Graphical statistics for the EqCM unemployment model



Returning to Figure 148:

A] we did not include indicators for the 4 mean shifts in $U_{r,t}$;

B] but there were a couple of 'outliers', or large residuals.

Then no need for A] would mean co-breaking between $U_{r,t}$ and d_t– but B] is a slight danger signal as follows. Say:

$$y_t = \mu_0 + (\mu_1 - \mu_0)\, 1_{\{t \geqslant T_1\}} \tag{68}$$

Then:

$$\Delta y_t = (\mu_1 - \mu_0)\, \Delta 1_{\{t \geqslant T_1\}} \tag{69}$$

As $\Delta 1_{\{t \geqslant T_1\}}$ must be zero when the indicator is zero for all $t < T_1$, and must also be zero when the indicator is unity for all $t > T_1$, then it is not zero only at the switch point, $t = T_1$, which is an 'impulse' indicator, $1_{\{t = T_1\}}$.

The change in a location shift is an impulse:

$$\Delta y_t = (\mu_1 - \mu_0)\, 1_{\{t = T_1\}} \tag{70}$$

First test of (67) is to add the 4 mean-shift indicators.

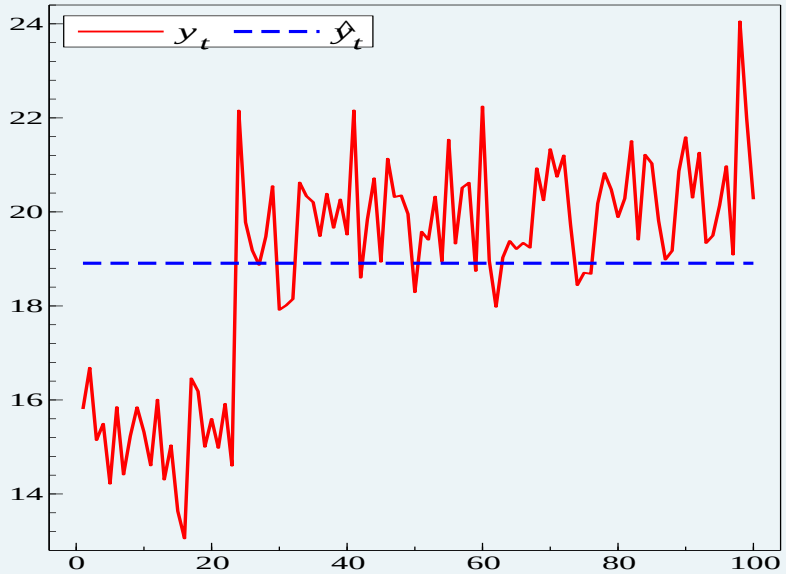
Doing so yields t-values of -1.5, 1.85, -0.73, and 1.0 none of which rejects the null that their effect is zero.
Thus, (67) seems to have 'captured' those location shifts.

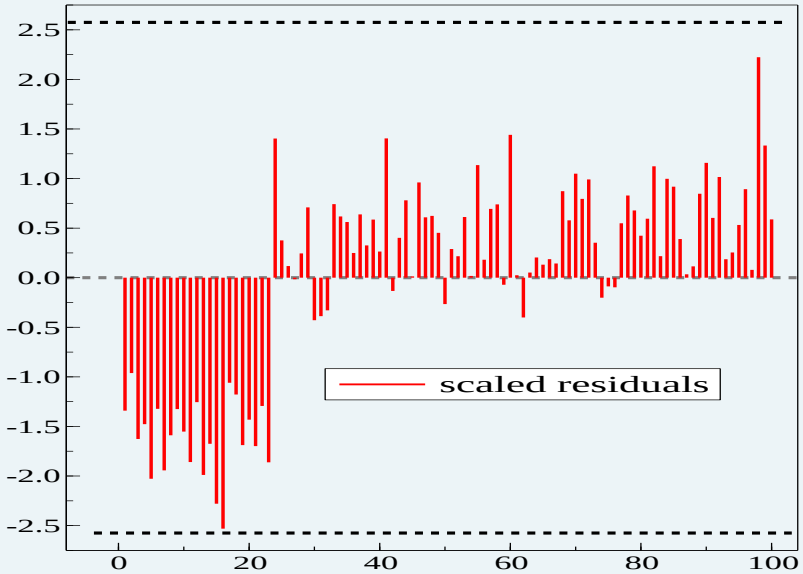
Given the lesson from (70), we next check for outliers.
Problematic indicators would correspond to the years where the jumps occurred: 1914, 1939, and 1980.
Could add those, but there may be other shifts relative to this model.

So allow for possible impulses at every data point to check for location shifts and outliers. Sounds impossible, but create a complete set of impulse indicators $\left\{1_{\{j=t\}},\ j=1,\ldots,T\right\}$ then add to model in feasible subsets: see Hendry and Nielsen (2010).

Called **impulse-indicator saturation (IIS)**.
In 'split-half' approach, add half the indicators & record significant outcomes; repeat for other half; then combine significant indicators.

Location shift of **5 standard errors** at 0.23T but no outliers at 1%.



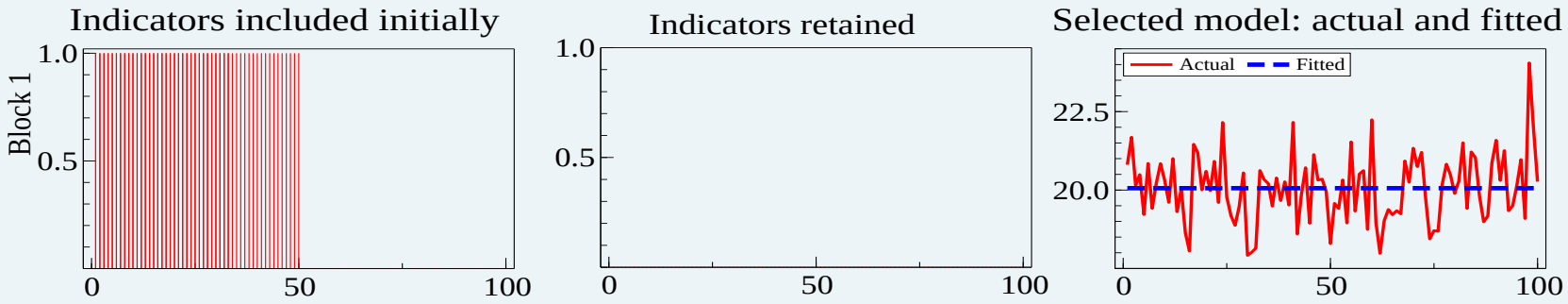


If just fit a constant, mean is in the 'middle', so leads to a large residual standard error: hence, despite large shift, **no outliers** as all residuals $\in [-2.5, 2.5]$SDs.

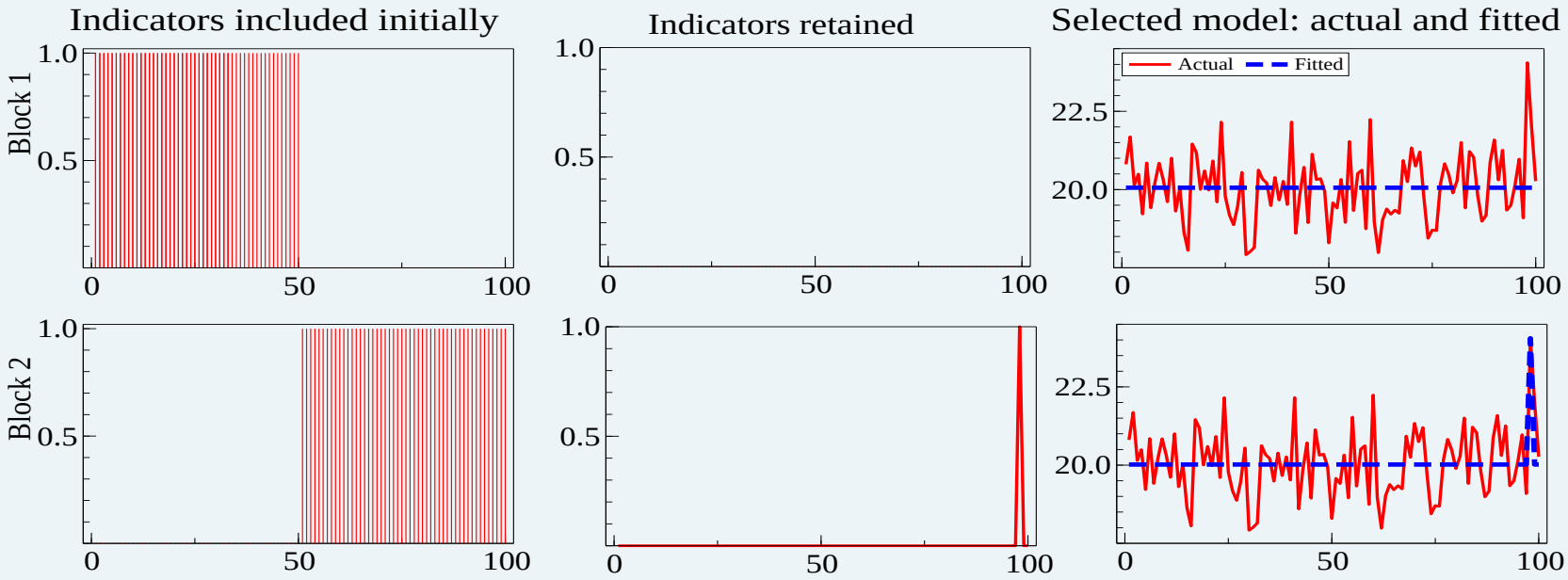
Absence of large residuals does not entail absence of shifts.

First illustrate what happens in this case when apply 'split-half' approach to IIS but there is no break, then repeat illustration of IIS for the above break.

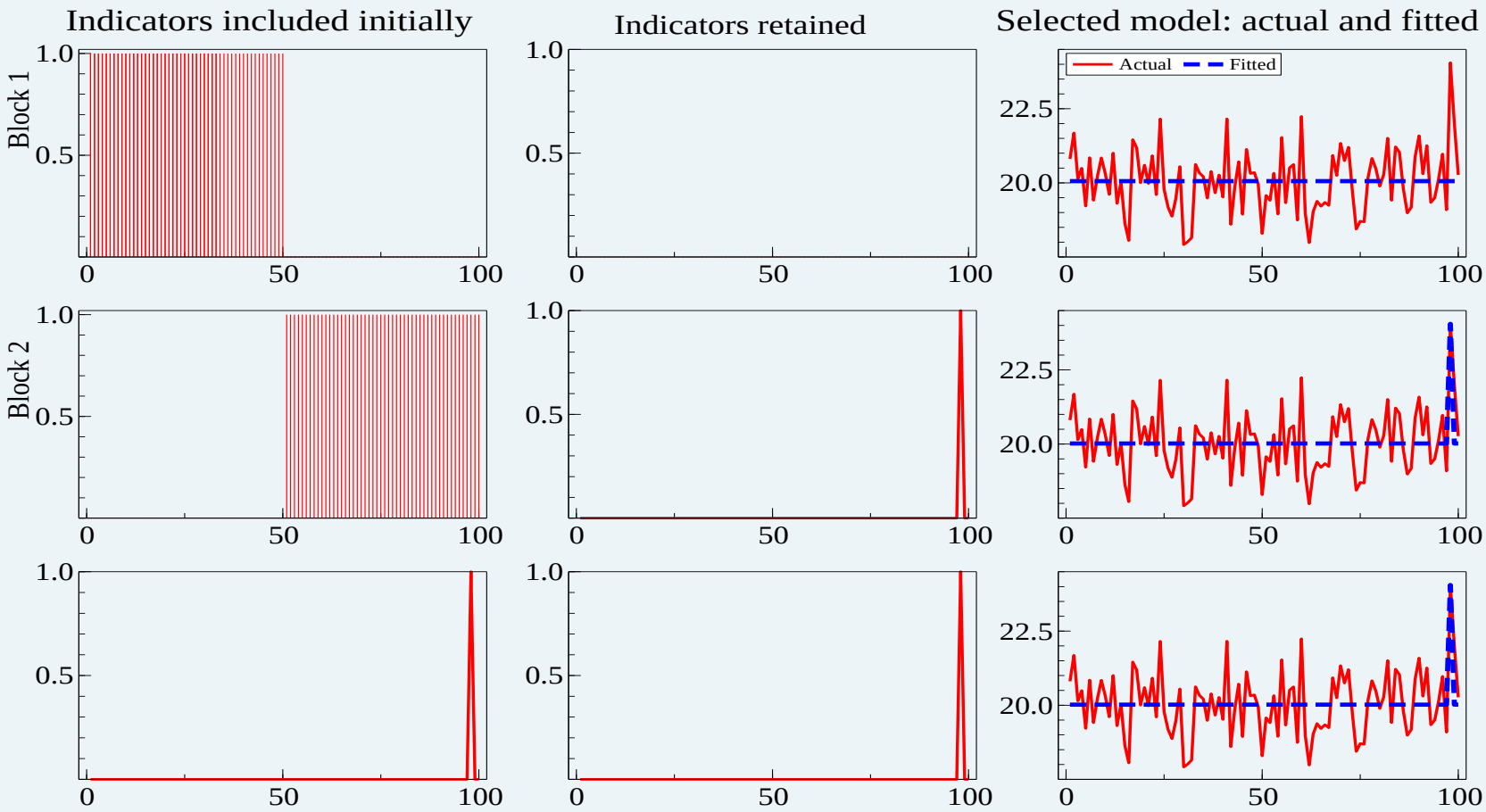
Create an indicator for every observation in candidate set and select by ‘split-half’ IIS at 1% when $T = 100$. First half:



Create an indicator for every observation in candidate set and select by 'split-half' IIS at 1% when $T = 100$. Second half:



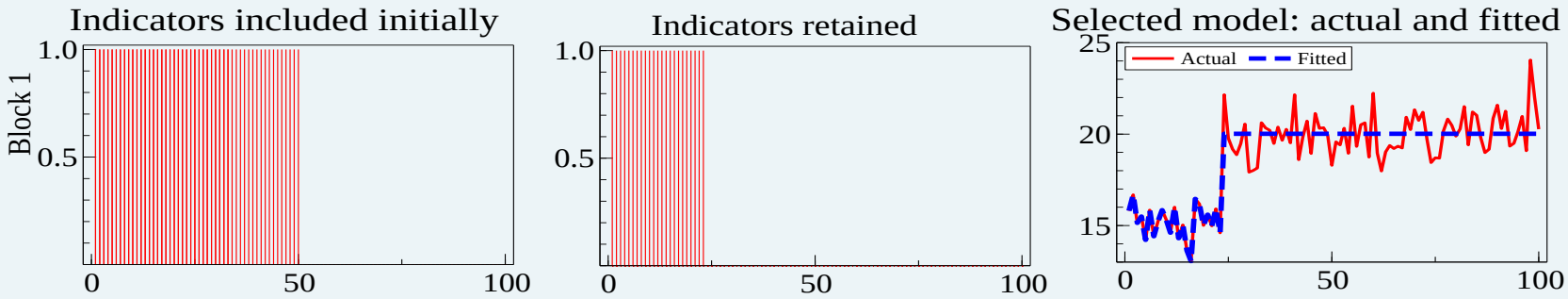
Illustrating 'split-half' IIS under the null of no breaks



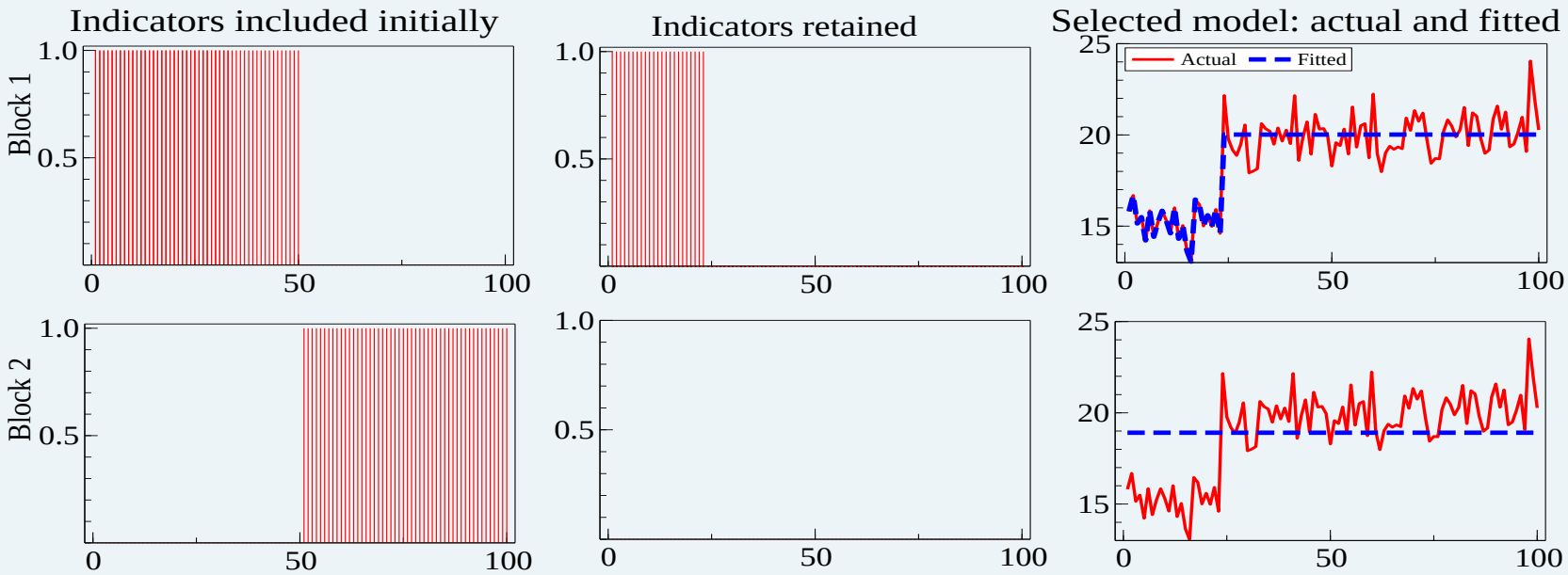
Have 100 observations without shifts, keep indicators significant at 1%: end by retaining 1 indicator, so lose 1 data point–

99% efficient despite adding 100 indicators.

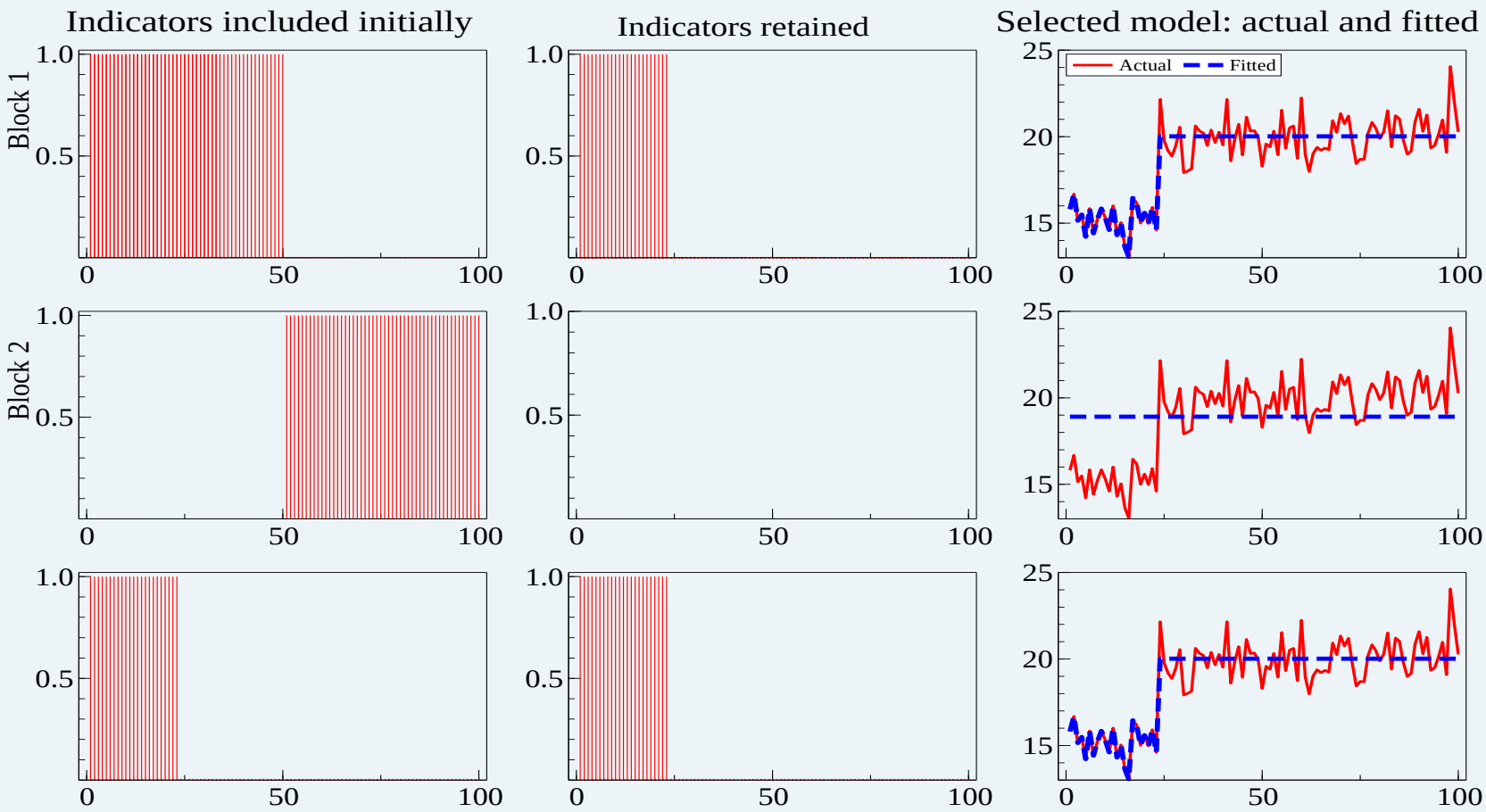
Again selecting at 1% when $T = 100$, first half leads to many retentions, as estimated mean is shifted by 5.



Second half now leads to no retentions, despite shift.



Illustrating 'split-half' IIS for a location shift



Combining both sets matches first set of indicators–so selection just retains these significant ones.

Same sign, similar magnitude impulses could be summarized by a single step indicator.

Can run *Autometrics* to see what it finds for U_r.

Check for outliers at every data point using impulse-indicator saturation at a tight significance level of 0.1%, so 1 chance in 1000 of finding an impulse where there was no genuine outlier:
yielded the following significant impulse indicators:
1880 (-4.9%) , 1884 (4.4%), 1921 (5.0%), 1922 (-5.3%), 1930 (3.5%), and 1939 (-3.6%).

Sudden 1921–1922 crash, and start of Great Depression in 1930 need indicators so are not captured by model.
But only 1939 matches the date for a danger signal, and is close to the size of the third shift $(\mu_3 - \mu_0)$ of about -3%.

Thus, despite the apparent close fit, the model may not explain the **average** low unemployment over the post-war reconstruction era.

Figure 158 records.

Graphical statistics of the IIS EqCM unemployment model

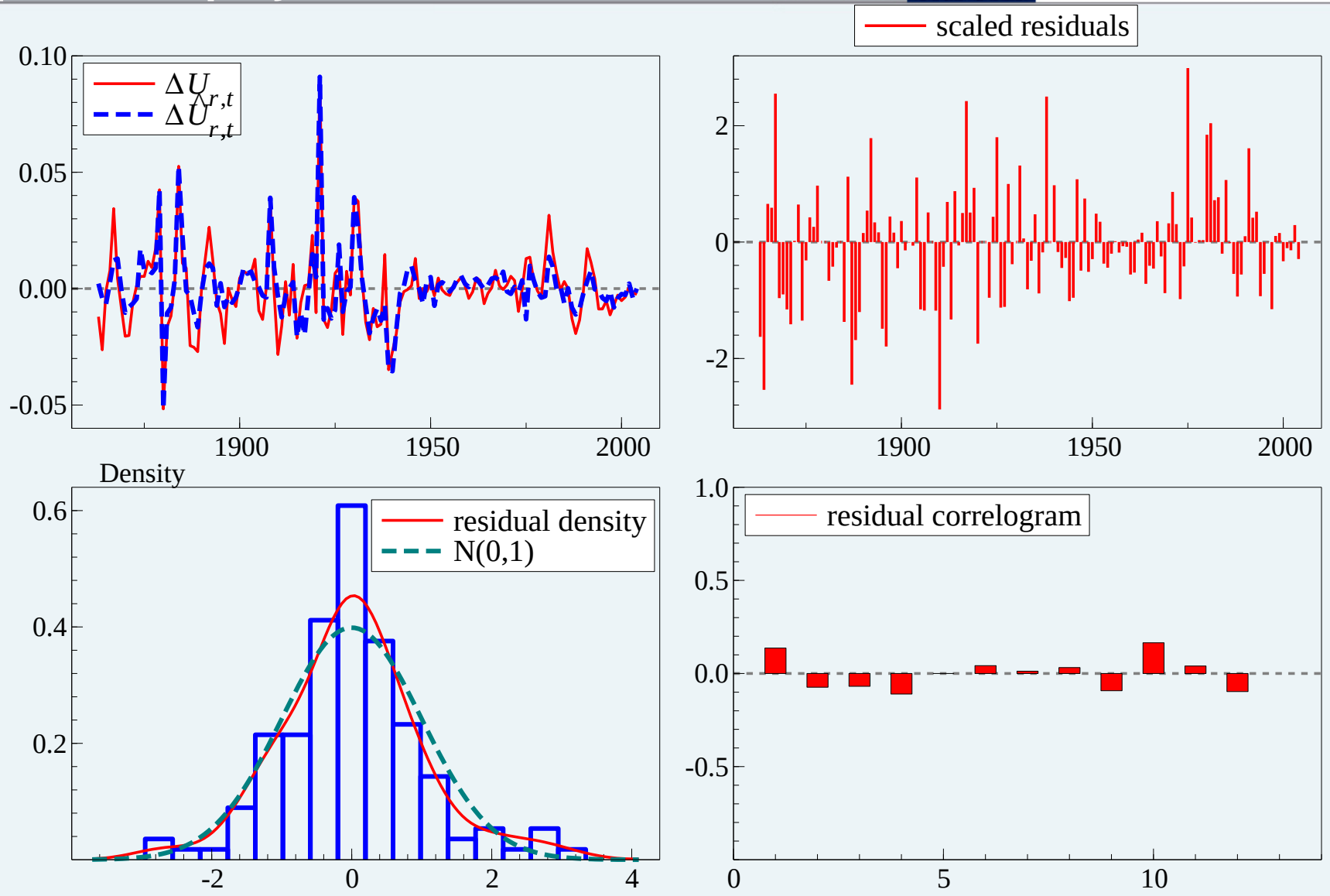


Figure: Graphical statistics of the IIS equilibrium-correction unemployment model.

All the models estimated above were developed a few years ago. There are 7 new annual outcomes on all the variables since then.

The period includes the sharp rise in unemployment during 2008, which nevertheless was much smaller than expected, given a fall of more than 6% in real GDP.

A strong test of the model is to check parameter constancy for these recent outcomes: compare out-of-sample actual and fitted values of the model after IIS in (71), over 2005–2011, not genuine statements about the future, which would require forecasts of d.

$$\begin{aligned}\Delta\widehat{\mathrm{U}}_{\mathrm{r},\mathrm{T}+\mathrm{h}|\mathrm{T}+\mathrm{h}-1} &= 0.16\Delta\mathrm{d}_{\mathrm{T}+\mathrm{h}} + 0.35\Delta\mathrm{U}_{\mathrm{r},\mathrm{T}+\mathrm{h}-1} \\ &\quad -0.08\left(\mathrm{U}_{\mathrm{r},\mathrm{T}+\mathrm{h}-1} - 0.05 - \mathrm{d}_{\mathrm{T}+\mathrm{h}-1}\right) \quad (71)\end{aligned}$$

where $\mathrm{T} = 2004$ and $\mathrm{h} = 1, \ldots, 7$, 1-step at a time.

Figure 160 records the outcome of that check.

The estimated model tracks the new data accurately: all ‘forecasts’ fall within the 95% intervals shown (bands in top chart, bars in lower).

‘Forecasts’ from the IIS EqCM unemployment model

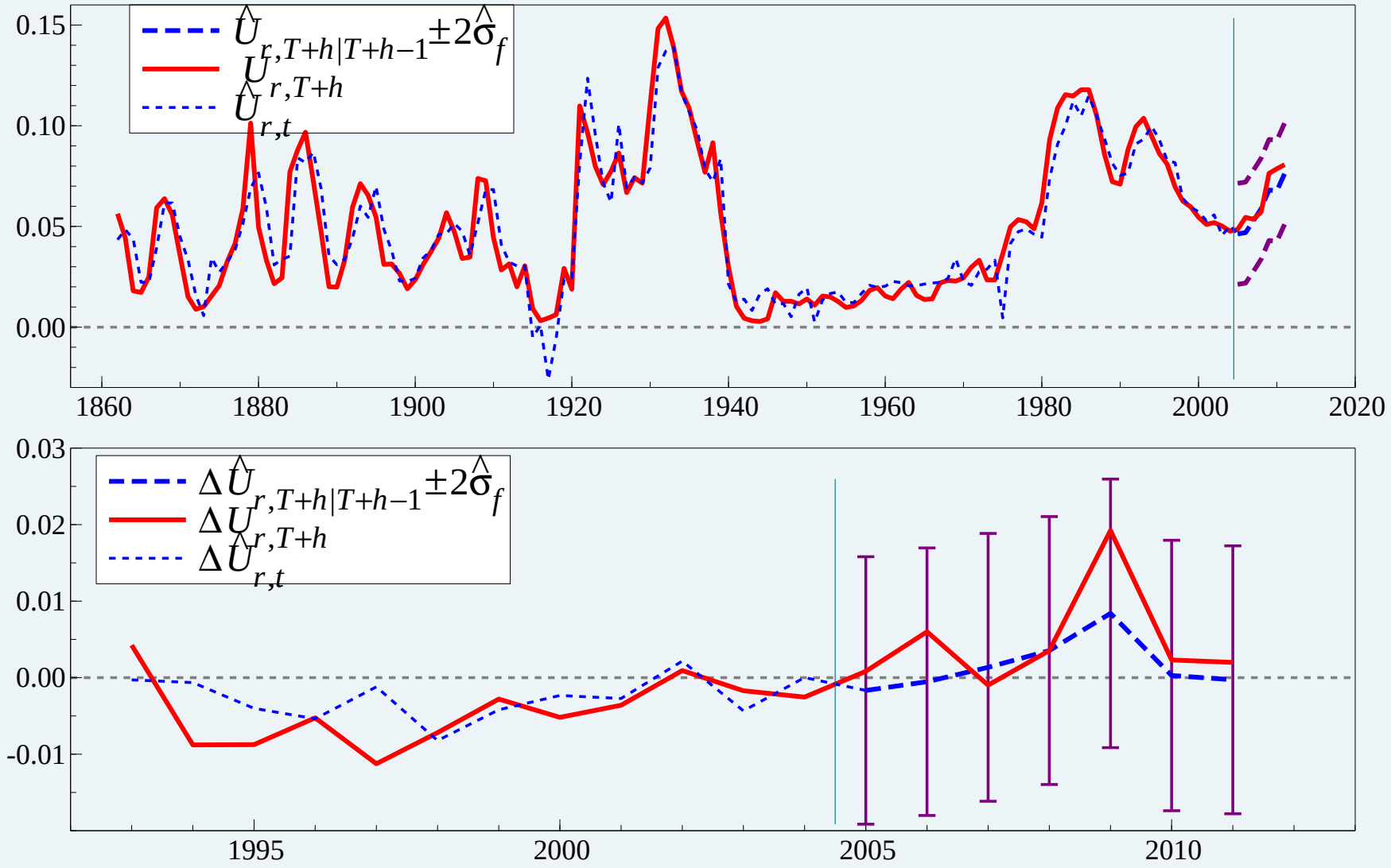


Figure: ‘Forecasts’ from the IIS equilibrium-correction unemployment model.

Some economic policy implications of (71)

(i) When the real long-term interest rate, $R_L - \Delta p$, equals the real growth rate, Δg, so $d = 0$, equilibrium unemployment is about 5%, close to the average unemployment rate.

The model does not explain why, merely that movements from that rate are associated with non-zero values of d.

(ii) To lower unemployment when $d > 0$ and return to that equilibrium requires lower real long-term interest rates or faster growth: both are policies currently in force, but difficult to maintain while imposing austerity, especially if that reduces the next generation's education.

(iii) Policy has lowered $R_L - \Delta p$ to offset the large fall in Δg, so d_t only rose briefly, 'explaining' why U_r rose less than anticipated.

(iv) Unemployment can be well below its equilibrium for long periods when $d < 0$ (e.g., 1939–1968).

A key issue is the possible consequence of (iv) for inflation—to which we turn after summarizing some lessons.

Summary of lecture 5 key points

(A) Economic-theory models rarely allow for sudden shifts, but may still help inform a statistical model specification.
(B) To represent the DGP requires a GUM that includes all the substantively relevant variables.
(C) Dynamic equations can be transformed to equilibrium-correction models (denoted EqCMs), which eliminate stochastic trends in cointegrated relations, and make interpretation easier.
(D) Multiple location shifts can be tackled by IIS with little impact on an empirical modelling exercise when there are no shifts or outliers.
(E) The GUMs that result from (B) and (D) usually contain more variables plus indicators than the sample size.
(F) Automatic computing software can handle such GUMs, and find a parsimonious, congruent and encompassing selection.
(G) The unemployment EqCM model based on the profits proxy d_t describes the evidence, and is constant on the data over 2005–2011.
(H) But that model is not a 'causal' explanation, as d_t is contemporaneous and its components may be affected by U_r.

Route map

(1) Macroeconomic data: evolution with abrupt change
(2) Taming trends and breaks
(3) Dependence between variables and over time
(4) Economic theory and statistical analysis: 2 key ingredients
(5) Modelling UK unemployment
(6) **Modelling UK wages**
(7) Modelling UK money demand
(8) Modelling UK prices
(9) Conclusion

Lecture 6 guide post

1. Empirical analyses must start from GUMs with all theory suggested variables, dynamics, shifts and non-linearities.
2. Location shifts are reasonably captured by a generalization of IIS called step-indicator saturation (denoted SIS).
3. Real wages respond non-linearly to inflation, consistent with ‘wage-price spirals’.
4. A non-linear response to unemployment is consistent with unemployment being involuntary.
5. Selecting location shifts by SIS does not preclude finding non-linearities nor conversely.
6. Little evidence that expectations of future values play an important role in the wage model.
7. Despite selecting from a GUM with more candidate variables than observations, the final real-wage model is readily interpretable, with a long-run equilibrium of a constant share of wages in GDP.
8. The model is constant over the Great Recession, encompasses other empirical models, and passes a test of super exogeneity.

What your 1860 wage would buy

Mackenzie (1921) records median, upper and lower quartiles of adult men's wages in the UK in 1860, 1880 and 1914.
Shows ranges of 14s.6d to 22s.6d in 1860 rising to 25s.2d–39s.4d in 1914: the last square with the data in Pember Reeves (1913)
(there were 20 shillings to a £ and 12 pence, denoted d, to a shilling).
Your wage would not buy you very much with bread costing about 1.5d per pound, milk 2d per pint, and (e.g.) bacon 1s per pound.

Estimates suggest that over half of total expenditure was allocated to food, 20%–30% to rent, about 12% to fuel, light and clothing.
Almost no income was left to spend, and malnutrition was rife.

We need to understand why 1860 workers were doing so badly on average, and why matters improved so greatly later.

Wage and price inflation

Many studies see labour market as main source of inflation:
(i) excess demand for labour;
(ii) competition over the profit share.

Factor markets determine wages, & prices of capital goods.
But factor demands are derived from final demands, so latter must be direct determinants of price inflation.

Basic economic theory-model has equations for:
prices (or profit markup of prices over costs),
wages,
unemployment.
These usually depend on **productivity** (output per employee), **costs**, usually unit labour costs (wages relative to productivity) and **import prices, commodity prices, taxes, exchange rates**, & special factors (such as oil), all with lagged reactions and perhaps expectations.

Role of unemployment

Policy emphasis on ‘natural rate of unemployment’ or ‘non-accelerating inflation rate of unemployment’ NAIRU.
When **voluntary** unemployment is below its ‘natural rate’, inflation will accelerate indefinitely: Friedman (1977).

UK contributions include Nickell (1990), and Layard, Nickell and Jackman (1991).

Opposite to Keynes (1936) and Phillips (1958): **involuntary** unemployment puts downward pressure on wages.

Wage-price dynamic interactions (spirals) important:
see Sargan (1964, 1980) for the UK;
and (e.g.) de Brouwer and Ericsson (1998) for Australia.

Rowlatt (1988) is good summary of forces driving UK inflation over 1969–1985.

New-Keynesian Phillips curve (NKPC) includes expected future inflation as a 'feed-forward' variable to explain current inflation:

$$\Delta p_t = \underset{\geqslant 0}{\gamma_f} E_t\left[\Delta p_{t+1}\right] + \underset{\geqslant 0}{\gamma_b} \Delta p_{t-1} + \underset{\geqslant 0}{\alpha} s_t + u_t \tag{72}$$

where Δp_t is rate of inflation, $E_t\left[\Delta p_{t+1}\right]$ is expected inflation one-period ahead conditional on information available today, and s_t are firms' real marginal costs. See Galí and Gertler (1999) and Galí, Gertler and Lopez-Salido (2001).

Also models with price-setting firms where wages are 'cost-push', raising final-goods prices: Dicks-Mireaux and Dow (1959), Godley and Nordhaus (1972).

All models still need adequate final demand to sustain their processes.

None allow for unanticipated shifts outside their theory.

Return to Table 2, adding U_{r} & $\Delta(\mathsf{g}-\mathsf{l})$, and showing whole period means (10). Five shifts in U_{r}–related to unstable Phillips curve.

Table: Means over nine sub-periods and overall in % p.a.

Shift	Sub-sample	Δw	$\Delta \mathsf{p}$	U_{r}	$\Delta(\mathsf{g}-\mathsf{l})$	$\Delta(w-\mathsf{p})$
(1)	1861–1913	1.00	0.20	4.2	1.09	0.80
(2)	1914–1920	14.6	14.0	1.5	−1.60	0.60
(3)	1921–1923	−12.2	−11.9	9.5	3.70	−0.30
(4)	1924–1938	0.50	−0.50	9.9	1.10	0.90
(5)	1939–1945	8.20	5.90	1.6	0.50	2.30
(6)	1946–1968	6.00	3.90	1.5	2.30	2.10
(7)	1969–1981	13.4	11.9	4.3	1.74	1.60
(8)	1982–2004	5.90	3.90	8.4	2.72	2.00
(9)	2005–2011	2.80	2.28	6.4	0.97	0.52
(10)	1861–2004	4.32	1.95	5.0	1.36	1.29

Figure 170 plots $\Delta \mathsf{p}$ with its location shifts.

Location shifts in UK price inflation

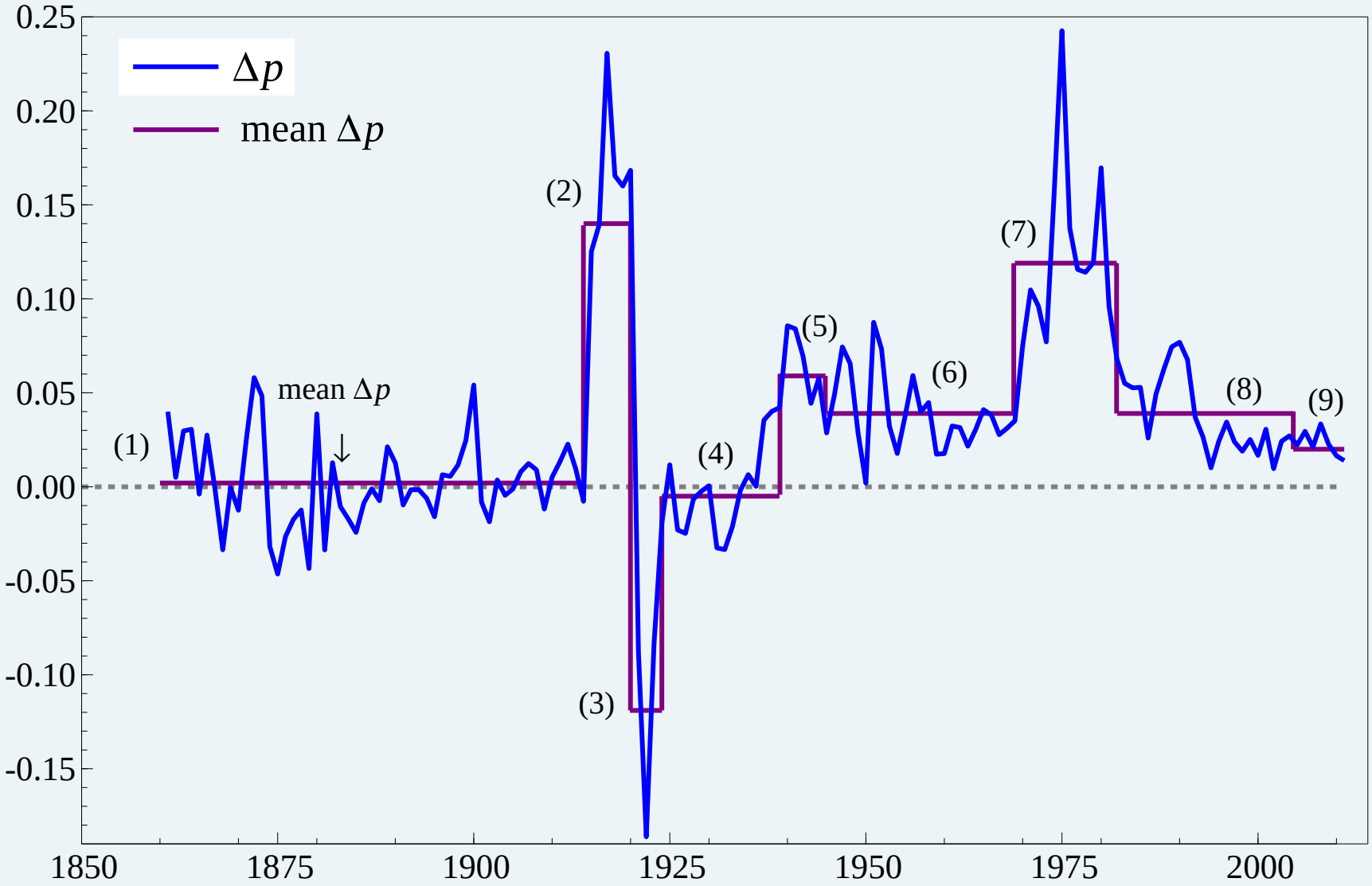


Figure: Location shifts in UK price inflation.

To detect location shifts automatically, introduce extension of IIS to step-indicator saturation (SIS):
complete set of step indicators $\{1_{\{t \leqslant j\}}, j = 1, \ldots, T\}$, where $1_{\{t \leqslant j\}} = 1$ for observations up to j, and zero otherwise.
Step indicators cumulate impulse indicators up to each next observation.

IIS: Impulses

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \ddots \end{bmatrix}$$

SIS: Step shifts

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where last column is the intercept dummy.
Castle, Doornik, Hendry and Pretis (2015) show SIS has a null retention frequency close to its nominal test size of α in congruent models.

The initial specification is:

$$y_t = \beta_0 + \boldsymbol{\beta}_1' \boldsymbol{z}_t + \epsilon_t \text{ where } \epsilon_t \sim \mathsf{IN}\left[0, \sigma_\epsilon^2\right] \tag{73}$$

The n conditioning variables $\boldsymbol{z}_t$ can be retained without selection for $n << T/2$.

Add the first $T/2$ indicators from the saturating set:

$$y_t = \beta_0 + \boldsymbol{\beta}_1' \boldsymbol{z}_t + \sum_{j=1}^{T/2} \delta_j 1_{\{t \leqslant j\}} + \epsilon_t \tag{74}$$

which equation can be estimated directly, indicators being retained when their estimated coefficients $\widehat{\delta}_j$ satisfy $\left|t_{\widehat{\delta}_j}\right| > c_\alpha$ where c_α is the critical value for significance level α.

Their locations are recorded, all those indicators are dropped and the second set is then investigated.

Combine selected indicators and re-select, with each of those m significant step indicators $\{1_{\{t \leqslant j\}}\}$ retained for adding to (73).

Under the null with $\alpha = 1/T$, at both sub-steps on average, $\alpha T/2$ (namely $1/2$ an indicator) will be retained by chance, so on average $\alpha T = 1$ indicator will be retained from the combined stage.

One degree of freedom is lost on average.
If $(n + T/2) > 2T/3$ say, divide total set of $N = n + T$ candidate variables into smaller sub-blocks, setting $\alpha = 1/N$ overall.
When m indicators are selected in a congruent representation at significance level α:

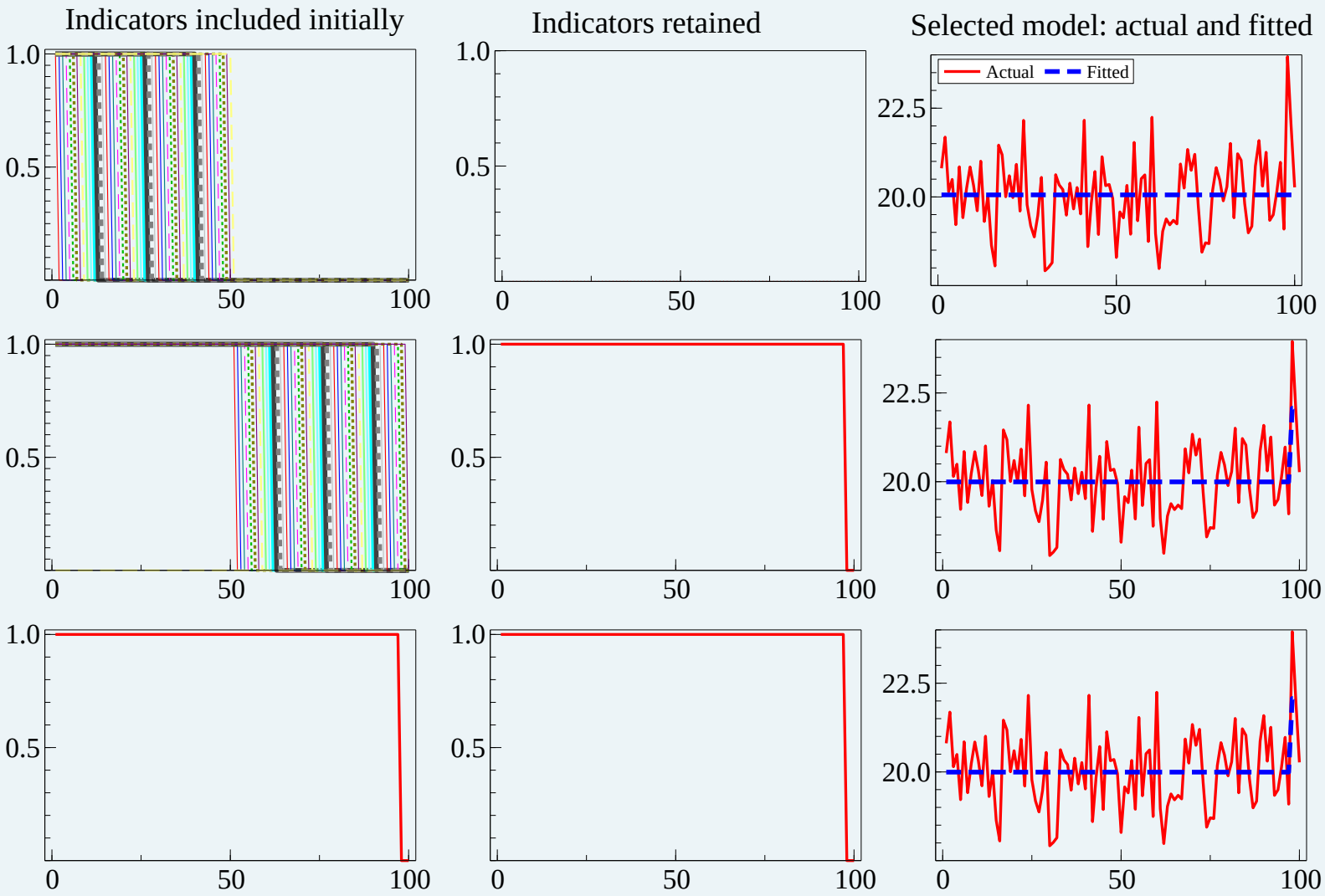
$$y_t = \beta_0 + \boldsymbol{\beta}_1' \boldsymbol{z}_t + \sum_{i=1}^{m} \phi_{i,\alpha} 1_{\{t \leqslant T_i\}} + \nu_t \tag{75}$$

where $\nu_t \sim \mathsf{IN}\left[0, \sigma_\nu^2\right]$, and coefficients of significant indicators are denoted $\phi_{i,\alpha}$.
Next two Figures illustrate the 'split-half' approach of SIS applied to:

$$y_t = 15 + \lambda \times 1_{\{t \geqslant 23\}} + \epsilon_t \text{ where } \epsilon_t \sim \mathsf{IN}\left[0, 1\right] \tag{76}$$

where $\lambda = 0$ (null) then $\lambda = 5$.

Illustrating 'split-half' approach to SIS when no location shifts



$T = 100$, **and no shifts, retains 1 significant step indicator, so lose 1 degree of freedom.**

Illustrating ‘split-half’ approach to SIS for a shift in (76)

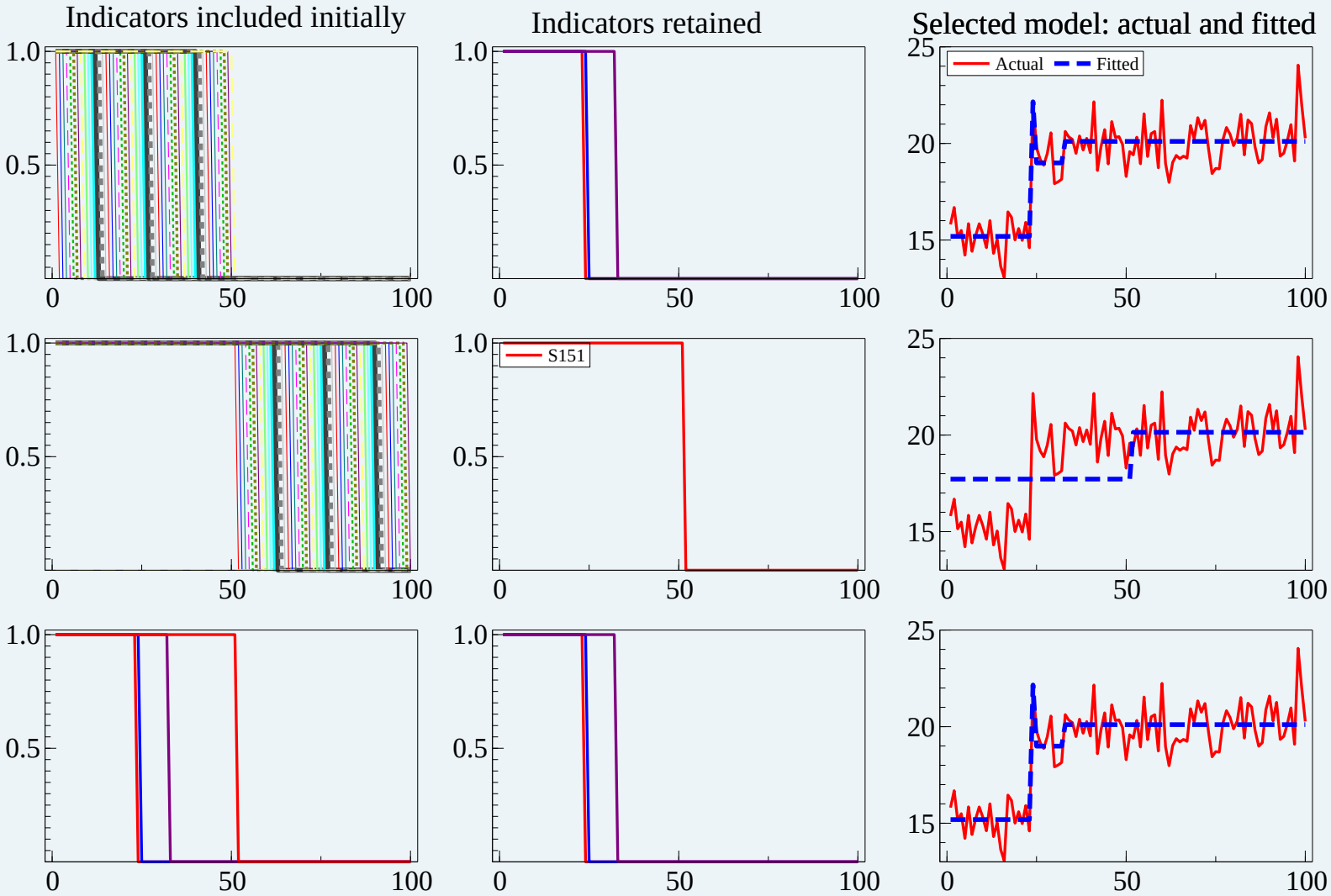


Figure: Illustrating SIS for a shift in (76) with $\lambda = 5$ at $T = 23$. First finds step indicators for shift (row 1), then finds indicator nearest shift (row 2), and combining only keeps first group: essentially combines indicators in Fig. 156.

Some location shifts found by SIS

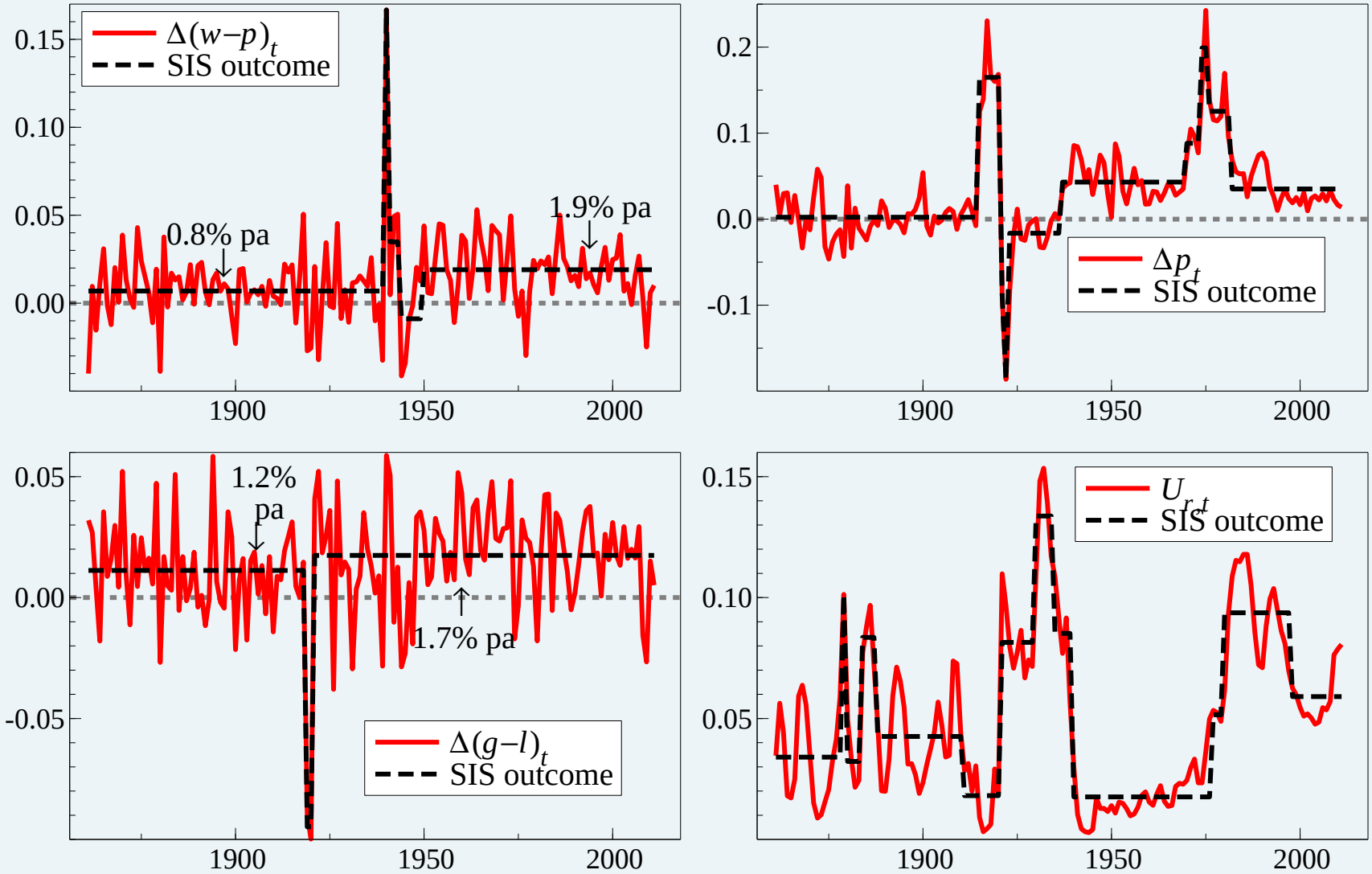


Figure: Location shifts in $\Delta(w-p)$, Δp, $\Delta(g-l)$ and U_r found by SIS.

Non-matching location shifts

A large mean shift is found for $\Delta(w - p)$ around 1945, following the spike in 1940, but with a small dip in between.

The outcomes for Δp are close to the step shifts described earlier.

Surprisingly for $\Delta(g - l)$, its mean shift starts in 1921 following the very sharp drop as WWI ended and the major flu' pandemic got under way.

As Table 5 suggested, $\Delta(w - p)$ and $\Delta(g - l)$ do not co-break. Consequently, any model linking them must allow for their different shifts, in addition to any direct connection.

For U_r, additional 'within regime' shifts are detected when the business cycles of the early and mid 1880s were especially severe, with the impact of the Great Depression treated as a further shift. These extra shifts do not alter the earlier notion of four main unemployment regimes.

Overall, SIS seems to match 'ocular' evidence and captures the main historical location shifts.

Wage and price adjustments

An economy can only have one aggregate nominal level, but w and p are two nominal variables, so transform to one real, $w - p$, which we will model, and one nominal, p. Can still recover both nominal.

Many forces affecting real & nominal wage adjustment including: staggered wage contracts; trade unions; insiders/outsiders; bargaining; institutional factors; unemployment; & indexation.

Factor demands derive from final demand, and final demand determines price inflation, Δp.

Determinants of Δp include capacity utilization or output 'gap'; 'natural rate of unemployment' or NAIRU; money growth; exchange rate and terms of trade shocks.

Obvious idea: include them all and see which work empirically.

Economic analysis can inform key aspects of model specification.

As price inflation rises, workers become more attentive and act to prevent further erosion of their real wages.

Suggests a non-linear reaction, $\Delta(w-p)_t = f_t \Delta p_t + \cdots$ where $f_t \simeq -1$ when $\Delta p_t \simeq 0$, so there is real wage erosion, at low cost to workers as little inflation; but

$f_t \simeq 0$ when Δp_t is large, so no erosion, though full compensation may need fought for by workers;

with some erosion in between. Castle and Hendry (2009) set :

$$f_t = \frac{-1}{1+\theta(\Delta p_t)^2} \tag{77}$$

where $\theta = 1000$, so almost no erosion by $\Delta p_t \geqslant 0.1$ ($\approx 10\%$ pa).

Figure 180 shows the response implied by (77) calculated from the historical data on Δp_t.

Non-linear wage reactions to price inflation and unemployment

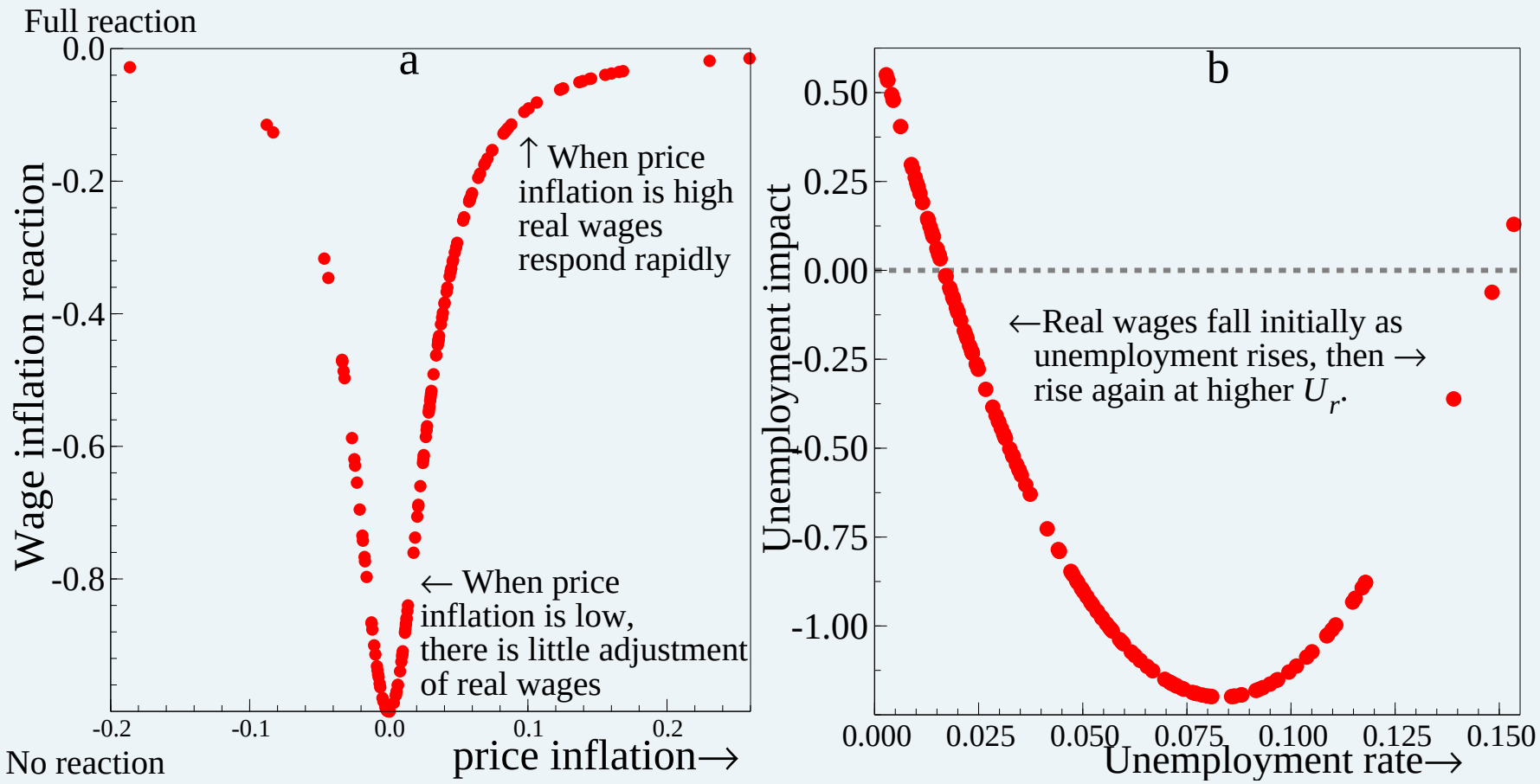


Figure: Non-linear wage reaction, f_t, to price inflation from (77) in panel *a*; and to unemployment in panel *b*.

Castle and Hendry (2014) show (77) is close to a logistic smooth transition. **Panel *b* is discussed below**.

Allow data evidence to determine adjustment process.

Initial Castle and Hendry (2009) model of $\Delta\left(w-p\right)_t$ included 25 candidate variables and yielded $\widehat{\sigma}=1.20\%$.

Had 4 indicator variables: 1918, 1940, 1975+1977, & WWII, measuring 3%, 14%, -5%, and 3% respectively.

No significant mis-specification tests.

Model then selected using *Autometrics* at 1%.

Only a few variables actually mattered as will see (indicators not shown).

$ulcp^*$ (real unit labour costs) is $w-p-g+l$ roughly adjusted for known changes in hours.

$\Delta_2 U_{r,t}=U_{r,t}-U_{r,t-2}$, the change over 2 years.

Results reported in (78), with graphs of model fit and residuals in Figure 183.

$$\Delta(w-p)_t = \underset{(0.13)}{0.76}\ f_t \times \Delta p_t - \underset{(0.04)}{0.14}\ \Delta_2 U_{r,t-1} + \underset{(0.05)}{0.39}\ \Delta(g-l)_t$$

$$+ \underset{(0.05)}{0.13}\ \Delta(g-l)_{t-2} - \underset{(0.01)}{0.08}\ (ulcp^* - \widehat{\mu})_{t-2} + \underset{(0.002)}{0.010}$$

$$R^2 = 0.75 \quad \widehat{\sigma} = 1.24\% \quad T = 1863\text{–}2004 \tag{78}$$

Coefficient of $f_t \Delta p_t$ significant but less than unity: some real wage erosion, & workers correct slowly from EqCM (8% pa).
Equilibrium correction also needed as real wage changes only reflect $0.39 + 0.13 \simeq 0.5$ of productivity changes.
Real wages converge to an equilibrium determined by $ulcp^* = \mu$, where μ is the long-run share, ($\widehat{\mu} = 1.85$):
remember Figures 25 (their 'common trend') and 53.
Unemployment *changes* have a small negative effect.
Intercept of 1% p.a. is unexplained real wage growth.
Fit of $\widehat{\sigma} = 1.24\%$ leaves a lot of variation unexplained:
interpret as 95% interval for errors of $\pm 2.5\%$ pa where the whole sample standard deviation of $\Delta(w-p)_t$ was 2.3%.

Graphical statistics of non-linear model

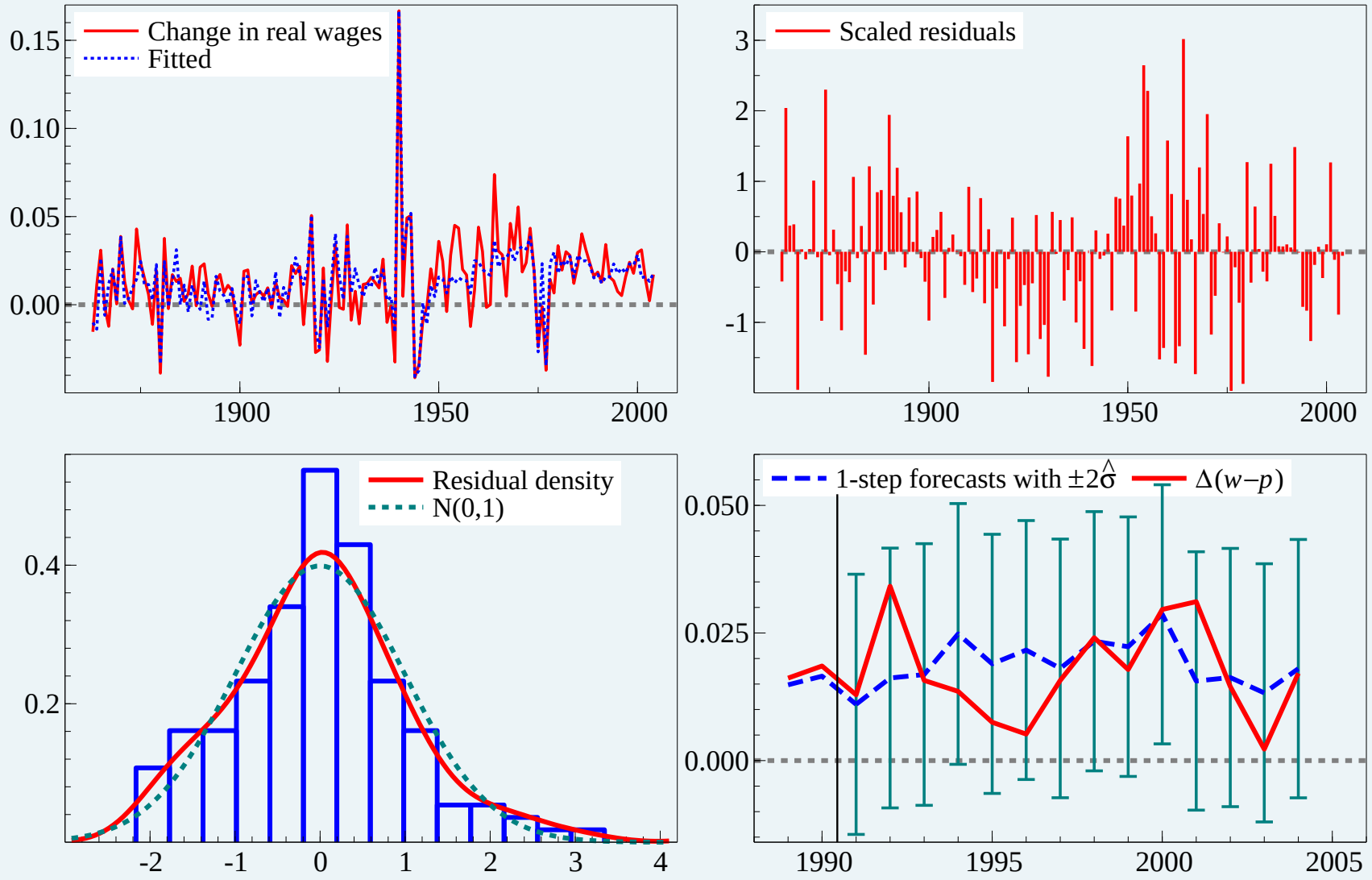
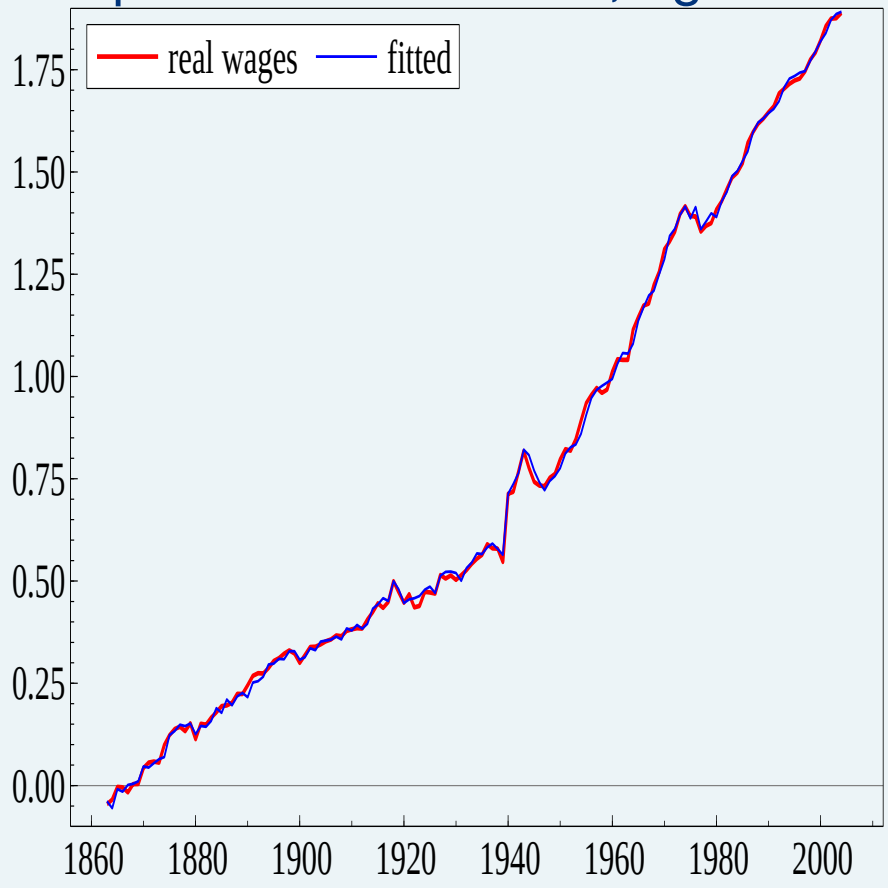
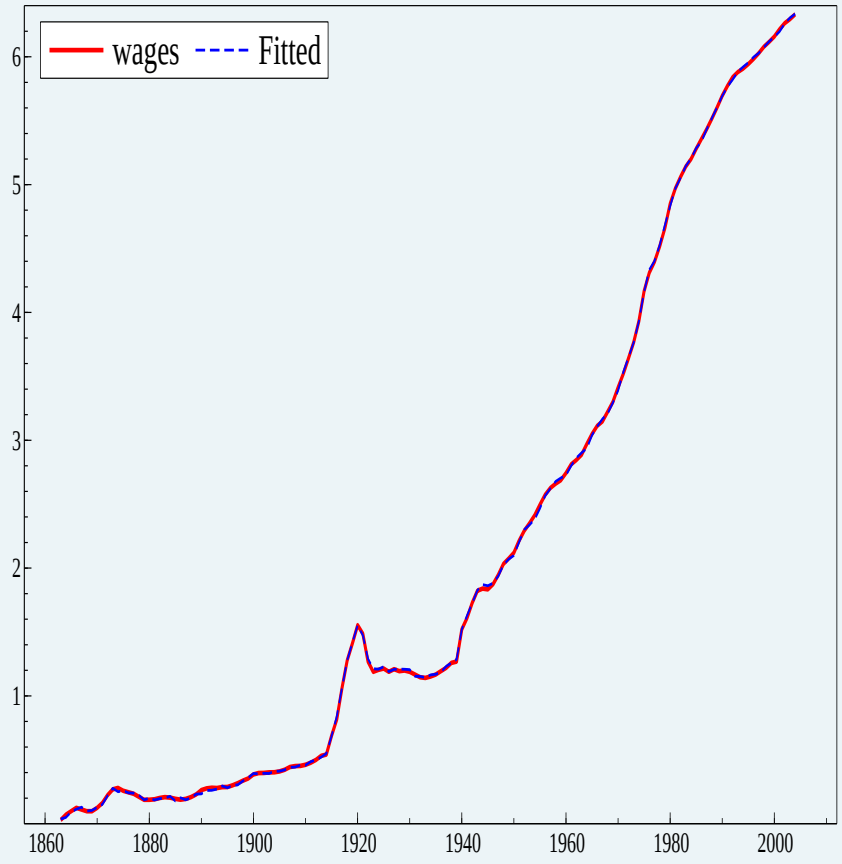


Figure: Model fit and residual statistics for (78) and 1-step 'forecasts' with $\pm 2\widehat{\sigma}$ over 1991–2004.

Graphical translation of real-wage inflation

Can 'translate' model of $\Delta(w-p)_t$ to one of:
$(w-p)_t$ given $(w-p)_{t-1}$;
or of w_t given p_t, w_{t-1}, p_{t-1}.
Left panel shows former, right the latter.





Both show 'near perfect' fit—an illusion from their strong trends.

To test the importance of expectations about future wages in (78), $E_{t-1}[\Delta w_{t+1}]$ was added by forecasting Δw_{t+1} from the variables $(R_S - R_L)_{t-i}$, $\Delta p_{o,t-i}$ (oil prices), $\Delta p_{w,t-i}$ (world prices), & Δp_{t-i} for $i = 1, 2$ (significant explanatory variables from the price inflation model). $\Delta(w-p)_t$ was split into Δw_t and Δp_t yielding:

$$\begin{aligned}\Delta w_t = & -\underset{(0.10)}{0.15}\,\widehat{\Delta w}_{t+1} - \underset{(0.04)}{0.02}\,\Delta w_{t-1} + \underset{(0.003)}{0.014} + \underset{(0.10)}{1.1}\,\Delta p_t \\ & + \underset{(0.17)}{0.70}\,f_t \times \Delta p_t + \underset{(0.08)}{0.46}\,\Delta(g-l)_t + \underset{(0.07)}{0.17}\,\Delta(g-l)_{t-2} \\ & - \underset{(0.02)}{0.11}\,(ulcp^* - \widehat{\mu})_{t-2} - \underset{(0.06)}{0.15}\,\Delta_2 U_{r,t-1} \qquad (79)\end{aligned}$$

$$\widehat{\sigma} = 1.4\% \quad T = 1876 - 2001$$

(indicators included but not shown). Adding $\widehat{\Delta w}_{t+1}$ leaves wage model almost unchanged, with 'wrong signed' coefficient: similar findings for other models of inflation are reported in Castle, Doornik, Hendry and Nymoen (2014).

New analysis of real-wages, using SIS and a new approach to modelling non-linearity, reported in Castle and Hendry (2014). Much more general, and re-selected here as:

$$\begin{aligned}\Delta(w-p)_t = {} & \underset{(0.003)}{0.031} + \underset{(0.041)}{0.303}\Delta(g-l)_t + \underset{(0.032)}{0.123}\Delta_2(g-l)_{t-1} + \underset{(0.012)}{0.725}(f_t\Delta p_t) \\ & - \underset{(0.028)}{0.158}(w-p-g+l-\widehat{\mu})_{t-2} - \underset{(0.035)}{0.201}U_{r,t} - \underset{(0.045)}{0.14}\Delta_2 U_{r,t} \\ & + \underset{(0.69)}{3.22}\left(U_{r,t}-\overline{U}_r\right)^2 - \underset{(0.029)}{0.145}\Delta^2 p_t - \underset{(0.011)}{0.149}S_{1939} + \underset{(0.014)}{0.166}S_{1940} \\ & - \underset{(0.008)}{0.044}S_{1943} - \underset{(0.008)}{0.025}(S_{2011}-S_{1946})\Delta u_{r,t} - \underset{(0.011)}{0.038}I_{1916} \\ & + \underset{(0.007)}{0.050}(I_{1942}+I_{1943}-I_{1944}-I_{1945}) - \underset{(0.011)}{0.045}I_{1977} \qquad (80)\end{aligned}$$

$$R^2 = 0.823; \quad \widehat{\sigma} = 1.03\%; \quad T = 1864-2004;$$

No diagnostic test rejects. The fit is considerably better than (78), where $\widehat{\sigma} = 0.0124$, yet variables in common have relatively similar coefficient estimates.

Interpreting (80)

Short-run impact of $\Delta(g - l)$ of 0.55, whereas the long-response is unity: shortfall removed by EqCM at about 16% p.a.
Reaction of real wages to price-inflation erosion from (77) shown in Figure 180*a* is now 73%.

Unemployment enters (80) non-linearly: expressed in percentages ($0.05 = 5\%$), combined effect is $-.5U_{r,t}(1 - 6.4U_{r,t})$, shown in Figure 180*b*. As unemployment increases, real wages fall till $U_r \approx 8\%$, then increase, changing sign when $U_r \geqslant 15\%$.

Workers first suffer a loss of bargaining power, then movements down the marginal product of labour curve raise real wages for workers still employed: **higher unemployment at higher real wages is clear evidence of involuntary unemployment**.

There is also a small negative impact from changes in unemployment and from $\Delta \log U_{r,t}$ in the post WWII epoch, adding to the negative impact on real wages of increases in unemployment.

Figure 188 shows a relatively constant fit for (80) over past 150 years.

Graphical statistics for (80)

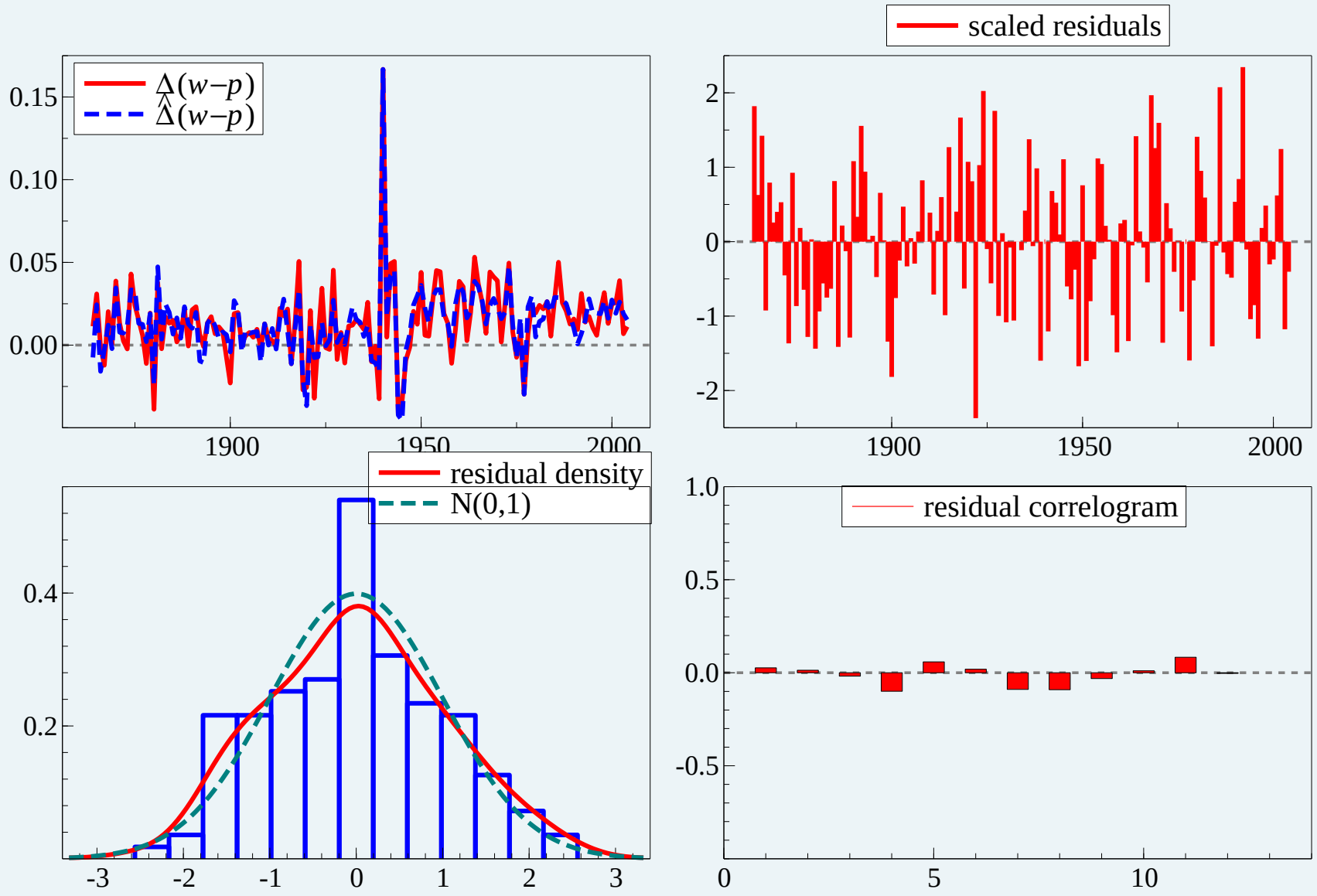


Figure: Graphical statistics for (80)

Super exogeneity requires that shifts in the DGPs of contemporaneous regressors do not alter the parameters of an estimated model. *Autometrics* can select dynamic models for the conditioning variables $(g-l)_t$, Δp_t, and $U_{r,t}$ (from 2 lags) using SIS at $\alpha = 0.005$, and test super exogeneity by all retained indicators being insignificant in (80).

Variable	q	null distribution	SIS test
$(g-l)_t$	2	$F(2,123)$	0.77
Δp_t	7	$F(7,118)$	1.87
$U_{r,t}$	14	$F(14,111)$	1.37

Table: SIS based super-exogeneity test of (80).

Table 6 records for each variable (first column) how many step indicators were retained (q, second column), the null distribution of the test statistic (third column), and the test statistic outcome (final column). No test rejects, so step shifts that significantly perturbed those variables did not affect the model in (80), consistent with those regressors being super exogenous for its parameters.

We now 'forecast' the 7 observations 2005–2011.

As d_t is contemporaneous, it would be unknown in a genuine forecast, so 'forecasting' here really checks parameter constancy.

y_{T+h} is to be forecast successively 1-step ahead over a period $h = 1, \ldots, H$ from a forecast origin at T with one known regressor $\{x_{T+h}\}$ using estimated parameters $\{\widehat{\beta}_i,\ i = 1, \ldots, 2\}$ as:

$$\widehat{y}_{T+h|T+h-1} = \widehat{\beta}_1 x_{T+h} + \widehat{\beta}_2 y_{T+h-1} \tag{81}$$

At $T + h - 1$, the right-hand side is known, so $\widehat{y}_{T+h|T+h-1}$ can be calculated, leading to 'forecast' errors given by the deviations $\widehat{u}_{T+h|T+h-1} = y_{T+h} - \widehat{y}_{T+h|T+h-1}$ from which statistics such as root mean square forecast errors (RMSFEs) can be calculated, where the RMSFE for y_{T+h} forecast by $\widehat{y}_{T+h|T+h-1}$ over $h = 1, \ldots, H$ is:

$$\text{RMSFE} = \frac{1}{H}\sum_{h=1}^{H}(y_{T+h} - \widehat{y}_{T+h|T+h-1})^2 = \frac{1}{H}\sum_{h=1}^{H}\widehat{u}^2_{T+h|T+h-1}.$$

Figure 192 reports two sets of 1-step ahead ‘forecasts’ for $y_{T+h} = \Delta(w-p)_{T+h}$ from (80) over 2005–2011 using (81).

Top row (panel *a*) shows all ‘forecasts’ lie within forecast intervals of $\pm 2\widehat{\sigma}_f$ calculated taking account of parameter-estimation uncertainty: $\text{RMSFE} = 0.0108$, close to the in-sample $\widehat{\sigma}$ of 0.0103, despite including the Financial Crisis and Great Recession and although the model was selected from $N > T$ candidate variables.

Figure 192 bottom row (panel *b*) shows ‘forecasts’ made including an intercept correction (IC). An IC is a step indicator equal to unity at and after the forecast origin, but zero before.

Clements and Hendry (1998) show that ICs can offset systematic forecast errors made after a location shift.

The IC improves the ‘forecasts’ as $\text{RMSFE} = 0.0098 < \widehat{\sigma}$, even though the IC is estimated from just one observation, increasing $\widehat{\sigma}_f$.

Both panels confirm that there has not been a substantive shift in (80) over the Great Recession.

‘Forecasts’ of real wages

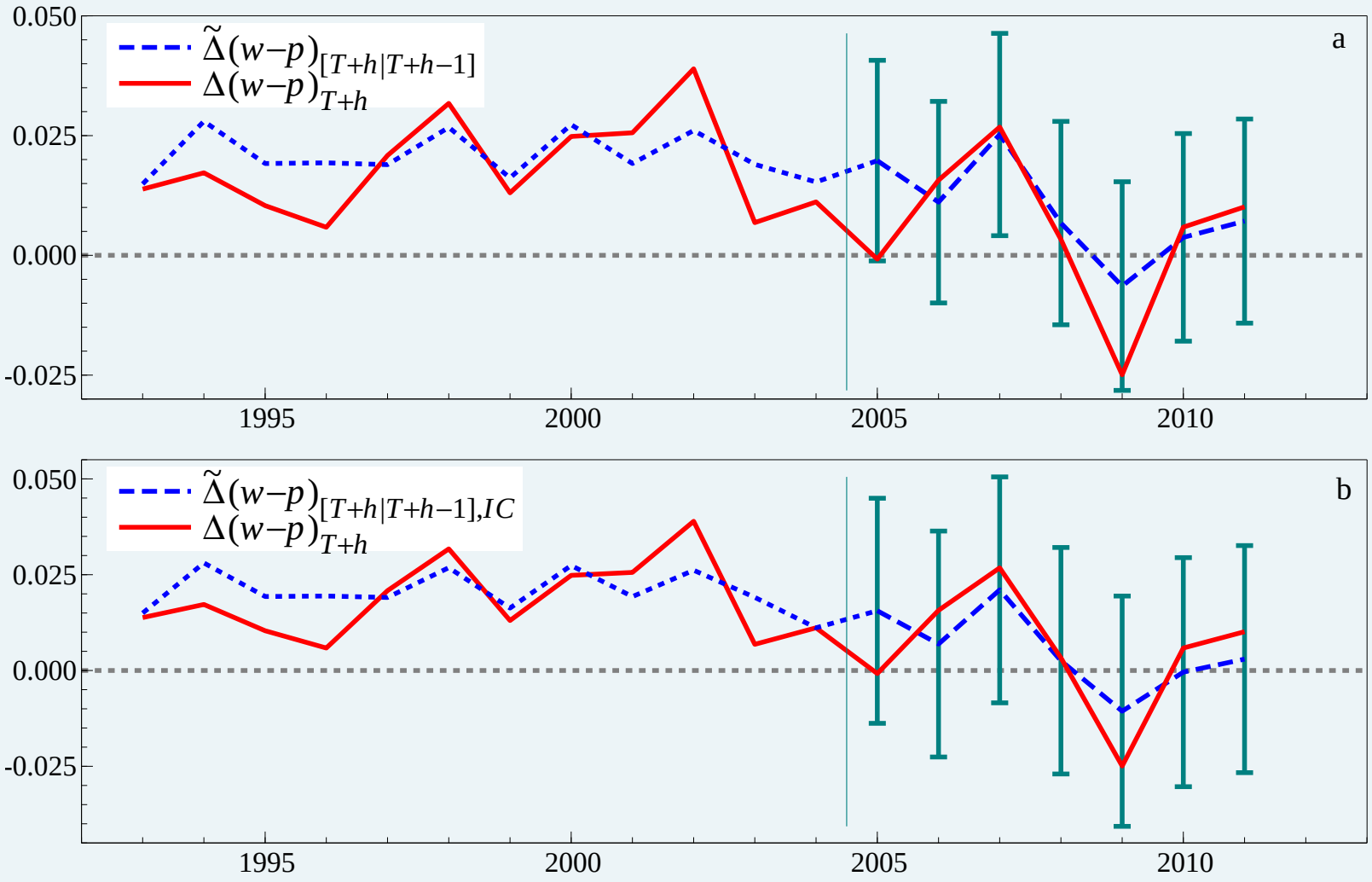


Figure: ‘Forecasting’ real wages without and with intercept corrections

Summary of lecture 6 key points

(A) Real wage growth occurredfrom increased output per worker.
(B) The selected models of real-wage growth, $\Delta(w-p)_t$, can be mapped back to (w_t, p_t) to explain nominal or real levels.
(C) The empirical analyses started from GUMs with all the theory-suggested variables, their dynamics, and non-linearities.
(D) Location shifts were reasonably captured by SIS.
(E) $\Delta(w-p)_t$ had a non-linear response to Δp_t, showing increasing reactions to real-wage erosion from inflation.
(F) Non-linear response to unemployment consistent with *involuntary* unemployment as more workers are unemployed at higher real wages.
(G) Selecting location shifts by SIS did not preclude finding non-linearities, and vice versa, so both had roles to play.
(H) Long-run equilibrium was a constant wage share, which corrected past real-wage 'losses' from previous incomplete adjustments.
(I) Despite selecting from more variables and indicators than observations, equation was interpretable, constant over the Great Recession and passed a stringent super-exogeneity test.

Route map

(1) Macroeconomic data: evolution with abrupt change
(2) Taming trends and breaks
(3) Dependence between variables and over time
(4) Economic theory and statistical analysis: 2 key ingredients
(5) Modelling UK unemployment
(6) Modelling UK wages
(7) **Modelling UK money demand**
(8) Modelling UK prices
(9) Conclusion

Lecture 7 guide post

1. There have been huge changes to financial systems since 1860.
2. To create a measure of money relevant throughout, here broad money, UK M4, various time series had to be spliced together.
3. An analysis of motives for holding money guides the GUM.
4. An earlier empirical model matches a theory with nominal short-run adjustment within bands and real long-run reactions shifting those bands in line with inflation and real income growth.
5. The long-run equilibrium relates the inverse velocity of circulation of money (namely the log ratio of real money to real GDP) to the net interest rate cost from holding money.
6. Re-estimating the money-demand model over the longer sample period to 2011 produces recognizably similar coefficients to the initial equation, and the cointegrating relation found up to 1975 is almost unchanged.
7. However, step-indicator saturation is needed to capture several important location shifts since 1970.

Workers' money in 1860

As we have seen in earlier chapters, a household in 1860 would have had little money, receiving it as a weekly wage, and spending most, including repaying short-term loans, before the next payday (surprisingly like 'payday loans' in the UK over 2008–2014).

Money was a flow to most people, held for brief periods:
only the wealthy had bank accounts, holding a stock of money as an asset, or as a precaution against negative events.

When economists discuss the demand for money, all three reasons—transactions, asset and precautionary—occur, noting that the outstanding stock has to be held by someone at every point in time.

So what happens when the demand for that stock does not match what is available?

Money supply and inflation

Historically, inflation was the process where prices of goods and services rose in terms of commodity money (gold and silver).

Really a fall in the price of ‘money’ from an increase in its volume: see the elegant discussion of 16th Century European inflation from the influx of South American gold in Hume (1752).

Deflation occurred when the supply of goods rose, as in the industrial revolution, without a concomitant increase in the supply of ‘money’.

In both cases, supply-side forces drove the outcomes, either from an ‘exogenous’ increase in precious metals (Californian gold rush, 1848, Australian, 1851, etc.), or conversely from prolific harvests or technological improvements.

Many financial innovations, from bills of exchange, paper money and fractional-reserve banking, through personal cheque books in the 1830s, telegraph in the 1860s, credit cards, to ATMs and on-line banking, and creation of new assets that count as ‘money’, have complicated that story—already noticed by Marshall (1926).

Changing financial institutions

At the start of our period, Building Societies (like Savings and Loans in the USA), were small cooperative institutions.

They grew greatly over the next century to rival commercial banks in size.

During the 1990s, many Building Societies converted to commercial banks, borrowing on wholesale money markets as bank liquidity ratios were allowed to decrease, setting the scene for the financial crisis—where many previous Societies were liquidated or taken over.

Changes in the spectrum of competing interest rates from financial innovation have plagued previous econometric models of money both broad (M4) and narrow, notes and coin plus demand deposits at commercial banks (M1), often called transactions money:
see Hacche (1974) and Hendry and Mizon (1978); Coghlan (1978) and Hendry (1979); as well as Goldfeld (1976) and Baba, Hendry and Starr (1992) for the USA.

Under the Gold Standard that operated till the 1930s, Central Banks mainly 'controlled the money supply'.

After a plethora of regimes, they now target inflation and unemployment.

In the UK, money creation is akin to that of virtual particles in physics, where an asset and a matching liability are simultaneously produced (or annihilated), discounting assets at the Central Bank if liquidity becomes scarce.

In effect, the quantity of money outstanding, M_t, is primarily determined by the demand to hold it.

Although unfunded budget deficits could have an impact, even the £375 billion purchase of Government debt by the Bank of England (Quantitative Easing) was initially accompanied by a fall in M4.

The demand for 'money' is now the main focus, but there is a tradition that inflation is mainly a monetary phenomenon (e.g., Friedman and Schwartz, 1982, criticized by Hendry and Ericsson, 1991).

Figure 201 panel *a* records the time series of m_t (discussed in the Appendix) and p_t.

Panel *b* plots $(m - p)_t$ (i.e., constant price money) and g_t, revealing major departures between them.

Real money increased markedly during Mrs Thatcher's 'monetary control' regime (which also coincided with deregulation of the banking system), then slowed during the late 1980s–early 1990s recession, before rising sharply again.

Panel *c* plots $(m - p - g)_t$, which rose almost 3-fold from its low in 1980, ending near its highest value.

Finally, panel *d* records the short-term interest rate (on 3-month Treasury bills) $R_{S,t}$, and the competing net interest rate $R_{n,t}$.

These were essential the same until 1985, then depart markedly, with the latter falling to near zero by the late 1990s.

As 'money' has become broader, a larger proportion pays interest: see e.g., Ericsson, Hendry and Prestwich (1998).

Money, inverse velocity and interest rates

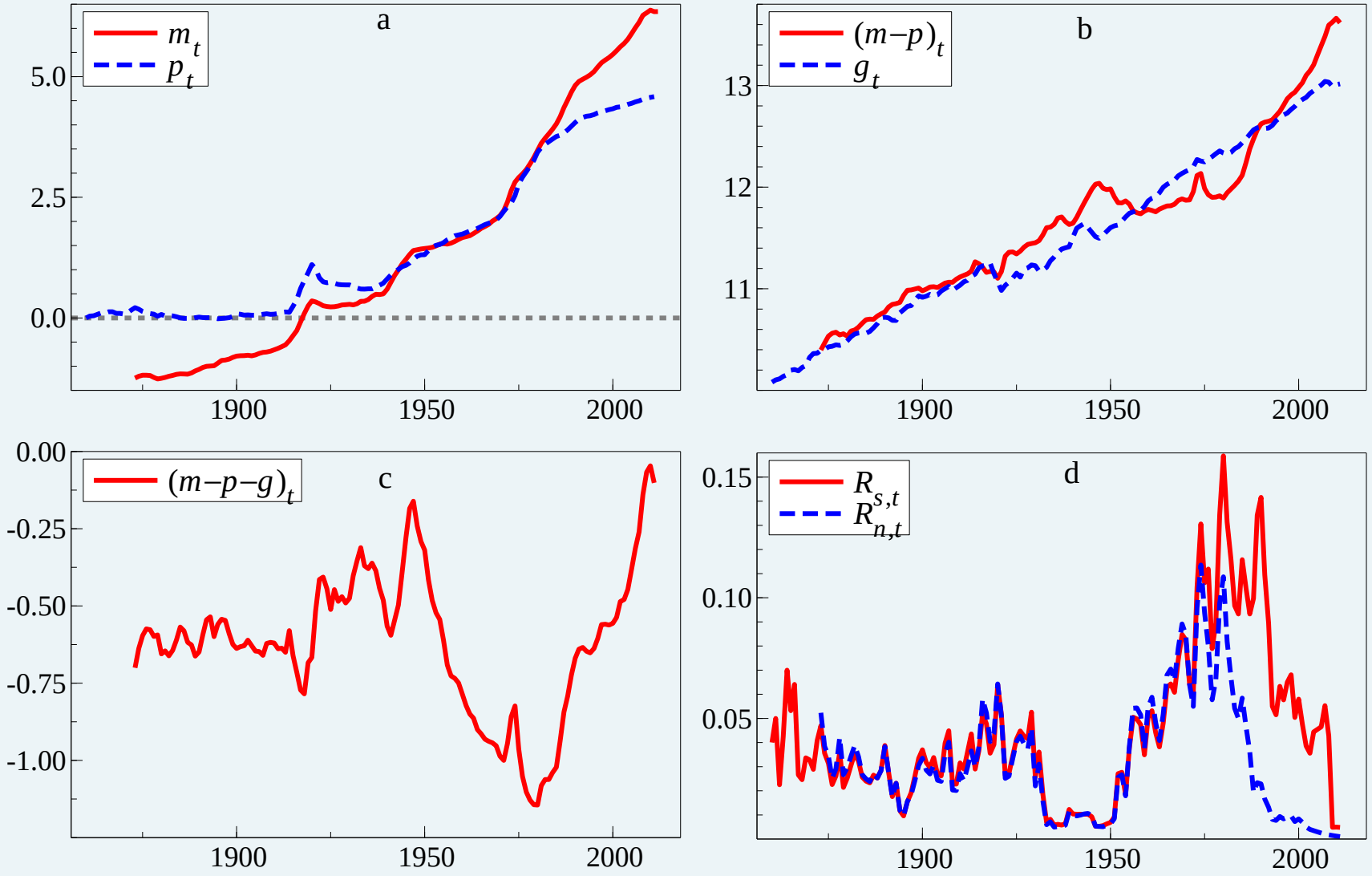


Figure: Time series of m_t and p_t (a), $(m-p)_t$ and g_t (b), $(m-g-p)_t$ (c), $R_{n,t}$ and $R_{S,t}$ (d).

Figure 203 panel *a* plots Δp_t with its mean shifts;
panel *b* shows the time series of Δp_t and Δm_t;
panel *c* records the corresponding location shifts in Δm_t; and
panel *d* a scatter plot of Δp_t against Δm_t with dates & a regression.
Panel *b* reveals many systematic departures between Δp_t and Δm_t with an absence of co-breaking, highlighted in panel *c* by the different location shifts in Δm_t found by SIS.

The upward slope of the regression in panel *d* cannot be interpreted as 'money causes inflation': as prices rise, more money is needed to buy the same quantities of goods and services.
Not only do m_t and p_t fail to cointegrate (even allowing for shifts), using only Δm_t to 'explain' Δp_t (with 4 lags of each, first selecting by SIS at 0.1%, then over regressors at 1%) yields the long-run relation $\Delta p = 0.39\Delta m$ which is well short of proportionality with $\widehat{\sigma} = 0.021$.

A simple autoregression in Δp_t selected with SIS has $\widetilde{\sigma} = 0.022$.

If money is a determinant of price inflation, it is clearly not the only one.

Location shifts in price inflation and money growth

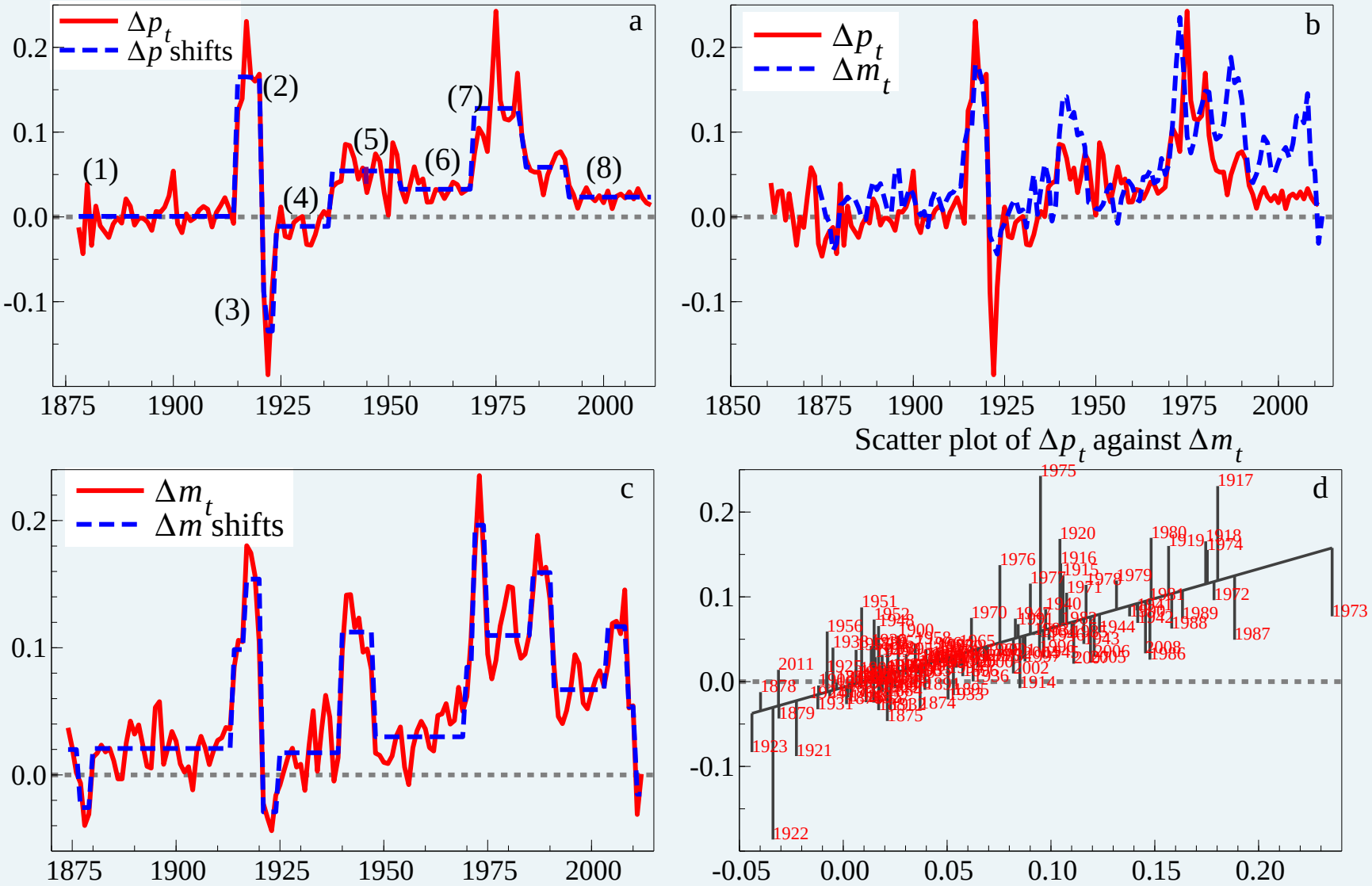


Figure: Location shifts in price inflation and money growth

Why hold paper money at all?

Paper money has no intrinsic value (unlike commodity money), is eroded by inflation, and depends on the willingness of others to accept it in exchange for goods and services. Starr (2012) suggests it is held to pay taxes, as Governments both issue money and agree to accept it back for tax purposes. Given that use, other transactions follow.

So how much money would economic agents then wish to hold?

There are a plethora of theories of money demand, but many are only relevant to transactions demand, summarized in Ericsson *et al.* (1998). Possible holding motives include: to finance transactions, to facilitate speculation in financial markets, as a store of value, and as a precaution against short-term fluctuations in incomes.

All depend on levels of expenditure or income (and maybe wealth), and possibly on the opportunity cost of holding idle money.

That cost is the interest foregone by not holding other safe interest bearing assets (like 3-month Treasury Bills), and depreciation from inflation.

These ideas lead to a general formulation of aggregate money demand (denoted m^d in logs) depending on GDP, g, prices, p, and the opportunity cost, denoted R_n, often measured by the interest rate on an outside asset, R_S less the interest rate paid on the proportion of M that earns interest R_o (like deposit accounts) and inflation Δp:

$$m^d = f(g, p, R_n, \Delta p) \tag{82}$$

or in a log-linear form:

$$m^d = \theta_1 g + \theta_2 p + \theta_3 R_n + \theta_4 \Delta p \tag{83}$$

where $\theta_1 > 0$, $\theta_2 > 0$, $\theta_3 < 0$, $\theta_4 < 0$, often with $\theta_1 = \theta_2 = 1$ so real money demand is proportional to real GDP.
(83) is an equilibrium relation, so at best corresponds to a cointegrating relationship:
a model of actual m_t would need to allow for dynamic adjustments, possible non-linearities, and shifts.

Hendry and Ericsson (1991) found that $(m - g - p)_t$ cointegrated with $R_{S,t}$ (which coincided with $R_{n,t}$ over most of 1878–1975), as:

$$\widetilde{m}^e_t = (m - p - g)_t + 0.309 + 7R_{S,t} \tag{84}$$

where $\{\widetilde{m}^e_t\}$ did not have a unit root.

Their selected model included a non-linear equilibrium correction in $\widetilde{m}^e$, denoted by $\widetilde{e}_{t-1} = (\widetilde{m}^e_{t-1} - 0.2)(\widetilde{m}^e_{t-1})^2$:

$$\begin{aligned}
\Delta(m-p)_t \;=\; & \underset{(0.06)}{0.47}\,\Delta(m-p)_{t-1} - \underset{(0.04)}{0.11}\,\Delta^2(m-p)_{t-2} - \underset{(0.04)}{0.59}\,\Delta p_t \\
& + \underset{(0.05)}{0.41}\,\Delta p_{t-1} - \underset{(0.006)}{0.017}\,\Delta r_{S,t} - \underset{(0.019)}{0.078}\,\Delta_2 r_{L,t} \\
& - \underset{(0.19)}{1.15}\,\widetilde{e}_{t-1} + \underset{(0.006)}{0.034}\,(D_1 + D_3) + \underset{(0.002)}{0.007} \\
& + \underset{(0.010)}{0.071}\,D_4 + \underset{(0.020)}{0.090}\,D_4\Delta r_{S,t} \qquad (85) \\
R^2 \;=\; & 0.88 \quad \widehat{\sigma} = 1.48\% \quad T = 1878 - 1975
\end{aligned}$$

In (85), $D_1 + D_3$ are dummy variables equal to unity during the two world wars respectively and zero elsewhere.

D_4 is a dummy which is unity only over the period 1971–1975 for changes to financial regulations called Competition and Credit Control, which also required changing R_S to R_n and adding both D_4 and its interaction with changes in interest rates, $D_4 \Delta r_{S,t}$.

The small net negative coefficient of inflation, obtained by combining the coefficients of Δp_t and Δp_{t-1}, shows some erosion of the value of real money holdings.

No mis-specification tests rejected, so (85) was congruent, and constant over the sample to 1970.

Friedman and Schwartz (1982) estimate demand for money equations with real money as the dependent variable, then solve them to express prices as dependent on nominal money treated as 'exogenous'.

However, 'inverting' (85) to have Δp_t as the dependent variable with Δm_t treated as exogenous was not constant.

Interpreting (85)

The formulation of $\widetilde{e}_{t-1}$ in (85) preserves the sign of the disequilibrium feedback, as the cubic component dominates.

There are two possible equilibria, one when $\widetilde{m}^e = 0$, the usual cointegration solution, and the other at $\widetilde{m}^e = 0.2$.

The second mainly accounts for the discrepancy between $m - p$ and g over 1921–1955 visible in Figure 201 panel *b*, although $\widetilde{e}_{t-1}$ operates over the whole sample period.

Real money growth is strongly autoregressive, and inflation and changes in (log) interest rates have net negative effects as expected.

(85) can be expressed in terms of nominal short-run money demand:

$$\Delta m_t \approx 0.41\Delta p_t + 0.11\Delta^2 p_{t-2} + 0.47\Delta m_{t-1} - 0.11\Delta^2 m_{t-2} \cdots$$

with long-run demand in real terms determined by (84).

That matches a theory of money demand where agents adjust holdings within bands in the short-run, but adjust those bands in the longer run as the price level and income change:

see Miller and Orr (1966) and Milbourne (1983).

Although there have been important data revisions since (85), it was possible to obtain a reasonably close replication:

$$\begin{aligned}
\Delta(m-p)_t &= \underset{(0.07)}{0.40}\ \Delta(m-p)_{t-1} - \underset{(0.05)}{0.14}\ \Delta^2(m-p)_{t-2} - \underset{(0.05)}{0.62}\ \Delta p_t \\
&+ \underset{(0.06)}{0.38}\ \Delta p_{t-1} - \underset{(0.006)}{0.025}\ \Delta r_{n,t} - \underset{(0.031)}{0.104}\ \Delta_2 r_{L,t-1} \\
&- \underset{(0.54)}{2.80}\ \widetilde{e}_{t-1} + \underset{(0.006)}{0.037}\ (D_1 + D_3) + \underset{(0.002)}{0.009} \\
&+ \underset{(0.010)}{0.065}\ D_4 + \underset{(0.028)}{0.087}\ D_4 \Delta r_{n,t} \qquad (86) \\
R^2 &= 0.87\ \ \widehat{\sigma} = 1.62\%\ \ T = 1878 - 1975
\end{aligned}$$

Most coefficients in (86) in common with (85) are similar, but $r_{n,t}$ replaces $r_{S,t}$, one lag has changed, and the coefficient of $\widetilde{e}_{t-1}$ is larger.

There have been a number of updates of equations like (85) by other authors, including Escribano (2004) who examines a range of alternative non-linear equilibrium correction formulations, and concludes with a constant-parameter equation to 2000 similar to (85). The long-run solution in Hendry (2001) which was estimated as a cointegrating relation over 1870–1991 holds essentially unchanged to 2011 when estimated as a static regression, despite continuing major changes to the UK's financial system:

$$\widetilde{m}^{e}_{t} = (m - p - g)_{t} + 0.38 + 7.4 R_{n,t} \tag{87}$$

where $R_{n,t}$ is the opportunity cost of holding money. Consequently, the previous non-linear equilibrium correction term $\widetilde{e}_{t-1}$ was retained.

The step shifts seen in Figure 203c and the marked departure between Δm_t and Δp_t after 1980 suggest applying SIS selecting at $\alpha = 0.001$ so only the most important shifts are retained, delivering:

$$\begin{aligned}
\Delta(m-p)_t = & \underset{(0.06)}{0.57}\ \Delta(m-p)_{t-1} - \underset{(0.04)}{0.12}\ \Delta^2(m-p)_{t-2} - \underset{(0.050)}{0.67}\ \Delta p_t \\
& + \underset{(0.06)}{0.51}\ \Delta p_{t-1} - \underset{(0.006)}{0.021}\ \Delta r_{n,t} - \underset{(0.024)}{0.060}\ \Delta r_{L,t-1} \\
& - \underset{(0.26)}{1.56}\ \widetilde{e}_{t-1} + \underset{(0.007)}{0.030}\ (D_1 + D_3) - \underset{(0.012)}{0.020} + \underset{(0.008)}{0.051}\ D_4 \\
& + \underset{(0.02)}{0.05}\ D_4 \Delta r_{n,t} + \underset{(0.06)}{0.17}\ \Delta g_t + \underset{(0.010)}{0.03}\ S_{1973} - \underset{(0.015)}{0.06}\ S_{1979} \\
& + \underset{(0.015)}{0.05}\ S_{1981} - \underset{(0.008)}{0.04}\ S_{1999} + \underset{(0.013)}{0.05}\ S_{2008} \qquad (88)
\end{aligned}$$

$$R^2 = 0.86 \quad \widehat{\sigma} = 1.81\% \quad T = 1878 - 2011$$

Changes to updated demand for money model

Five step shifts are retained, but coefficient estimates are little altered.

All diagnostic tests are insignificant, though the fit remains less good than over the shorter period to 1975.

Figure 213 shows the graphical outcome, where panel *a* also reports the combination of the step indicators.

The main periods not captured by the economic variables are 1980–1999, where a negative location shift was needed, followed by a large positive step over 2000–2008.

Location shifts have manifestly not ceased.

Updated demand for money model

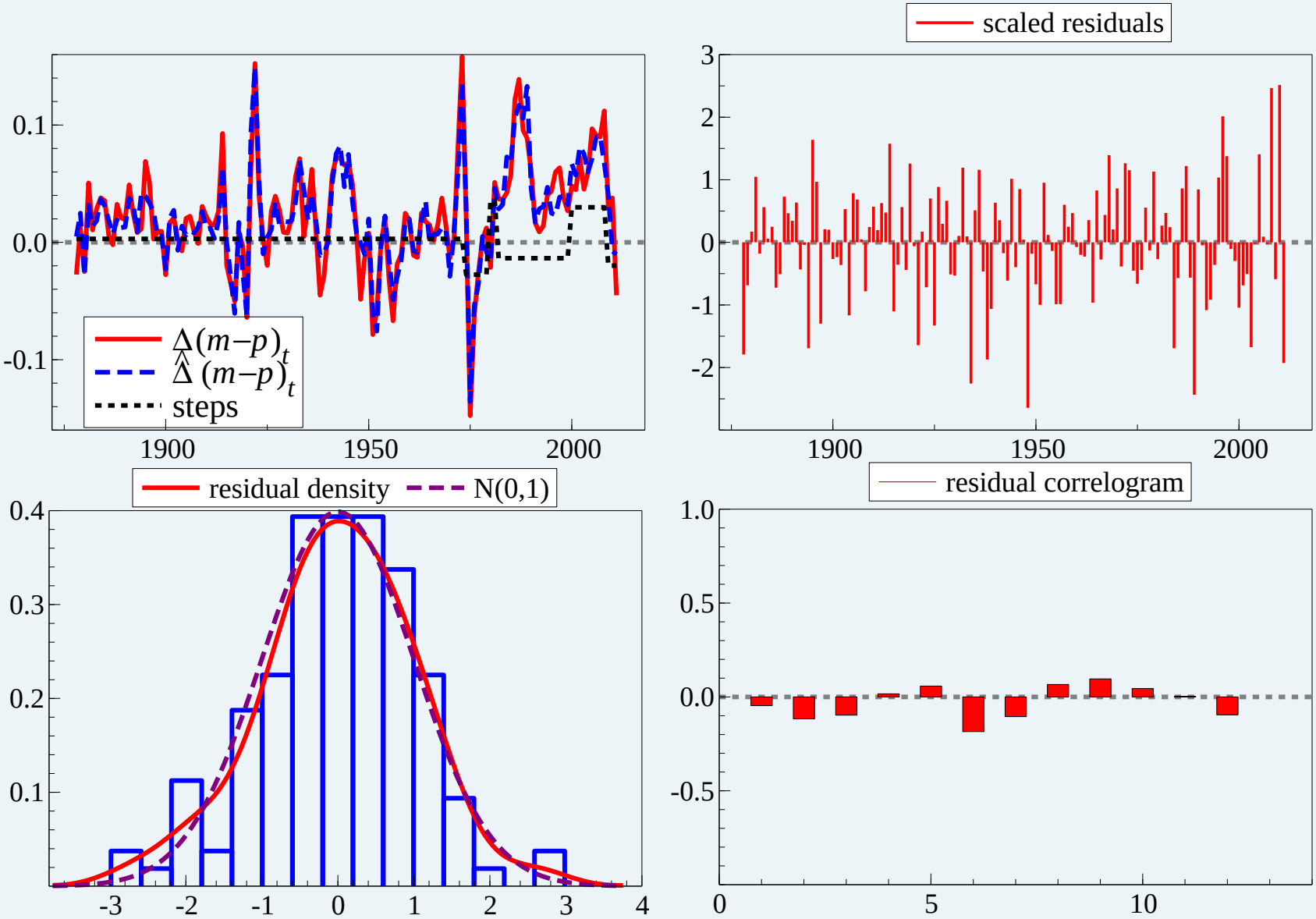


Figure: Graphical statistics for the money model extension using SIS.

(A) There have been huge changes in the financial system over the last 150 years.
(B) This necessitated creating a spliced time series to measure money, where the broad component M4 was used here.
(C) Updating the earlier model in Hendry and Ericsson (1991) for data revisions delivered similar results to theirs.
(D) The empirical models reasonably matched a theory of money demand with nominal short-run adjustment within bands and real long-run reactions shifting those bands in line with inflation and real income growth.
(E) The cointegrating relation was almost unchanged when extending the sample period from 1976 to 2011.
(F) Re-estimating the dynamic money demand model over the longer sample revealed recognizably similar coefficients to the initial equation.
(G) However, step-indicator saturation was needed to capture several important location shifts since 1970.

Route map

(1) Macroeconomic data: evolution with abrupt change
(2) Taming trends and breaks
(3) Dependence between variables and over time
(4) Economic theory and statistical analysis: 2 key ingredients
(5) Modelling UK unemployment
(6) Modelling UK wages
(7) Modelling UK money demand
(8) **Modelling UK prices**
(9) Conclusion

Lecture 8 guide post

1. There are many potential theory-relevant domestic and foreign determinants of inflation.
2. A previous empirical analysis starting from a GUM that included all theory-suggested variables and their dynamics found most were relevant yet the model still had many large shifts.
3. To update and extend that equation to 2011 requires modeling all the excess demand influences on inflation.
4. Location shifts are captured by impulse and step indicators.
5. The price inflation model has two equilibrium corrections, one from the excess demand for goods and services, and the other from the price markup over home and imported costs
6. Despite the role of unemployment in some theories of price inflation, little evidence of its importance is found.
7. Expectations of future inflation are not found to matter greatly.
8. Overall, viable empirical models of price inflation can be developed despite its turbulent history.

Price inflation

We have noted many major differences between life in 1860 and that over the ensuing 150 years, and the final one concerns inflation.

During a working life from 1860 to 1900, as Figure 16 showed, the price level would have risen by less than 10%, and most of that rise occurred in 1900 itself (probably from demand pressures during the Boer War).

During the equivalent 1960–2000, the price level rose more than 1300%.

To complete the explanation of the nominal level in the UK economy, we clearly need to model prices.

Price inflation determinants

Price inflation is the ‘steam valve’ for all excess demands: money, cost push, demand pull, devaluation, profligate governments. **No single-variable explanation is sufficient**.

(a) excess demands for goods and services;
(b) excess demand for factors of production;
(c) excess money holdings;
(d) direct shocks on exchange rates and prices of imports;
(e) excess government demands (unfunded deficits);
(f) special factors: commodity prices, wars, price controls.

Measure these respectively by:
(a) deviation of output from capacity;
(b) wage inflation, unemployment, excess labour demand;
(c) excess money demand;
(d) purchasing power parity deviations, imported inflation;
(e) national debt, long- and short-run interest rates;
(f) commodity prices, indicators, lagged rates of change.

Price inflation equation from Hendry (2001)

$$\Delta p_t = \underset{(0.03)}{0.18}\, g^d_{t-1} - \underset{(0.02)}{0.19}\, \pi^*_{t-1} - \underset{(0.09)}{0.83}(R_S - R_L + .0065)_{t-1}$$

$$+ \underset{(0.11)}{0.62}\, \Delta R_{S,t-1} + \underset{(0.03)}{0.19}\, \Delta m_{t-1} + \underset{(0.03)}{0.27}\, \Delta p_{w,t}$$

$$+ \underset{(0.03)}{0.27}\, \Delta p_{t-1} + \underset{(0.01)}{0.04}\, \Delta p_{o,t-1} + \underset{(0.002)}{0.04}\, I_{d,t} \qquad (89)$$

$$R^2 = 0.975 \quad \widehat{\sigma} = 1.14\% \quad T = 1875 - 1991$$

g^d= excess demand (deviation of g from the production function)
π^*= markup over ulc^* (nominal unit labour costs adjusted for hours):

$$\pi^*_t = (p_t - 0.675 ulc^*_t - 0.25 p_{\pounds,t} - 0.075 p_{o,t}) + 0.11 I_{2,t} + 0.25. \qquad (90)$$

$p_{\pounds,t} = p_{w,t} - e_t$ denotes world prices in sterling;
Δp_w, Δp_o: world and oil-price inflation.
$I_{2,t}$: indicator equal to unity after WWII; $I_{d,t}$: indicator for outliers.

$I_{d,t}$ collects impulse indicators for huge, large, and medium outliers:

$$I_b = \begin{cases} 1 & \text{for } 1915,\ 1917,\ 1919 \\ -1 & \text{for } 1921,\ 1922 \end{cases}$$

$$I_l = \begin{cases} 1 & \text{for } 1916,\ 1918,\ 1920,\ 1975 \\ -1 & \text{for } 1943,\ 1945,\ 1973 \end{cases}$$

$$I_m = \begin{cases} 1 & \text{for } 1880,\ 1900,\ 1939,\ 1940,\ 1970,\ 1971,\ 1980 \\ -1 & \text{for } 1881,\ 1942,\ 1944. \end{cases}$$

$I_{b,t}$, $I_{l,t}$, $I_{m,t}$ are indicators for outliers of roughly 12%, 6% and 4% in years shown. Combined into one index I_d with weights of 3, 1.5, and 1: $I_{d,t} = 3I_{b,t} + 1.5I_{l,t} + I_{m,t}$. Yet still also need $I_{2,t}$.

Reveals that 22 largest annual changes in UK inflation are unexplained by any variables suggested from economic theory.

The close fit, homoskedastic, serially uncorrelated, near normal residuals, due in large part to $I_{d,t}$: dropping it from (89) increases the residual standard deviation markedly to $\widehat{\sigma} = 2.9\%$

Most theories of inflation are part of the explanation in (89). Significant proximate determinants were:
excess demand for goods and services, g^d; **markup**, π^*; **world price inflation**, Δp_w; **PPP deviations**, $p_{\pounds}$; **nominal-money growth**, Δm; **long-short spread**, $R_S - R_L$; **commodity prices**, Δp_o, and **interest rate changes**, ΔR_S, all mattered: excess money, debt, and labour demands did **not** matter.

Indicators crucial despite all these theoretically-suggested variables. Several major episodes not explained: high inflation in WWI and collapse in 1920–22; rapid inflation in 1940, low inflation for rest of WWII (rationing and price controls); high inflation during 1970s oil crises, and 'stagflation'. All need indicators in addition to usual drivers. **No 'single-cause' explanation sufficies.**
Fiat money not sole cause of inflation in modern economy, even if in 15th–18th centuries under commodity money.

Figure 222 records the Graphical statistics of (89)

Graphical statistics of earlier price inflation model

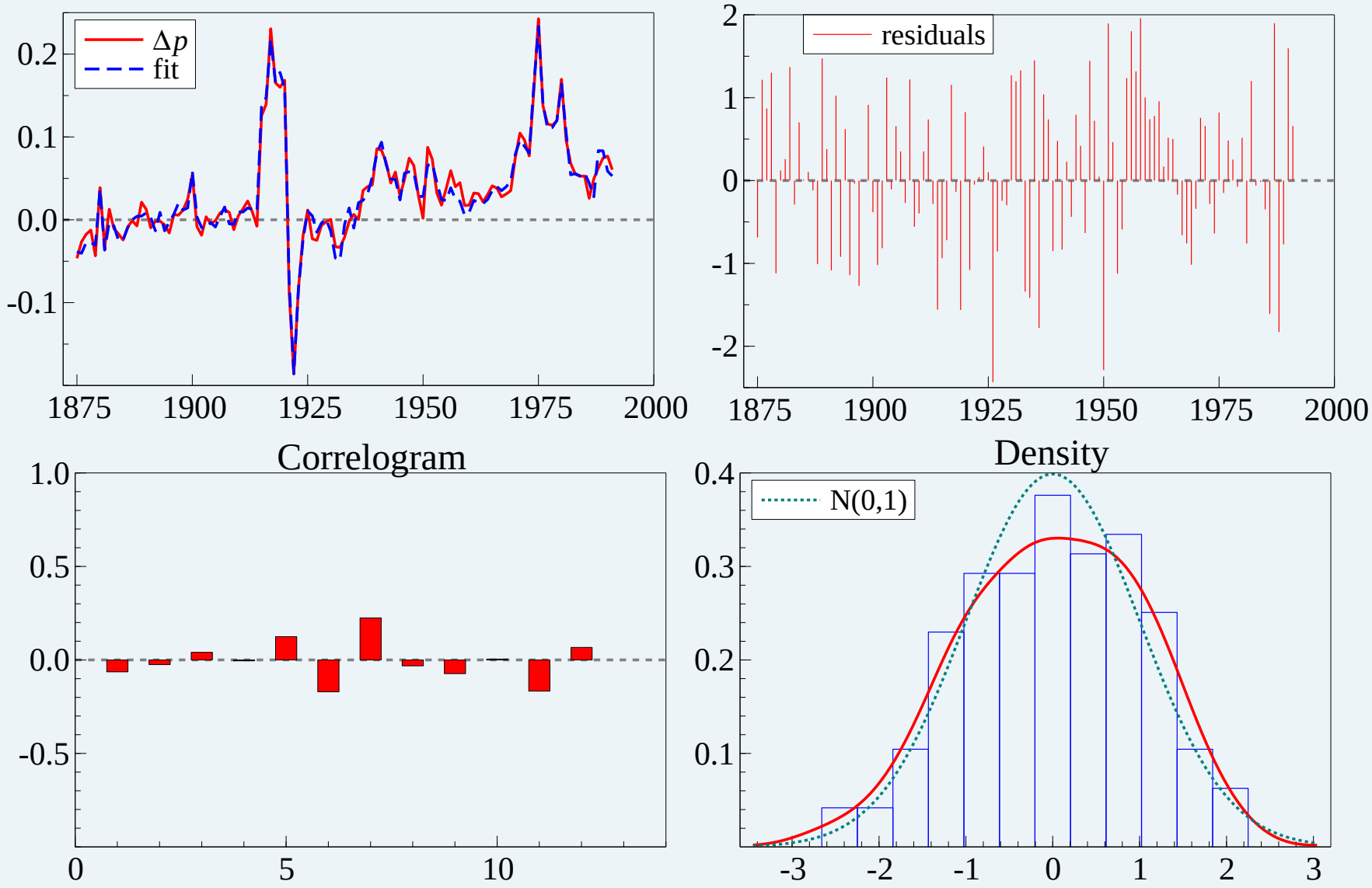


Figure: Price inflation model fit and residuals.

Have 20 new observations, and many advances in econometric technology since (89) was modelled, so develop an extended version.

First need 'auxiliary' models for the price inflation determinants (a)–(f).

Then, seek to replicate the price inflation model (89) despite data revisions and new equations for (a) g^d, (b) $\widetilde{m}^e$, (c) π^*, (d) Δp_w, $p_{\pounds}$, (e) n^d, $R_S - R_L$, and (f) Δp_o,
using the indicator saturation methods explained earlier.

Finally develop an updated and extended equation to investigate both changes from and similarities with the previous study.

Begin with (a): 'production function' for $(g - l)_t$. GUM generalized (4) with four lags of $(g - l)_t$ and $(k - wpop)_t$ (log-ratio of capital to working population used to avoid sudden shifts from unemployment). Those regressors, constant and trend were fixed, and SIS implemented for 1862–1944 at 0.1%, then lagged regressors were selected at 1%, retaining the 'static' production function.
Same procedure was then used for 1945–2004.

Resulting equations were similar to Hendry (2001, section 3.1).

$$\begin{aligned}(g-l)_t &= \underset{(0.068)}{0.705}\,(g-l)_{t-1} + \underset{(0.055)}{0.083}\,(k-wpop)_t + \underset{(0.27)}{0.651} \\ &\quad + \underset{(0.0009)}{0.002}\,t - \underset{(0.015)}{0.116}\,S_{1918} - \underset{(0.015)}{0.092}\,S_{1920} \\ R^2 &= 0.992 \quad \widehat{\sigma} = 1.89\% \quad T = 1862-1944 \end{aligned} \tag{91}$$

For 1945–2004, static production function with location shifts was:

$$\begin{aligned}(g-l)_t &= \underset{(0.31)}{1.69} + \underset{(0.049)}{0.39}\,(k-wpop)_t + \underset{(0.0013)}{0.015}\,t + \underset{(0.01)}{0.035}\,S_{1946} \\ &\quad + \underset{(0.008)}{0.044}\,S_{1973} + \underset{(0.008)}{0.028}\,S_{1978} + \underset{(0.007)}{0.042}\,S_{1989} + \underset{(0.007)}{0.016}\,S_{1995} \\ R^2 &= 0.999 \quad \widehat{\sigma} = 1.23\% \quad T = 1945-2004 \end{aligned} \tag{92}$$

(91) has the long-run solution (including, but not reporting, step indicators), where $t_{ur} = -4.34^*$:

$$e_{g-l,1,t} = (g-l)_t - 2.21 - 0.0068t - 0.28\,(k - wpop)_t \qquad (93)$$

Next (92) leads to (again including, but not reporting, step indicators):

$$e_{g-l,2,t} = (g-l)_t - 1.69 - 0.015t - 0.39\,(k - wpop)_t \qquad (94)$$

This was then extended to 2011. Capacity (cap_t) was calculated by combining the right-hand sides of (93) and (94), so the measure of excess demand for goods and services was
$g_t^d = e_{g-l,1,t}1_{\{1860 \leqslant t \leqslant 1944\}} + e_{g-l,2,t}1_{\{1945 \leqslant t \leqslant 2011\}}$.
Figure 226 panel *a*, shows the plot of cap_t with $(g-l)_t$; and panel *b* the resulting excess demand (or supply), g_t^d.
The depressions at the start of period, the inter-war epoch and the 'Great Recession' from 2008 on are very visible, as are the peaks of the Boer War, WWI and WWII, and the so-called 'stop-go' economic policies of the post-war till the mid 1970s.

Capacity and excess demand for goods and service

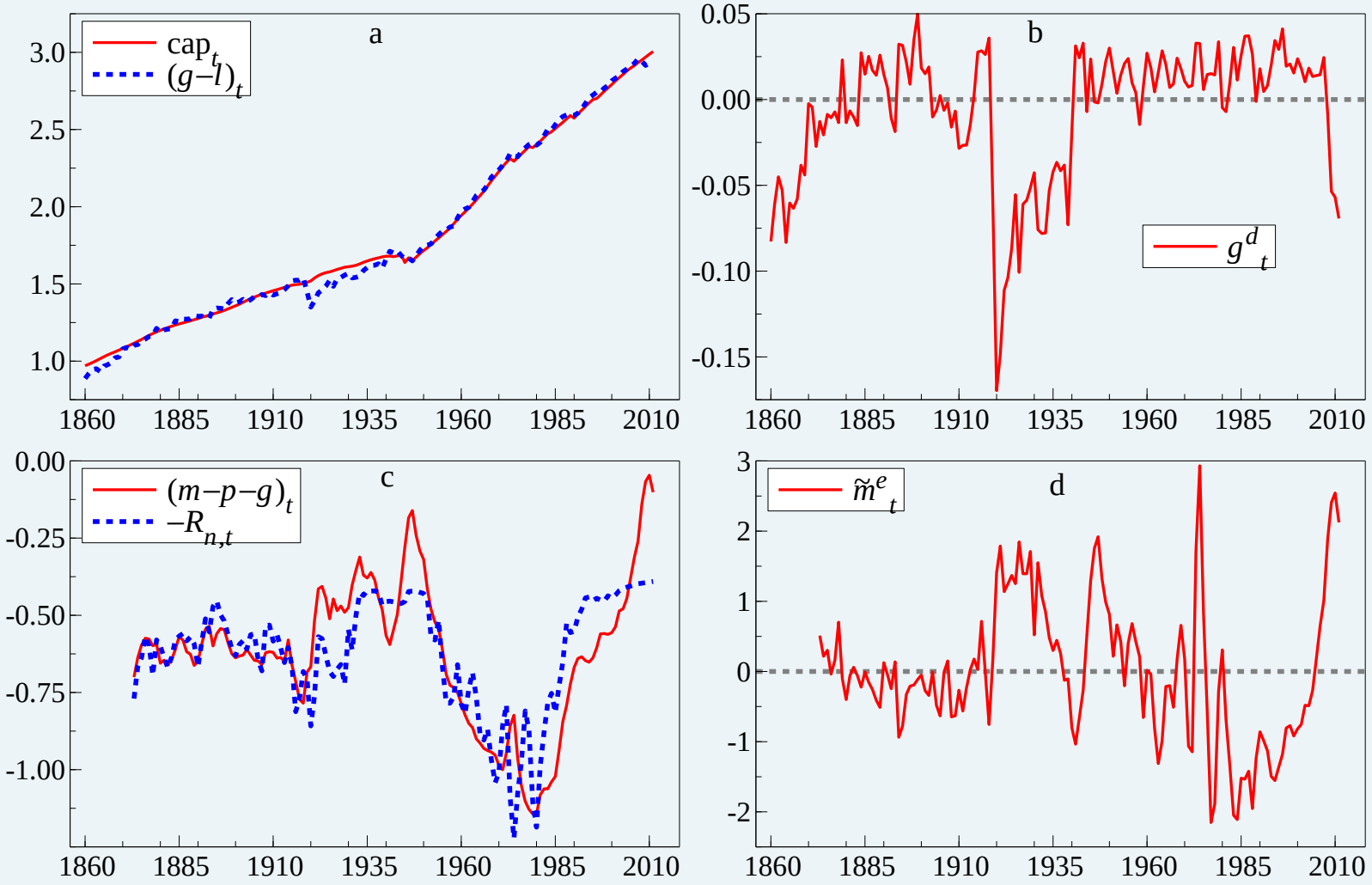


Figure: Capacity, excess demand for goods and services, velocity and interest rates, and excess demand for money.

(b) excess demand for money

The excess demand for money was computed as $\widetilde{m}^e$ from (87).

Figure 226 panel *c* shows the graph of inverse velocity $(m - p - g)_t$ and $-7.4R_{n,t}$.

Panel *d* records $\widetilde{m}^e_t$.

The increase in $\widetilde{m}^e_t$ starting from the trough in the mid 1980s has been very large.

As discussed in both Ericsson *et al.* (1998) and Escribano (2004), the earlier dynamic model of UK money demand in Hendry and Ericsson (1991) remained constant for their extended samples, updated to 2011 in the previous lecture, although the latest period required additional step indicators.

Unit labour costs $\mathrm{ulc}_t = (w + l - g)_t$ were introduced in Lecture 6, and formed a key component of the markup π^*_t adjusted for hours defined in (90), which was highly significant in (89).

Here we use essentially the same formulation:

$$\pi_t = p_t - 0.675\,(\mathrm{ulc}_t + 0.005t) - 0.25p_{\pounds,t} - 0.075p_{o,t} + 0.05l_{2,t} - 1.55. \tag{95}$$

where $p_{\pounds,t} = p_{w,t} - e_t$ as before, the implicit trend adjustment for changes in hours is 0.5% pa, and the remaining coefficients are as in (90): the constant reflects revisions and changes in base years, resulting in $l_{2,t}$ having a smaller effect.

Figure 229 panel *a*, plots π_t. The sharp drop in the markup since 2000 is consistent with competition from China, and the lower rate of inflation the UK has experienced this Century compared to last, closer to 19th Century experiences.

Markup, UK prices and world prices in Sterling, and inflation rates

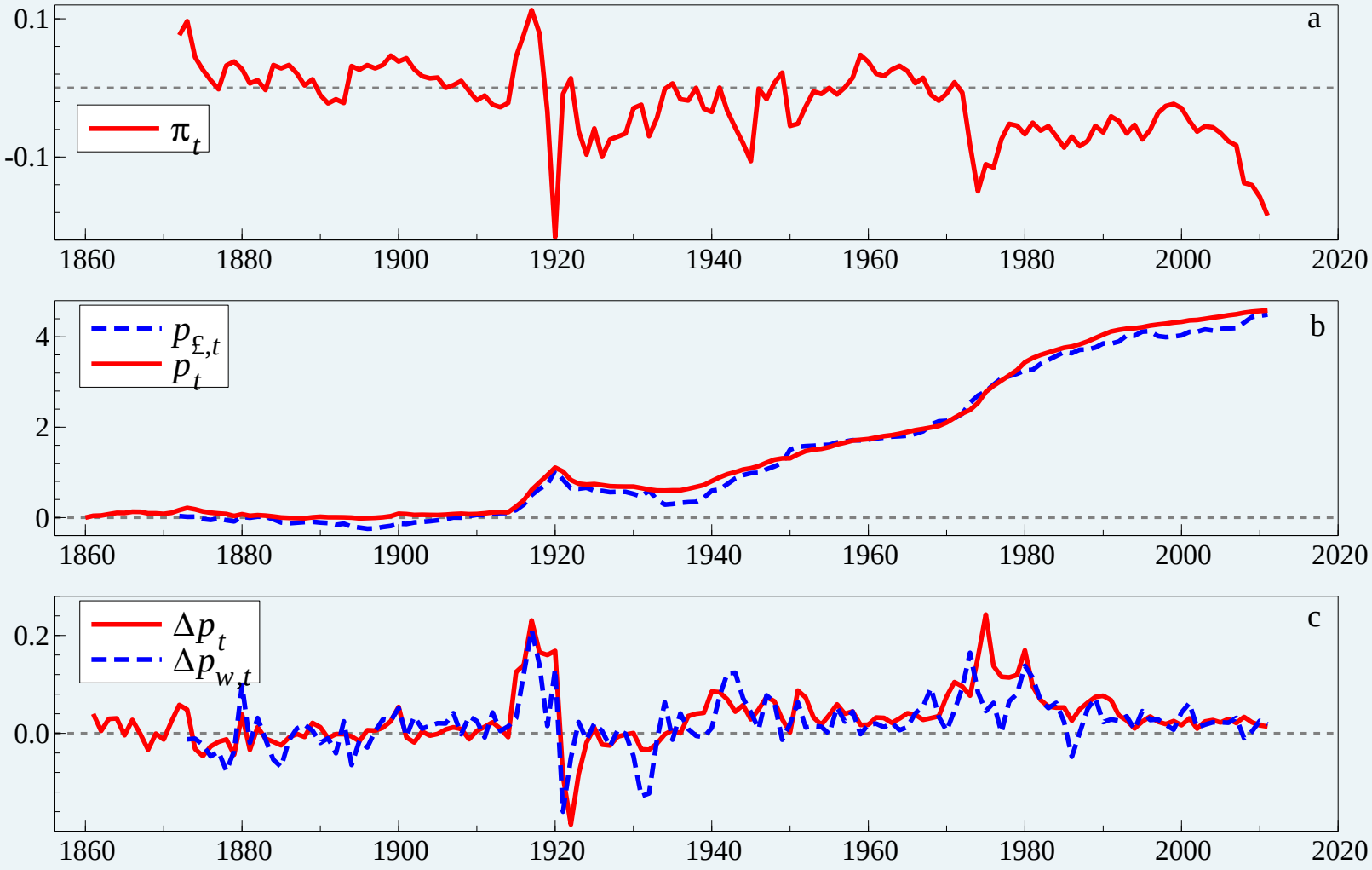


Figure: Markup, UK prices and world prices in $\pounds$, and their inflation rates, Δp_t and $\Delta p_{w,t}$

(d) world price inflation, PPP and interest rates

Figure 58 presented graphs for the £ Sterling exchange rate index, e_t, the logs of the UK price level, p_t, and world prices, $p_{w,t}$ (middle panel), and purchasing power parity given by:

$ppp_t = e_t + p_t - p_{w,t}$.

Here, Figure 229 records the comparative levels of UK and world prices in Sterling (panel *b*), and their inflation rates, Δp_t and $\Delta p_{w,t}$ (panel *c*).

Expressed in a common currency, p_t and $p_{£,t}$ behave quite similarly, although in their own currencies, UK and world price inflation exhibit many substantial departures.

$\Delta p_{w,t}$ helps explain Δp_t below.

Figure 59 showed the UK's long-term and short-term interest rates, R_L and R_S, and their spread, $R_L - R_S$.

Figure 232 records four transformations of N of potential relevance: $(N/PG)_t$ (panel *a*), Δn_t (*b*), $\Delta(n-p)_t$ (*c*) and $\Delta(n-p-g)_t$ (*d*).

(N/PG) in (*a*) is shown from 1820, and is now at levels seen in the 19th Century when the Napoleonic Wars' Debt was still being repaid. (N/PG) remains well below its peak after WWII as nominal GDP grew rapidly in the 1950s, and is still below its historical average.
At much higher levels, the UK financed the Industrial Revolution (see Allen, 2009), and built a large empire.

Δn reflects War Finance: Boer War, WWI, and WWII are manifest, as are 3 similar-sized increases during the 1974–1978 Oil Crisis, 1990–1994 recession under John Major, & Great Recession from 2007. Given the large fall in GDP, the last increase is surprisingly small.

The final two rises in $\Delta(n-p)$ are comparable to WWII.

The latest rise in $\Delta(n-p-g)$ is the largest of the 3, as GDP fell considerably. **However, no role was found for National Debt in explaining UK inflation.**

UK National Debt

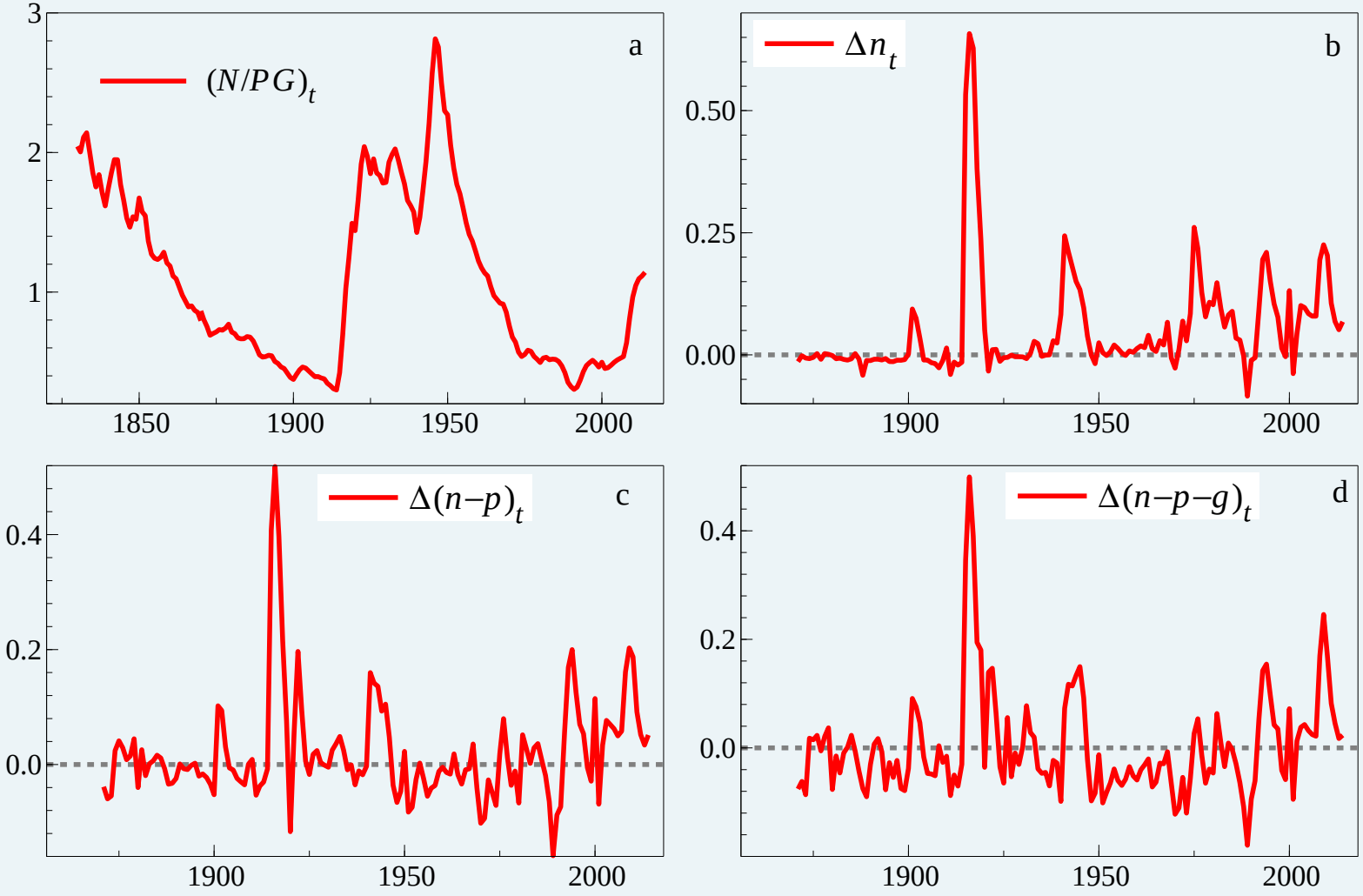


Figure: UK National Debt: (*a*) $(N/PG)_t$, (*b*) Δn_t, (*c*) $\Delta(n-p)_t$ and (*d*) $\Delta(n-p-g)_t$.

Figure 234 panels (*a*) and (*b*) show the log level and changes in an index of nominal raw material prices (including fuel), denoted $\mathrm{p}_{\mathrm{o,t}}$.

Nominal price changes have been large, with WWI, the post-war crash, the Great Depression, WWII, the two Oil Crises, the rise of China during the first decade of the 21st Century, and the Great Recession all too visible, albeit these are not the only large shifts.

Figure 234 panels (*c*) and (*d*) show the levels of nominal, denoted $\mathrm{p}_{\pounds\mathrm{o,t}}$, and real, $\mathrm{p}_{\pounds\mathrm{o,t}} - \mathrm{p}_{\mathrm{t}}$, commodity prices measured in $\pounds$.

The level of $\mathrm{p}_{\pounds\mathrm{o,t}}$ has risen much more than $ prices because of the relative fall in the value of the $\pounds$, whereas the latter has not really trended since the end of the 19th Century, with a marked recent recovery, probably due to China's large demands to sustain its rapid growth.

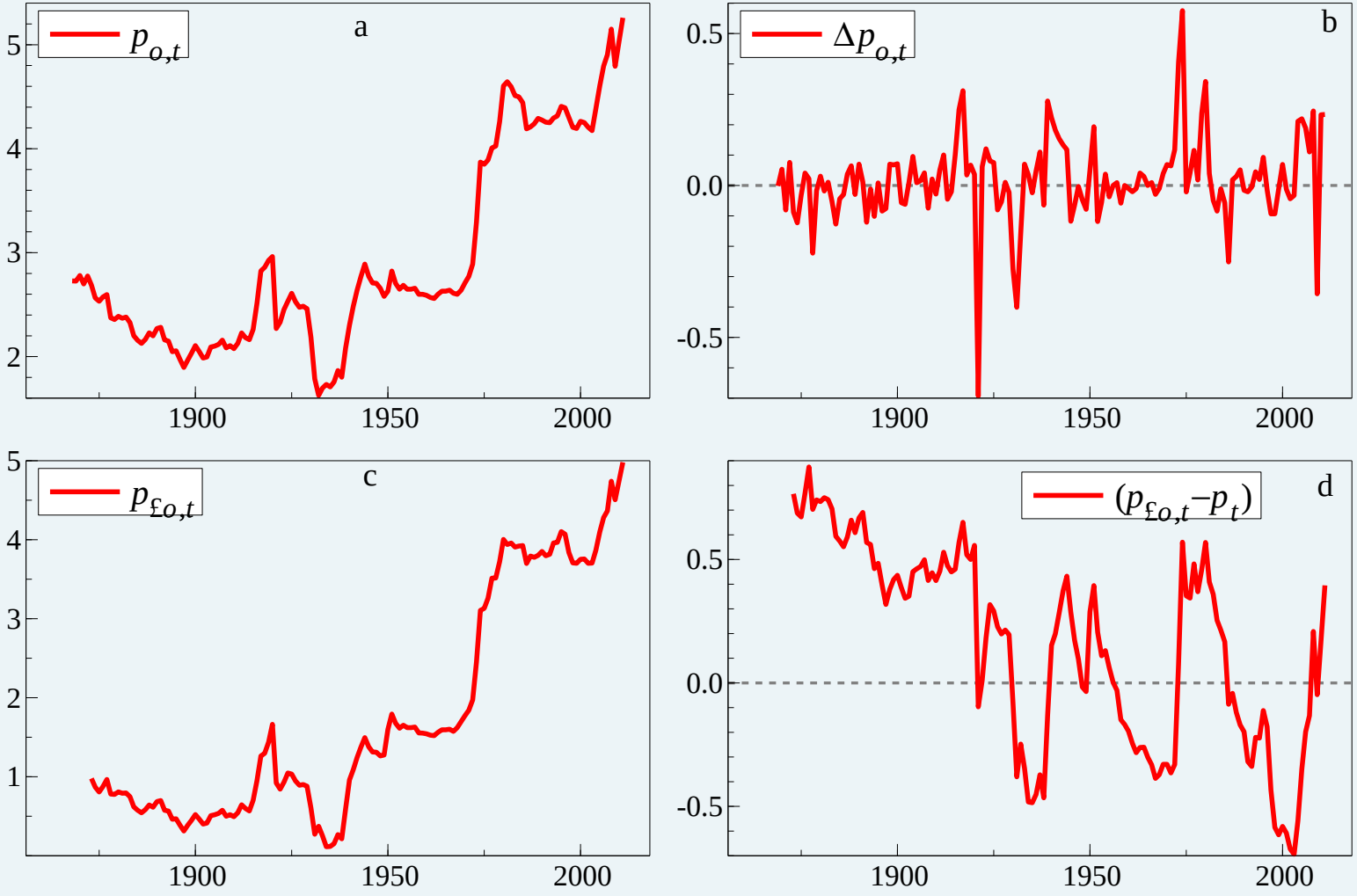


Figure: Commodity prices

A price index for commodities raises an interesting econometric issue, namely the meaning of a constant relationship. By construction:

$$p_{o,t} = \sum_{i=1}^{n} w_{i,t} p_{i,t}$$

where the $\{p_{i,t}\}$ are the prices of individual commodities, including many different metals, fibres, foods, and fuels.
The $\{w_{i,t}\}$ are index weights, often value shares, sometimes trade.
In 1860, commodities like coal and whale oil were important fuels, whereas others, like petroleum, were not.
By 2011, weights have changed substantively.
The impact on overall inflation of a change in a commodity price alters with its importance in production, so it is unlikely that any individual $p_{i,t}$ would have a constant relation to p_t over 150 years.
That does not preclude a constant relationship between $p_{o,t}$ and p_t, but all the change is 'hidden' in the shifting weights $\{w_{i,t}\}$.
More general settings are discussed in Hendry (1996).

Over the same sample 1875–1991, (89) could be closely replicated on the revised and extended data, using π_{t-1} rather than π^*_{t-1}.

$$\Delta p_t = \underset{(0.04)}{0.19}\, g^d_{t-1} - \underset{(0.04)}{0.24}\, \pi_{t-1} - \underset{(0.09)}{0.74}\, (R_S - R_L + .0065)_{t-1} + \underset{(0.03)}{0.19}\, \Delta m_{t-1}$$

$$+ \underset{(0.12)}{0.59}\, \Delta R_{S,t-1} + \underset{(0.03)}{0.28}\, \Delta p_{w,t} + \underset{(0.03)}{0.27}\, \Delta p_{t-1} + \underset{(0.01)}{0.03}\, \Delta p_{o,t-1} + \underset{(0.002)}{0.04}\, I_{d,t}$$

$$(R^*)^2 = 0.962 \quad \widehat{\sigma} = 1.30\% \quad F_{ar}(2,106) = 0.39 \quad F_{arch}(1,115) = 0.51$$

$$\chi^2(2) = 0.14 \quad F_{reset}(2,106) = 2.0 \quad F_{het}(18,98) = 2.3^{**} \tag{96}$$

The non-linearity test in Castle and Hendry (2010) yielded $F_{nl}(27,81) = 1.72^*$.

Despite similarities in estimated coefficients, the heteroskedasticity and non-linearity tests reveal that the data revisions and different measures of excess demand and markup have somewhat altered the relationship.

Retaining all of the variables in (96) SIS was used at 0.5%, selecting 7 step indicators: 1936 (−1.7%), 1949 (3.8%), 1950 (−7.0%), 1951 (3.8%), 1973 (−3.3%), 1975 (3.0%), 1993 (2.0%).
Only the last is after the earlier sample: these are denoted $\{S_{i,t}\}$.
Data over 2000–2011 provide a parameter constancy 'forecast test'.

$$\begin{aligned}
\Delta p_t &= \underset{(0.03)}{0.12}\, g^d_{t-1} - \underset{(0.04)}{0.13}\,\pi_{t-1} - \underset{(0.10)}{0.54}\,(R_S - R_L + .0065)_{t-1} \\
&+ \underset{(0.03)}{0.14}\,\Delta m_{t-1} + \underset{(0.11)}{0.47}\,\Delta R_{S,t-1} + \underset{(0.03)}{0.25}\,\Delta p_{w,t} \\
&+ \underset{(0.03)}{0.29}\,\Delta p_{t-1} + \underset{(0.01)}{0.03}\,\Delta p_{o,t-1} + \underset{(0.002)}{0.04}\, I_{d,t} + \{S_{i,t}\} \qquad (97) \\
(R^*)^2 &= 0.968 \;\; \widehat{\sigma} = 1.1\% \;\; F_{ar}(2,107) = 0.09 \;\; T = 1875 - 1999 \\
\chi^2(2) &= 3.6 \;\; F_{arch}(1,123) = 0.85 \;\; F_{reset}(2,107) = 4.0^* \\
F_{het}(23,99) &= 1.42 \;\; F_{nl}(27,82) = 1.38 \;\; F_{chow}(12,109) = 0.97
\end{aligned}$$

The improved fit over (96) is due to SIS, but the heteroskedasticity and non-linear tests no longer reject (a constant is insignificant).

Interpreting (97)

(97) is consistent with more benign inflation than over 1875–1991, where 22 large impulses had needed indicators.

In late 1992, the UK left the European Exchange Rate Mechanism (ERM) on 'Black Wednesday', and started one of its longest and fastest periods of economic growth till 2008, when the Financial Crisis and Great Recession hit.

The final step indicator of 2.0% up to 1993 is consistent with a lower inflation rate thereafter. The Bank of England's independence in 1998 came with an inflation target (now around 2.0%), but there was also downward pressure on UK prices from China's competitive exports. Nevertheless, (89) has not shifted substantively over the last 20 years. Figure 239 records the graphical statistics with $\{S_{i,t}\}$ shown as a dashed line, accounting for little of the explanation.

Although some 'forecasts' are over-estimates of inflation, the Chow test value is less than unity, so the fit is better to the last period than on average over the sample with indicators. All but the last forecast lie well within the 95% intervals shown as error bands in (*a*).

Price inflation model (97) graphical statistics

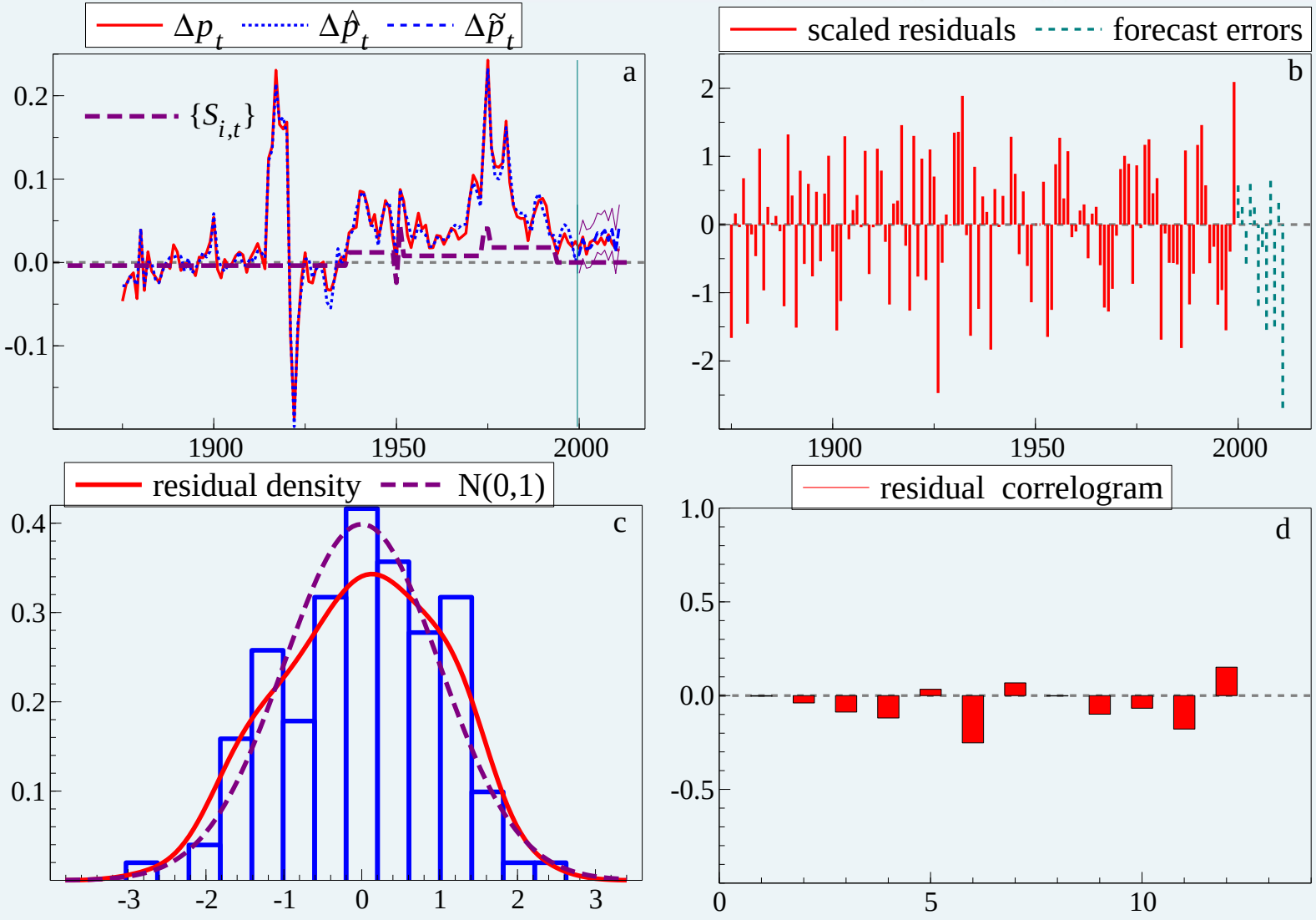


Figure: Price inflation model (97) graphical statistics with $\{S_{i,t}\}$ shown in panel (*a*).

The 7 step indicators significant at 0.5% mainly account for a slightly higher average inflation rate over the post-war period till 1993.
In contrast, the 22 impulse indicators in $I_{d,t}$ represent very large jumps and drops, from 4% to 12%, at what would otherwise be outliers.
Both not only adjust for aspects of Δp_t not explained by any regressors, but also correct for shifts in regressors not matching those of the dependent variable, though primarily the former here.

Most theories of inflation are part of the explanation in (89) and (97).
The significant proximate determinants include g^d_{t-1}, representing (a), Δm_{t-1}, (b), π_{t-1}, (c), $\Delta p_{w,t}$, (d), $(R_S - R_L + 0.0065)_{t-1}$ and $\Delta R_{S,t-1}$, (e), and $\Delta p_{o,t-1}$, plus indicators, (f).

Despite all the theoretically-suggested variables, indicators are crucial.
The important episodes not explained in (89) remain that way.
The ERM departure adds one more indicator in the 20 years since.

Inflation is not very inertial, as Δp_{t-1} has a coefficient of only 0.29, similar to that in (89) with no step indicators.

The price inflation model in (97) has two equilibrium corrections, one from the excess demand for goods and services, and the other from the price markup over home and imported costs.

The first affects inflation so long as $g^d \neq 0$; or rounding and setting $u_r = \log(0.05) \approx -3$, until $g \simeq 0.6l + 0.4k + 0.015t + 0.5$, so actual output equals potential as determined by the production function in (94). This matches one of the variables that the Bank of England seems to use in judging the state of the economy.

The second EqCM can be written as:

$$\begin{aligned} -\pi_t &= 0.675\left((w - p + l - g)_t + 0.005t\right) + 0.25\left(p_{\pounds,t} - p_t\right) \\ &+ 0.075\left(p_{o,t} - p_t\right) - 0.05 I_{2,t} + 1.55 \end{aligned} \tag{98}$$

The results confirm that no 'single-cause' explanation of price inflation suffices, with seven groups of forces all mattering. However, the variables for ppp deviations, excess money, national debt, and labour demand (unemployment and wages, other than through the markup) were not found to matter.

The New-Keynesian Phillips curve (NKPC) includes expected future inflation as a 'feed-forward' variable to explain current inflation:

$$\Delta p_t = \underset{\geqslant 0}{\gamma_1} E_t\left[\Delta p_{t+1} \mid \mathcal{I}_t\right] + \underset{\geqslant 0}{\gamma_2} \Delta p_{t-1} + \underset{\geqslant 0}{\gamma_3} s_t + u_t \tag{99}$$

where $E_t\left[\Delta p_{t+1} | \mathcal{I}_t\right]$ is today's expected inflation one-period ahead given available information, $\mathcal{I}_t$, firms' real marginal costs are s_t, and anticipated signs of coefficients are shown: see Galí and Gertler (1999), Galí *et al.* (2001) and Castle *et al.* (2014).
As $E_t[\Delta p_{t+1} | \mathcal{I}_t]$ is unobservable, it is replaced by:

$$E_t\left[\Delta p_{t+1} \mid \mathcal{I}_t\right] = \Delta p_{t+1} + \nu_{t+1} \tag{100}$$

where by taking expectations on both sides of (100):

$$E_t\left[\Delta p_{t+1} \mid \mathcal{I}_t\right] = E_t\left[\Delta p_{t+1} \mid \mathcal{I}_t\right] + E_t[\nu_{t+1} \mid \mathcal{I}_t] \tag{101}$$

so $E_t[\nu_{t+1} \mid \mathcal{I}_t] = 0$, and hence ν_{t+1} must be unpredictable from available information.

Substituting (100) into (99):

$$\Delta p_t = \beta_1 \Delta p_{t+1} + \beta_2 \Delta p_{t-1} + \beta_3 s_t + \epsilon_t \tag{102}$$

where $\epsilon_t \sim D[0, \sigma_\epsilon^2]$. Since ν_{t+1} in (100) is not independent of Δp_{t+1}, neither is ϵ_t in (102), so estimation requires a set of 'exogenous' variables z_t as instruments. Lacking data on s_t, we use real unit labour costs $(w - p - g + l)_t$, treated as endogenous. The estimates of (102) using as additional instruments 1 and 2 lags of $(w - p - g + l)$, $\Delta(g - l)$, and R_L, from the real-wage model, were:

$$\begin{aligned} \Delta p_t &= \underset{(0.09)}{0.65}\ \widehat{\Delta p}_{t+1} + \underset{(0.060)}{0.023}\ \widehat{(w - p - g + l)}_t + \underset{(0.08)}{0.50}\ \Delta p_{t-1} \\ &\quad - \underset{(0.07)}{0.11}\ \Delta p_{t-2} - \underset{(0.11)}{0.05} \end{aligned} \tag{103}$$

$$\begin{aligned} \widehat{\sigma} &= 2.89\% \quad F_{ar}(2, 141) = 1367.2^{**} \quad F_{arch}(1, 146) = 1.78 \\ \chi^2_{nd}(2) &= 70.54^{**} \quad \chi^2_{Sar}(4) = 8.95 \quad F_{het}(8, 139) = 5.34^{**} \end{aligned}$$

Signs are as anticipated, though the coefficients of inflation add to more than unity, and $(w - p - g + l)_t$ is insignificant.

Both normality and heteroskedasticity tests strongly reject in (103), probably from unmodelled location shifts. A new statistic, $\chi^2_{\mathsf{Sar}}(k)$, tests the validity of the instruments (see Sargan, 1964).
Such estimates are similar to others reported.
Checking for location shifts, SIS at 0.5%, retaining all regressors in (103), found 18 significant step indicators, denoted $\{S_{i,t}\}$, leading to:

$$\begin{aligned}
\Delta p_t &= -\underset{(0.06)}{0.034}\,\widehat{\Delta p}_{t+1} + \underset{(0.09)}{0.37}\,(w - \widehat{p - g} + l)_t \\
&\quad + \underset{(0.05)}{0.28}\,\Delta p_{t-1} - \underset{(0.05)}{0.10}\,\Delta p_{t-2} - \underset{(0.17)}{0.68} + \{S_{i,t}\} \qquad (104) \\
\widehat{\sigma} &= 1.74\% \quad F_{\mathsf{ar}}(2,123) = 2.75 \quad F_{\mathsf{arch}}(1,146) = 2.72 \\
\chi^2_{\mathsf{nd}}(2) &= 6.81^* \quad \chi^2_{\mathsf{Sar}}(18) = 33.56^* \quad F_{\mathsf{het}}(24,121) = 0.92
\end{aligned}$$

Now $\widehat{\gamma}_1$ is negative and insignificant, $(w - p - g + l)_t$ is highly significant, and inflation inertia has disappeared.
Most of the mis-specification tests are also insignificant.

$\chi^2_{Sar}(18)$ rejects at 5% due to omitting $S_{\{1974\}}$ from the model but including it in the instruments: adding $S_{\{1974\}}$ back to the model produces $\chi^2_{Sar}(17) = 26.61$.

The non-normality reflects the stringent significance level of 0.5% used by SIS, and the omission from (104) of the important regressors in (97) where $\widehat{\sigma} = 1.1\%$.

As Castle *et al.* (2014) found in their analysis of NKPC models (based on IIS), the apparent significance of Δp_{t+1} in equations like (103) is due to the future value acting as a proxy for the unmodelled shifts.

It would have taken remarkable prescience for anyone in 1914 to have anticipated the dramatically higher inflation of 1915, or even in 1916 anticipated a doubling in 1917; and they would have had to anticipate price controls restraining inflation during the Second World War.

Summary of lecture 8 key points

(A) Viable empirical models of price inflation can be developed.
(B) Δp_t depended on most theory-relevant domestic and foreign variables, yet the model still had many large shifts.
(C) The empirical analyses started from GUMs with all the theory-suggested variables, their dynamics, shifts and trends.
(D) Location shifts were reasonably captured by impulse and step indicators.
(E) The price inflation model had two equilibrium corrections, one from the excess demand for goods and services, and the other from the price markup over home and imported costs.
(F) Despite its emphasis in some theories, there was little direct role for unemployment in the price inflation process.

Route map

(1) Macroeconomic data: evolution with abrupt change
(2) Taming trends and breaks
(3) Dependence between variables and over time
(4) Economic theory and statistical analysis: 2 key ingredients
(5) Modelling UK unemployment
(6) Modelling UK wages
(7) Modelling UK money demand
(8) Modelling UK prices
(9) **Conclusion**

Conclusions I

Huge changes over last 150 years: technological, medical and financial innovations, education, roles of women, laws, social reforms, wars, policy-regime shifts, industrial composition, etc.

These resulted in great improvements in productivity & real wages, with large rises in the price level, and varying unemployment.

Breaks, shifts and stochastic trends were pervasive, and must be modelled to understand how economies work, enhancing general economic analyses by accounting for key empirical phenomena that would otherwise distort estimated relationships.

We used differences, cointegration, indicator saturation, co-breaking, dynamics, non-linearities and related variables to tackle abrupt shifts, inter-dependence, evolution, and changing relationships.

Modelling methodology also yields insights into how to undertake and evaluate empirical analyses.

Conclusions II

Large numbers of potentially relevant variables requires computer software to choose empirical models.
Indeed, indicator saturation can only be undertaken by a computer program—but *Autometrics* does so successfully.

Models of unemployment, real wages, money and price inflation explain much of observed movements over almost 150 years, although most models needed indicators for the largest shifts, which therefore remain unexplained by any of the 'economic variables' included.

The unemployment rate was subject to major location shifts which help explain apparent changes in 'Phillips curves'.
However, neither unemployment nor expectations of future inflation played direct roles in either real-wage or price-inflation equations.

Money demand was difficult to model as financial innovations changed its measurements intermittently.
No role was found for 'excess money' in inflation, although money growth was significant.

Conclusions III

The teaching of empirical macro-econometrics often eschews the complications that are present in macroeconomic data because they are ‘too difficult’.

Having seen the prevalence of strong yet changing trends, abrupt shifts, high correlations between variables and with their own lags, this short introduction could not avoid tackling their joint interactions.

Consequently, we addressed the main concepts and some of the tools.

The power, flexibility and ease of use of computer software that can reduce a high-dimensional initial formulation to an interpretable, parsimonious and encompassing final selection makes it feasible to adopt such an approach to intermediate level teaching.

All the variables and many of the models presented above can be illustrated live in the classroom, hopefully empowering future generations of empirical modellers.

Conclusions IV

Notwithstanding the difficulties many experienced after the 2008 Financial Crisis and Great Recession,
and valid worries about inequality, climate change and new pandemics, the present is one of the best ever times in history to be alive:
I cannot imagine many readers wanting to live a median life in 1860.

Although life 150 years ago was not quite as bad as "solitary, poor, nasty, brutish, and short" (to quote from Thomas Hobbes's *Leviathan*, 1651), about 15% of infants died in their first year after birth.

The vast improvements in almost every aspect of living are beautifully captured visually in http://www.ourworldindata.org/:
change is the norm in them all, just as we have seen in macroeconomics.

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P_t	=	implicit deflator of GDP, $(1860 = 1)$
G_t	=	real GDP, \$million, 1985 prices
$U_{r,t}$	=	unemployment rate, fraction
W_t	=	average weekly wage earnings index, $(1860 = 1)$
L_t	=	employment $= \text{Wpop} - U_r$
Wpop_t	=	working population
K_t	=	total capital stock
$R_{L,t}$	=	bond rate
$R_{S,t}$	=	short-term interest rate
E_t	=	£ exchange rate index
$P_{w,t}$	=	trade-weighted world price index
TU_t	=	trade union membership
B_t	=	'replacement ratio' from unemployment benefits
S_t	=	hours lost through strikes, p.a., from 1890
Δx_t	=	$(x_t - x_{t-1})$ for any variable x_t & $\Delta^2 x_t = \Delta x_t - \Delta x_{t-1}$

Wage index: hourly wage rates prior to 1946, weekly wage rates afterwards. Latter standardized by dividing by normal hours, for trend rate of decline of 0.5% p.a. (based on a drop from 56 to 40 over 1913 to 1990, plus an increase in paid holidays), so unit labour costs were adjusted accordingly.

Instrumental variables uses another variable, $w_1 \ldots w_T$, say, so multiplies:

$$y_t = \beta_1 z_t + \epsilon_t \tag{105}$$

by w_t to get:

$$w_t y_t = \beta_1 w_t z_t + w_t \epsilon_t \tag{106}$$

and averaging over the sample data:

$$\frac{1}{T}\sum_{t=1}^{T} w_t y_t = \beta_1 \left(\frac{1}{T}\sum_{t=1}^{T} w_t z_t\right) + \frac{1}{T}\sum_{t=1}^{T} w_t \epsilon_t \tag{107}$$

Assuming $E[w_t \epsilon_t] = 0$, set the sample average to zero:

$$\widetilde{\beta}_1 = \frac{\sum_{t=1}^{T} w_t y_t}{\sum_{t=1}^{T} w_t z_t}. \tag{108}$$

Squared residuals **not now minimized**, but are defined by:

$$\widetilde{\epsilon}_t = y_t - \widetilde{\beta}_1 z_t \tag{109}$$